Asymptotic preserving scheme for transport of charged particles under high magnetic fields

Céline Caldini-Queiros, Mihai Bostan, Nicolas Crouseilles, Mohammed Lemou

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Outline

- Gyro-average operator
- Micro-macro schemes for Vlasov equation

Section 1

Gyro-average operator

Highly penalized transport [1]

Linear transport problem, where a part of the transport operator is highly penalized

$$\begin{cases} \partial_t u^\varepsilon + a(t,y)\cdot\nabla_y u^\varepsilon + \dfrac{b(y)}{\varepsilon}\cdot\nabla_y u^\varepsilon = 0, & (t,y)\in\mathbb{R}_+\times\mathbb{R}^m \\ u^\varepsilon(0,y) = u_0^\varepsilon(y), & y\in\mathbb{R}^m. \end{cases}$$

1. M. Bostan J. Differential Equations, 249(7) :1620–1663, 2010.

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By Hilbert method : formal expansion

$$u^\varepsilon = u + \varepsilon u_1 + \varepsilon^2 u_2 + \dots$$

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Hilbert expansion

$$\varepsilon^{-1} : b(y)\cdot\nabla_y u = 0,$$
$$\varepsilon^{0} : \partial_t u + a(t,y)\cdot\nabla_y u + b(y)\cdot\nabla_y u_1 = 0,$$
$$\varepsilon^{1} : \partial_t u_1 + a(t,y)\cdot\nabla_y u_1 + b(y)\cdot\nabla_y u_2 = 0,$$
$$\vdots$$

Crucial role of $\mathcal{T} = b(y)\cdot\nabla_y$. We assume $\mathrm{div}_y b = 0$. Let P be the projection onto $\mathcal{T}$ kernel. We obtain the model for u :

$$\partial_t u + P\left(a\cdot\nabla_y u\right) = 0,\ (t,y)\in\mathbb{R}_+\times\mathbb{R}^m$$

We determine u_1 up to a function v_1 in $\mathcal{T}$ kernel, solution of

$$\partial_t v_1 + P\left(a\cdot\nabla_y v_1\right) + P\left(\partial_t w_1 + a\cdot\nabla_y w_1\right) = 0,\ (t,y)\in\mathbb{R}_+\times\mathbb{R}^m.$$

We need to determine P

Gyro-average along a flow

$b : \mathbb{R}^m \mapsto \mathbb{R}^m$, field

$$b \in W^{1,\infty}_{\text{loc}}(\mathbb{R}^m),$$

$$\text{div}_y b = 0$$

Flow : $Y = Y(s;y)$:

$$\frac{\mathrm{d}Y}{\mathrm{d}s} = b(Y(s;y)),\ (s,y) \in \mathbb{R} \times \mathbb{R}^m,$$

For a T_c-periodic flow, we define

$$\langle u \rangle (y) = \frac{1}{T_c} \int_0^{T_c} u\left(Y(s;y)\right) \mathrm{d}s, y \in \mathbb{R}^m.$$

Proposition

The average operator is linear continuous. Moreover it coincides with the orthogonal projection on the kernel of $\mathcal{T}$, *i.e.*,

$$\langle u \rangle \in \ker \mathcal{T} \text{ and } \int_{\mathbb{R}^m} (u - \langle u \rangle)\varphi \mathrm{d}y = 0, \ \ \forall \varphi \in \ker \mathcal{T}.$$

Section 2

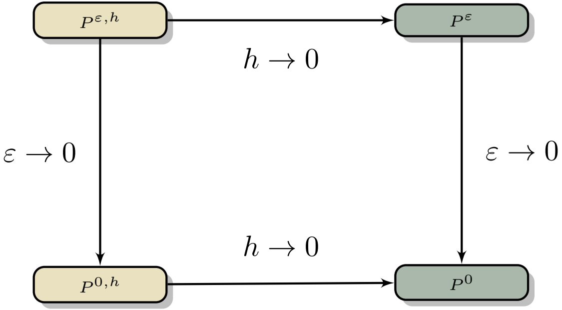
Micro-macro schemes for Vlasov equation

Goal : construct a Asymptotic Preserving scheme which

- is free from the constraint $\Delta t = O(\varepsilon^2)$
- is consistent with all regimes $\varepsilon = O(1)$ AND $\varepsilon \ll 1$

