Password project

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Physical context and model

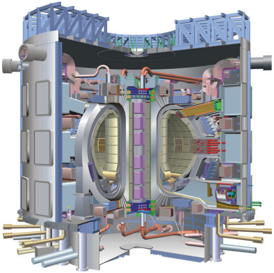


FIGURE : ITER

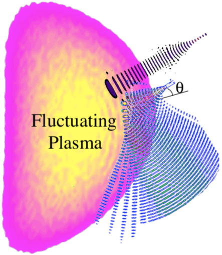


FIGURE : waves

Equations : Maxwell-Newton

$$\begin{cases} -\frac{1}{c^2}\partial_t E + \nabla \wedge B = \mu_0 J \\ \partial_t B + \nabla \wedge E = 0 \\ m_e \partial_t u_e = -e(E + u_e \wedge B_0) - m_e \nu u_e \end{cases} \qquad J = eN_e u_e$$

Some references on numerical simulation

- Décomposition de domaines pour la simulation "full wave" dans un plasma froid, T. Hattori (thesis)
- Analyse mathématique et numérique de problèmes d'ondes apparaissant dans les plasmas magnétiques, L.-M. Imbert-Gérard (thesis)
- Hybrid resonance of Maxwell's equations in slab geometry, B.Després, L.M. Imbert-Gérard and R. Weder, in JMPA
- Stable coupling of the Yee scheme with a linear current model, F. Da Silva, M. Campos-Pinto, B. Després, S. Heuraux, HAL preprint 2014.
- Full-wave modeling of the O-X mode conversion in the Pegasus Toroidal Experiment, A. Kohn, J. Jacquot, M.W. Bongard, S. Gallian, E.T. Hinson and F.A. Volpe, arXiv :1104.0743 [physics.plasm-ph](2011).

In 1D : $\partial_y = i\theta$. Unknowns : $\mathbf{u} = (E_1, E_2) = (E_x, E_y)$

Variational formulation :

$$\int_{-L}^{H} (E_2' - i\theta E_1)\overline{(\tilde{E}_2' - i\theta \tilde{E}_1)} - \int_{-L}^{H} (\varepsilon_0 + i\nu Id)\mathbf{E} \cdot \overline{\tilde{\mathbf{E}}}$$
$$-i\sqrt{\alpha(-L)}E_2(-L)\tilde{E}_2(-L) = -g_{inc}(-L)\overline{(\tilde{E}_2(-L))}$$

$$a(\mathbf{u},\mathbf{v}) = a_1(\mathbf{u},\mathbf{v}) + ia_2(\mathbf{u},\mathbf{v}) \quad \text{and} \quad l(\mathbf{v}) = -g_{inc}(-L)\overline{(v_2(-L))}$$

$$\begin{cases} a_1(\mathbf{u},\mathbf{v}) = \int_{-L}^{H} (u_2' - i\theta u_1)\overline{(v_2' - i\theta v_1)} - \int_{-L}^{H} \varepsilon_0 \mathbf{u} \cdot \overline{\mathbf{v}}, \\ a_2(\mathbf{u},\mathbf{v}) = -\nu \int_{-L}^{H} \mathbf{u} \cdot \overline{\mathbf{v}} - \sqrt{\alpha(-L)}u_2(-L)\overline{v_2(-L)}, \end{cases}$$

where $a_1 = a_1^*$ and $a_2 = a_2^*$ are hermitian.

$$\mathbf{u}, \mathbf{v} \in V = \{(E_1, E_2) |\ E_1, E_2, E_2' \in L^2\}$$

$$\|u\|_V^2 = \|E_1\|_{L^2}^2 + \|E_2\|_{L^2}^2 + \|E_2'\|_{L^2}$$

Frequency domain study

$$|a(\mathbf{u},\mathbf{u})| \geq \frac{1}{4}\sqrt{\frac{\nu}{\hat{\varepsilon}+\theta^2}}\|u_2'\|_{L^2}^2 + \frac{\nu}{4}\|\mathbf{u}\|_{L^2}^2,$$

- The problem is well-posed (existence/uniqueness)
- Passing to the limit $\nu \to 0^+$ is the limit absorption principle

Matlab code, P^1 FEM

For physically adapted coefficients N_e, B_0 it yields the hybrid resonance

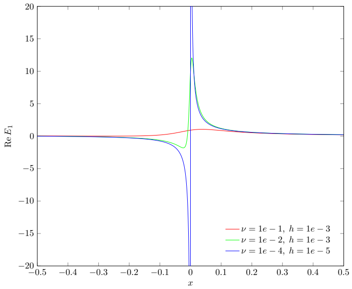


FIGURE : Convergence $E_x^\nu(x)$

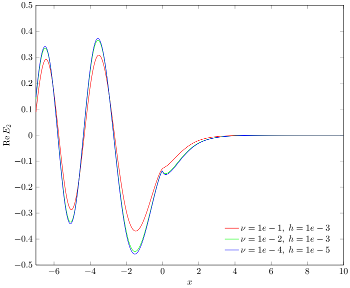
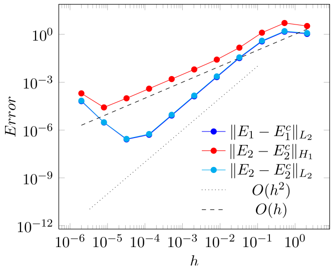
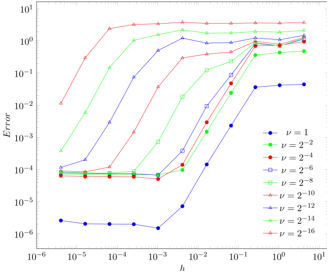


FIGURE : Convergence $E_y^\nu(x)$

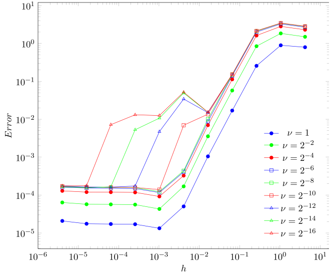
$$E_x^\nu \approx \frac{c}{x + i\nu}$$



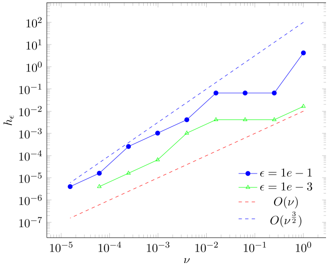
(a) Convergence non sigular case



(b) $E_x(x)$ error (singular case)



(c) $E_y(x)$ error (singular case)



(d) h_ε vs ν (sing.)

Time dependent problem

Harmonic forcing on the boundary : recover the limiting amplitude principle[1] at the limit $t \to \infty$

$$\begin{cases} -\varepsilon_0 \partial_t E_x = e N_e u_x \\ -\varepsilon_0 \partial_t E_y - \partial_x E_y = e N_e u_y \\ \partial_t H_z + \partial_x E_y = 0 \\ m_e \partial_t u_x = e(E_x + u_y B_0) - \nu m_e u_x \\ m_e \partial_t u_y = e(E_x - u_x B_0) - \nu m_e u_y \end{cases}$$

Finite difference, 1D energy conservative leap-frog scheme[2]

◊

1. C. Morawetz, CPAM, 1962
2. Stable coupling of the Yee scheme with a linear current model, F. Da Silva, M. Campos-Pinto, B. Després, S. Heuraux, HAL preprint 2014

Thank you for your attention !