CEMRACS 2015:
Coupling Multi-Physics Models involving Fluids

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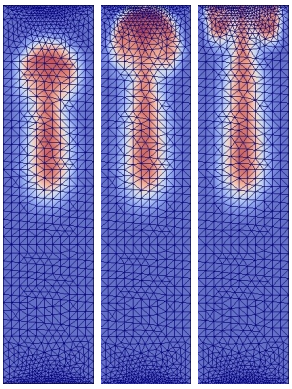
PROJECT PROPOSAL

Project label: DIPLOMA

Project full name: Study of a DepressurIsation Process at LOw MAch number in a nuclear core reactor

Context

In the aftermath of the CEMRACS'11 session, a research group named CDMATH[1] was founded to pursue studies upon the modelling of the coolant fluid at low Mach number in a nuclear core. This initial work was dedicated to the analysis of the LMNC[2] model coupled to the Stiffened Gas law. This equation of state was replaced in subsequent works by interpolation formulae to fit experimental data. Some theoretical results were provided (explicit solutions, locations of vaporisation, asymptotic behaviour) which were confirmed by numerical studies in 1D. The group then focused on numerical algorithms in dimensions 2 and 3. These works lead to several publications.



Goals

① to derive a robust algorithm in 3D, in particular in the case of complex data (geometry, power density) which could induce recirculations of the coolant;

② to extend the study in 1D to a time-varying thermodynamic pressure which could imply a noticeable extra computational cost.

[1] CDMATH stands for "Ce sont Des Mathématiques Appliquées à la ThermoHydraulique". Refer to http://cdmath.jimdo.com/.

[2] LMNC stands for "Low Mach Nuclear Core".

Setting of the mathematical problem

On the occasion of the 2011 CEMRACS session, S. Dellacherie and Y. Penel proposed a project entitled "Discrétisation d'un modèle bas Mach adapté à la modélisation simplifiée d'un cœur de réacteur nucléaire à eau" – whose acronym was Basmac. This was a preliminary study of the Lmnc model [3]:

$$\begin{cases} \nabla\cdot \boldsymbol{u} = \dfrac{\beta(h,p_0)}{p_0}\Phi, & \text{(1a)} \\ \rho(h,p_0)\times\big[\partial_t h + \boldsymbol{u}\cdot\nabla h\big] = \Phi, \qquad (t,\boldsymbol{x})\in\mathbb{R}^+\times\Omega_d, & \text{(1b)} \\ \rho(h,p_0)\times\big[\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\big] - \nabla\cdot\sigma(\boldsymbol{u}) + \nabla\bar{p} = \rho(h,p_0)\boldsymbol{g}. & \text{(1c)} \end{cases}$$

Φ and $\boldsymbol{g}$ are some data (respectively the power density and the gravity field). Variables $\boldsymbol{u}$, h and $\bar{p}$ denote the velocity field, the enthalpy and the dynamic pressure. Indeed, two pressure fields are involved in System (1): the thermodynamic pressure p_0 which is here a constant and corresponds to a reference value of the pressure at the exit of the core, and the dynamic pressure $\bar{p}$ which is a small perturbation. This duality is due to the low Mach number assumption under which System (1) is derived from the Navier-Stokes equations through an asymptotic expansion which amounts to filtering out the acoustic waves. The resulting system (1) has a different nature, coupling elliptic (1a), hyperbolic (1b) and parabolic (1c) equations.

The Cauchy stress tensor $\sigma(\boldsymbol{u})$ is expressed by $\sigma(\boldsymbol{u}) = \mu(h,p_0)\times\big(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\big) + \eta(h,p_0)\times(\nabla\cdot\boldsymbol{u})\mathcal{I}d$. We denote by $\rho(h,p_0)$ and $\beta(h,p_0)$ the density and the compressibility coefficient which are deduced from the equation of state (EOS) that relates thermodynamic variables. Related boundary conditions are such that $\boldsymbol{u}$ and h are prescribed at the entry ($z=0$) and a free outflow condition is considered at the exit ($z=L_z$).

In the 2011 project, the monophasic fluid was assumed to satisfy the Stiffened Gas law

$$\rho(h,p) = \frac{p}{\beta\times\big[h-q\big]} \text{ with } \beta,\ q \text{ some constants.} \qquad (2)$$

In the resulting proceeding [1], an explicit 1D unsteady solution was derived in the case $\Phi = cst$ and 1D academic numerical simulations were carried out relying on the method of characteristics.

Phase change was then taken into account in [2] where each pure phase was modelled through the Stiffened Gas EOS with tuned parameters (in this case, β and q in (2) are piecewise constant). The modelling of the mixture phase based on thermodynamic equilibria is detailled in the aforementioned paper and turns out to be similar to the stiffened gas EOS with specific parameters. Explicit 1D unsteady solutions can still be derived in the case of constant power densities. In the general case, 1D steady solutions are provided. The numerical scheme designed in [1] was adapted to phase change and applied to more physical tests. Even though the two first equations are coupled, at each time step the method of characteristics enables to deal with Eq. (1b) independently and a simple integration of Eq. (1a) provides the velocity field subsequently. Eq. (1c) is (in 1D) post-processed to compute $\bar{p}$.

The extension to dimension 2 was performed in [4] involving the software FreeFem++. The weak formulation was derived using the method of characteristics to handle the transport operators. In dimension 2, the issue consisted in dealing with the coupling of all three equations in System (1) while in dimension 1, equations can be treated one after the other (theoretically in single phase flows and numerically when phase change occurs, as explained above).