Principle of Asymptotic Preserving Schemes [2]

Let P_ε be a continuous problem that converges to P_0 when $\varepsilon \to 0$. We seek and approximation $P_{\varepsilon,h}$ such that



2. S. Jin, SIAM J. Sci. Comput. 99

2D guiding center model

$$B^\varepsilon = \left(0, 0, \frac{B(x)}{\varepsilon}\right), \; B > 0$$

$x = (x_1, x_2)$, $v = (v_1, v_2)$ and ${}^\perp v = (v_2, -v_1)$.

$$\frac{T_c}{T_{obs}} = \varepsilon \ll 1.$$

Vlasov equation in this regime

$$\varepsilon \partial_t f^\varepsilon + \left(v \cdot \nabla_x + \frac{q}{m} E(x) \cdot \nabla_v\right) f^\varepsilon + \frac{\omega_c}{\varepsilon} {}^\perp v \cdot \nabla_v f^\varepsilon = 0$$

$(t, x, v) \in \mathbb{R}_+ \times \mathbb{R}^2 \times \mathbb{R}^2$.

2D guiding center model

$\mathcal{T} = \omega_c \, {}^{\perp}v \cdot \nabla_v$

Flow characteristics are solution of

$$\frac{\mathrm{d}X}{\mathrm{d}s} = 0, \quad \frac{\mathrm{d}V}{\mathrm{d}s} = \omega_c(X(s;x,v)) \, {}^{\perp}V(s;x,v), \quad (X,V)(0;x,v) = (x,v).$$

Associated flow characteristics

$$X(s) = x, \quad V(s) = R(-\omega_c s)v, \quad (X,V)(0;x,v) = (x,v).$$

Gyro-average operator

$$\begin{aligned}
\langle u \rangle (x,v) &= \frac{1}{T_c(x)} \int_0^{T_c(x)} u(X(s;x,v), V(s;x,v)) \, \mathrm{d}s \\
&= \frac{1}{2\pi} \int_0^{2\pi} u(x, \mathcal{R}(\alpha)v) \, \mathrm{d}\alpha
\end{aligned}$$

2D guiding center model

We denote $a(x, v) \cdot \nabla_{x,v} = v \cdot \nabla_x + q/mE \cdot \nabla_v$ and $\mathcal{T} f^\varepsilon = \mathrm{div}_v(f^\varepsilon \omega_c(x)\, {}^\perp v)$. The model writes

$$\partial_t f^\varepsilon + \frac{1}{\varepsilon} a(x, v) \cdot \nabla_{x,v} f^\varepsilon + \frac{1}{\varepsilon^2} \mathcal{T} f^\varepsilon = 0.$$

Limit model derivation

$$\mathcal{T}f = 0, \tag{1}$$

$$a(x,v)\cdot\nabla_{x,v} f + \mathcal{T}f^1 = 0, \tag{2}$$

$$\partial_t f + a(x,v)\cdot\nabla_{x,v} f^1 + \mathcal{T}f^2 = 0, \tag{3}$$

$$\mathcal{T}f = 0 \iff \exists\, g = g(t,x,r) \text{ such as } f(t,x,v) = g(t,x,r=|v|)$$

$$\langle a\cdot\nabla_{x,v} f\rangle = \langle v\rangle\cdot\nabla_x g(t,x,|v|) + \frac{q}{m}E(x)\cdot\frac{\langle v\rangle}{|v|}\partial_r g(t,x,|v|) = 0$$

$$a\cdot\nabla_{x,v} f + \mathcal{T}f^1 = 0 \iff \mathcal{T}(f^1 - \langle f^1\rangle) = -a(x,v)\cdot\nabla_{x,v} f$$

$$f^1 - \langle f^1\rangle = -\mathcal{T}^{-1}(a(x,v)\cdot\nabla_{x,v} f)$$

Limit model derivation

$$\mathcal{T} f = 0, \tag{1}$$

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$$\langle a \cdot \nabla_{x,v} f \rangle = \langle v \rangle \cdot \nabla_x g(t,x,|v|) + \frac{q}{m} E(x) \cdot \frac{\langle v \rangle}{|v|} \partial_r g(t,x,|v|) = 0$$

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$$f^1 - \langle f^1 \rangle = -\mathcal{T}^{-1}(a(x,v) \cdot \nabla_{x,v} f)$$

Proposition

There is a free divergence a vector field $\mathcal{B}$ such as for all $f \in \ker \mathcal{T}$ we have

$$\mathcal{T}^{-1}(a \cdot \nabla_{x,v} f) = \mathcal{B} \cdot \nabla_{x,v} f$$

$$\mathcal{T} f = 0, \tag{1}$$

$$a(x,v) \cdot \nabla_{x,v} f + \mathcal{T} f^1 = 0, \tag{2}$$

$$\partial_t f + a(x,v) \cdot \nabla_{x,v} f^1 + \mathcal{T} f^2 = 0, \tag{3}$$

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$$a \cdot \nabla_{x,v} f + \mathcal{T} f^1 = 0 \iff \mathcal{T}(f^1 - \langle f^1 \rangle) = -a(x,v) \cdot \nabla_{x,v} f$$

$$f^1 - \langle f^1 \rangle = -\mathcal{T}^{-1}(a(x,v) \cdot \nabla_{x,v} f)$$

$$\mathcal{B} = \left(-\frac{{}^\perp v}{\omega_c}, \frac{{}^\perp E}{B} - \left(\frac{\nabla_x B}{B} \cdot \frac{v}{\omega_c} \right) {}^\perp v \right),$$

We have to eliminate f^2 from equation

$$\partial_t f + a(x,v) \cdot \nabla_{x,v} f^1 + \mathcal{T} f^2 = 0$$

We apply the gyro-average operator

$$\partial_t f + \left\langle a(x,v) \cdot \nabla_{x,v} f^1 \right\rangle = 0$$

Or

$$\left\langle a \cdot \nabla_{x,v} \left\langle f^1 \right\rangle \right\rangle = 0$$

then

$$\partial_t f + \left\langle a(x,v) \cdot \nabla_{x,v} (f^1 - \left\langle f^1 \right\rangle) \right\rangle = 0$$

Limit model derivation

We have to eliminate f^2 from equation

$$\partial_t f + a(x,v) \cdot \nabla_{x,v} f^1 + \mathcal{T} f^2 = 0$$

We apply the gyro-average operator

$$\partial_t f + \left\langle a(x,v) \cdot \nabla_{x,v} f^1 \right\rangle = 0$$

Or

$$\left\langle a \cdot \nabla_{x,v} \left\langle f^1 \right\rangle \right\rangle = 0$$

or else

$$\partial_t f - \langle a \cdot \nabla_{x,v} (\mathcal{B} \cdot \nabla_{x,v} f) \rangle = 0$$

Limit model

Proposition

There is a free divergence vector field $\mathcal{C}$ such that $\forall\ f \in \ker \mathcal{T}$

$$-\langle a \cdot \nabla_{x,v}(\mathcal{B} \cdot \nabla_{x,v} f)\rangle = \mathcal{C} \cdot \nabla_{x,v} f.$$

This vector field writes

$$\mathcal{C} = \left(\frac{{}^{\perp}E}{B} - \frac{|v|^2}{2\omega_c} \frac{{}^{\perp}\nabla_x B}{B}, \frac{1}{2} \left(\frac{{}^{\perp}E}{B} \cdot \frac{\nabla_x B}{B} \right) v \right)$$

Limit model for $(f^{\varepsilon})_{\varepsilon}$

$$\partial_t f + \mathcal{C} \cdot \nabla_{x,v} f = 0, \quad f(0) = \left\langle f^{\text{in}} \right\rangle, \quad \mathcal{T} f = 0$$