However, although the aformentioned algorithm provided a relevant hint of the behaviour of the coolant fluid, discrepancies were noticed between numerical results (for instance values of the temperature at which mixture appears) and experimental values. That is why another approach was investigated. The analytical EOS was replaced by formulae fitting tabulated values provided in [6]. Hence each thermodynamic variable and each constitutive parameter were considered polynomial functions of the enthalpy and of the thermodynamic pressure in both liquid and vapor phases. The modelling of the mixture remains unchanged. This approach will be presented in a forthcoming paper [5] and a better assessment to experimental data will be shown.

Part I: robustness of a 3D algorithm for the p_0–Lmnc model

When thermal conduction effects are modelled, System (1) becomes

$$\begin{cases} \nabla\cdot\boldsymbol{u} = \dfrac{\beta(h,p_0)}{p_0}\left[\nabla\cdot\big(\lambda(h,p_0)\nabla T(h,p_0)\big) + \Phi\right], & \text{(3a)} \\ \rho(h,p_0)\times\left[\partial_t h + \boldsymbol{u}\cdot\nabla h\right] = \nabla\cdot\left[\lambda(h,p_0)\nabla T(h,p_0)\right] + \Phi, \qquad (t,\boldsymbol{x})\in\mathbb{R}^+\times\Omega_d, & \text{(3b)} \\ \rho(h,p_0)\times\left[\partial_t\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right] - \nabla\cdot\sigma(\boldsymbol{u}) + \nabla\bar{p} = \rho(h,p_0)\boldsymbol{g}. & \text{(3c)} \end{cases}$$

The temperature is not a post-processing anymore and is involved in the resolution of the model. We underline that T is constant in the mixture phase.

As the order of magnitude of the tabulated thermal conduction coefficient is small, it seems difficult to predict the role played by λ. A first purpose of this part will thus be to determine its influence.

The second purpose will consist in extending the 2D FreeFem++ algorithm to dimension 3. Even if the weak formulation is the same, the computational time increases and it enables to deal with more general geometries. In particular, dividing walls may make recirculations appear, which would make the problem ill-posed (inward flow at the exit). A particular attention will be paid to the computational time.

Part II: extension to the $P_0(t)$–Lmnc model

To enrich the modelling and to deal with more general experiments, the thermodynamic pressure can be considered time-varying. Model (1) then reads [3]

$$\begin{cases} \nabla\cdot\boldsymbol{u} = -\dfrac{P_0'(t)}{\rho\big(h,P_0(t)\big)c^2\big(h,P_0(t)\big)} + \dfrac{\beta\big(h,P_0(t)\big)}{P_0(t)}\Phi, & \text{(4a)} \\ \rho\big(h,P_0(t)\big)\times\left[\partial_t h + \boldsymbol{u}\cdot\nabla h\right] = \Phi + P_0'(t), \qquad (t,\boldsymbol{x})\in\mathbb{R}^+\times\Omega_d, & \text{(4b)} \\ \rho\big(h,P_0(t)\big)\times\left[\partial_t\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right] - \nabla\cdot\sigma(\boldsymbol{u}) + \nabla\bar{p} = \rho\big(h,P_0(t)\big)\boldsymbol{g}. & \text{(4c)} \end{cases}$$

As in parent models, $P_0(t)$ is a given function. In Eq. (4a), c^2 denotes the squared speed of sound and is computed from the EOS. The main consequence is that even in dimension 1 for monophasic flows modelled with the stiffened gas law, equations are strongly coupled. However, Eq. (4c) remains decoupled in dimension 1 and the computation of $\bar{p}$ is still a post-processing.

The study would consist initially in three steps which correspond to the way the constant pressure case was investigated:

① As the change from p_0 to $P_0(t)$ induces a global coupling, one might investigate whether it is possible to derive explicit solutions in simple cases.

② When the depressurization is fast, the low Mach hypothesis may not be satisfied anymore. Thus, it would be interesting to study the sensibility of the solution and especially of the Mach number with respect to the magnitude of the variations of $P_0(t)$. It would be done first in the monophasic case and then in the diphasic case.

③ When coupled to the SG law, the model includes parameters which must be tuned depending on the pressure field. As many pressure values are involved, one might determine relevant intervals over which values can be kept constant. Numerical simulations could then be performed in 1D or 2D.

④ In the tabulated case for a single phase flow, polynomials must be updated at each time step (as P_0 evolves). Two strategies are straightaway contemplated: either a 2-variable interpolation is processed or several 1-variable interpolation polynomials for each sampled pressure values are computed initially. The goal is to avoid too restrictive extra computational time in order to preserve the attractiveness of the low Mach approach.

Organisation

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References

[1] M. Bernard, S. Dellacherie, G. Faccanoni, B. Grec, O. Lafitte, T.-T. Nguyen, and Y. Penel. Study of low Mach nuclear core model for single-phase flow. In *ESAIM:Proc*, volume 38, pages 118–134, 2012.

[2] M. Bernard, S. Dellacherie, G. Faccanoni, B. Grec, and Y. Penel. Study of a low Mach nuclear core model for two-phase flows with phase transition I: stiffened gas law. *Math. Model. Numer. Anal.*, 48(6):1639–1679, 2014.

[3] S. Dellacherie. On a low Mach nuclear core model. In *ESAIM:Proc*, volume 35, pages 79–106, 2012.

[4] S. Dellacherie, G. Faccanoni, B. Grec, E. Nayir, and Y. Penel. 2D numerical simulation of a low Mach nuclear core model with stiffened gas using FreeFem++. In *ESAIM:ProcS*, volume 45, pages 138–147, 2014.

[5] S. Dellacherie, G. Faccanoni, B. Grec, and Y. Penel. Study of a low Mach nuclear core model for two-phase flows with phase transition II: tabulated values.

[6] E. W. Lemmon, M. O. McLinden, and D. G. Friend. *Thermophysical Properties of Fluid Systems*. National Institute of Standards and Technology, Gaithersburg MD, 20899.