Polar coordinates

We take $\omega_c = 1,\ q = 1,\ m = 1.$
We switch to polar coordinates : $(v_1, v_2) \to (|v|, \theta)$ with $|v| \geq 0$ and $\theta \in [0, 2\pi]$ such as

$$\begin{aligned} a(x,v) \cdot \nabla_{x,v} =& r\cos\theta \partial_{x_1} f^\varepsilon + r\sin\theta \partial_{x_2} f^\varepsilon \\ &+ (E_1 \cos\theta + E_2 \sin\theta)\partial_r f^\varepsilon \\ &+ \frac{1}{r}(-E_1\cos\theta + E_2 \sin\theta)\partial_\theta f^\varepsilon \end{aligned}$$

and

$$\mathcal{T} f^\varepsilon = -\partial_\theta f^\varepsilon$$

Decomposition

$$\partial_t f^\varepsilon + \frac{1}{\varepsilon} a(x,v) \cdot \nabla_{x,v} + \frac{1}{\varepsilon^2} \mathcal{T} f^\varepsilon = 0.$$

When $\varepsilon \to 0$ f^ε converge to a function in the kernel of $\mathcal{T}$. We then decompose our function as

$$f^\varepsilon(t, x_1, x_2, r, \theta) = r^\varepsilon(x_1, x_2, r, \theta) + g^\varepsilon(x_1, x_2, r)$$

where

$$g^\varepsilon = \langle f^\varepsilon \rangle, r^\varepsilon = f^\varepsilon - \langle f^\varepsilon \rangle$$

We obtain the following micro-macro scheme

$$\begin{cases} \partial_t g^\varepsilon & + \frac{1}{\varepsilon} \langle a \cdot \nabla_{x,v} r^\varepsilon \rangle = 0, \\ \partial_t r^\varepsilon & + \frac{1}{\varepsilon} a \cdot \nabla_{x,v} g^\varepsilon + \frac{1}{\varepsilon} a \cdot \nabla_{x,v} r^\varepsilon - \frac{1}{\varepsilon} \langle a \cdot \nabla_{x,v} r^\varepsilon \rangle - \frac{1}{\varepsilon^2} \partial_\theta r^\varepsilon = 0. \end{cases}$$

Discretisation

- time : $g^\varepsilon(t^n) \leftrightarrow g^n$ et $r^\varepsilon(t^n) \leftrightarrow r^n$.
- space : $\Phi(F^n) \approx a \cdot \nabla_{x,v} f^\varepsilon$ Lax-Richtmyer, centered scheme, order 2
- Fourier decomposition in θ for $\mathcal{T}$ (inversible on zero mean functions)

$$\begin{cases} \frac{r^{\varepsilon,n+1}-r^{\varepsilon,n}}{\Delta t} & +\frac{1}{\varepsilon}\Big((I-\langle\cdot\rangle)(r^{\varepsilon,n}+g^{\varepsilon,n})\Big) - \frac{1}{\varepsilon^2}\partial_\theta r^{\varepsilon,n+1} = 0, \\ \frac{g^{\varepsilon,n+1}-g^{\varepsilon,n}}{\Delta t} & +\frac{1}{\varepsilon}\left\langle a\cdot\nabla_{x,v} r^{\varepsilon,n+1}\right\rangle = 0. \end{cases} \quad (4)$$

We have

$$\begin{aligned} r^{n+1} &= \left(I + \frac{\Delta t}{\varepsilon^2}\mathcal{T}\right)^{-1} \left(r^n - \frac{\Delta t}{\varepsilon}(I - \langle\cdot\rangle)a \cdot \nabla_{x,v}(g^n + r^n)\right) \\ &= \left(I + \frac{\Delta t}{\varepsilon^2}\mathcal{T}\right)^{-1} \left(-\frac{\Delta t}{\varepsilon}a \cdot \nabla_{x,v}g^n\right) + \varepsilon^2\mathcal{F}(r^n) \\ &= -\varepsilon\mathcal{B}^{\lambda_\varepsilon} \cdot \nabla_{x,v}g^n + \varepsilon^2\mathcal{F}(r^n). \end{aligned}$$

Preliminary computations

We have

$$r^{n+1} = \left(I + \frac{\Delta t}{\varepsilon^2}\mathcal{T}\right)^{-1}\left(r^n - \frac{\Delta t}{\varepsilon}(I - \langle\cdot\rangle)a\cdot\nabla_{x,v}(g^n + r^n)\right)$$
$$= \left(I + \frac{\Delta t}{\varepsilon^2}\mathcal{T}\right)^{-1}\left(-\frac{\Delta t}{\varepsilon}a\cdot\nabla_{x,v}g^n\right) + \varepsilon^2\mathcal{F}(r^n)$$
$$= -\varepsilon\mathcal{B}^{\lambda_\varepsilon}\cdot\nabla_{x,v}g^n + \varepsilon^2\mathcal{F}(r^n).$$

$$\mathcal{B}^{\lambda}\cdot\nabla_{x,v} = \frac{\mathcal{O}_\lambda v}{\sqrt{\lambda^2+\omega_c^2}}\cdot\nabla_x + \frac{q}{m\sqrt{\lambda^2+\omega_c^2}}{}^t\mathcal{O}_\lambda E\cdot\nabla_v + \left(\frac{{}^\perp v\otimes v}{\lambda^2+\omega_c^2}{}^t\mathcal{O}_\lambda{}^2\nabla_x\omega_c\right)\cdot\nabla_v$$

with

$$\mathcal{O}_\lambda = \begin{pmatrix} \frac{\lambda}{\sqrt{\lambda^2+\omega_c^2}} & -\frac{\omega_c}{\sqrt{\lambda^2+\omega_c^2}} \\ \frac{\omega_c}{\sqrt{\lambda^2+\omega_c^2}} & \frac{\lambda}{\sqrt{\lambda^2+\omega_c^2}} \end{pmatrix}.$$

Then, we obtain for the macro equation :

$$\begin{aligned}
g^{n+1} =& g^n - \frac{\Delta t}{\varepsilon}\left\langle a\cdot\nabla_{x,v} r^{n+1}\right\rangle \\
=& g^n - \frac{\Delta t}{\varepsilon}\left\langle a\cdot\nabla_{x,v}(-\varepsilon\mathcal{B}^{\lambda_\varepsilon}\nabla_{x,v}g^n)\right\rangle - \frac{\Delta t}{\varepsilon}\left\langle a\cdot\nabla_{x,v}(\varepsilon^2\mathcal{F}(r^n))\right\rangle \\
=& g^n - \Delta t(\mathcal{C}^{\lambda_\varepsilon}\cdot\nabla_{x,v}g) + \varepsilon^2\left\langle\mathcal{B}^{\lambda_\varepsilon}\cdot\nabla_{x,v}(\mathcal{B}^{\lambda_\varepsilon}\cdot\nabla_{x,v}g^n)\right\rangle \\
& -\Delta t\epsilon\left\langle a\cdot\nabla_{x,v}\mathcal{F}(r^n)\right\rangle
\end{aligned}$$

Then, we obtain for the macro equation :

$$
\begin{aligned}
g^{n+1} =& g^n - \frac{\Delta t}{\varepsilon} \left\langle a \cdot \nabla_{x,v} r^{n+1} \right\rangle \\
=& g^n - \frac{\Delta t}{\varepsilon} \left\langle a \cdot \nabla_{x,v}(-\varepsilon \mathcal{B}^{\lambda_\varepsilon} \nabla_{x,v} g^n) \right\rangle - \frac{\Delta t}{\varepsilon} \left\langle a \cdot \nabla_{x,v}(\varepsilon^2 \mathcal{F}(r^n)) \right\rangle \\
=& g^n - \Delta t(\mathcal{C}^{\lambda_\varepsilon} \cdot \nabla_{x,v} g) + \varepsilon^2 \left\langle \mathcal{B}^{\lambda_\varepsilon} \cdot \nabla_{x,v}(\mathcal{B}^{\lambda_\varepsilon} \cdot \nabla_{x,v} g^n) \right\rangle \\
& - \Delta t \epsilon \left\langle a \cdot \nabla_{x,v} \mathcal{F}(r^n) \right\rangle
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{C}^\lambda \cdot \nabla_{x,v} = & \left[\frac{\omega_c^2}{\lambda^2 + \omega_c^2} \frac{{}^\perp E}{B} + \frac{|v|^2}{2} {}^\perp \nabla_x \left(\frac{\omega_c}{\lambda^2 + \omega_c^2} \right) \right] \cdot \nabla_x \\
& - \frac{1}{2} \frac{{}^\perp E}{B} \cdot \nabla_x \left(\frac{\omega_c}{\lambda^2 + \omega_c^2} \right) \omega_c v \cdot \nabla_v .
\end{aligned}
$$

Algorithms

First step : prediction.

$$\left\{\begin{array}{l} r^{n+1/2}_{i,j,k,l} = \left(I + \frac{\Delta t}{2\varepsilon^2}\mathcal{T}\right)^{-1}\left[\bar{r}^n_{i,j,k,l} - \frac{\Delta t}{2\varepsilon}\Big((\Phi(r^n))_{i,j,k,l} - \langle \Phi(r^n)\rangle_{i,j,k,l}\Big)\right] \\ -\varepsilon(\mathcal{B}^{\lambda_\varepsilon} \cdot \nabla_{x,v} g^n)_{i,j,k,l}, \\ g^{n+1/2}_{i,j,k} - \bar{g}^n_{i,j,k} + \frac{\Delta t}{2}(\mathcal{C}^{\lambda_\varepsilon} \cdot \nabla_{x,v} g^n)_{i,j,k,l} \\ -\varepsilon^2 \Big\langle \mathcal{B}^{\lambda_\varepsilon} \cdot \nabla_{x,v}(\mathcal{B}^{\lambda_\varepsilon} \cdot \nabla_{x,v} g^n)\Big\rangle_{i,j,k,l} + \frac{\Delta t \varepsilon}{2} \langle a \cdot \nabla_{x,v}\mathcal{F}(r^n)\rangle_{i,j,k,l} = 0. \end{array}\right.$$

Second step : correction.

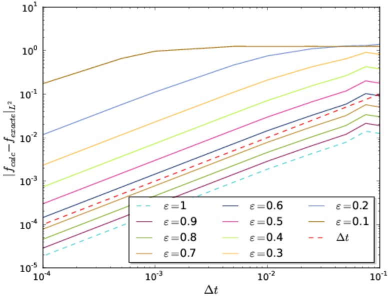
$$\left\{\begin{array}{l} r^{n+1}_{i,j,k,l} = \left(I + \frac{\Delta t}{\varepsilon^2}\mathcal{T}\right)^{-1}\Big[r^n_{i,j,k,l} - \frac{\Delta t}{\varepsilon}\Big(\Phi(r^{n+1/2})_{i,j,k,l} \\ \qquad - \Big\langle \Phi(r^{n+1/2})\Big\rangle_{i,j,k,l}\Big)\Big] - \varepsilon(\mathcal{B}^{\lambda_\varepsilon} \cdot \nabla_{x,v} g^{n+1/2})_{i,j,k,l}, \\ g^{n+1}_{i,j,k} - g^n_{i,j,k} + \Delta t(\mathcal{C}^{\lambda_\varepsilon} \cdot \nabla_{x,v} g^{n+1/2})_{i,j,k,l} \\ \qquad -\varepsilon^2 \Big\langle \mathcal{B}^{\lambda_\varepsilon} \cdot \nabla_{x,v}(\mathcal{B}^{\lambda_\varepsilon} \cdot \nabla_{x,v} g^{n+1/2})\Big\rangle_{i,j,k,l} \\ \qquad + \Delta t \varepsilon \Big\langle a \cdot \nabla_{x,v}\mathcal{F}(r^{n+1/2})\Big\rangle_{i,j,k,l} = 0. \end{array}\right.$$

Numerical results

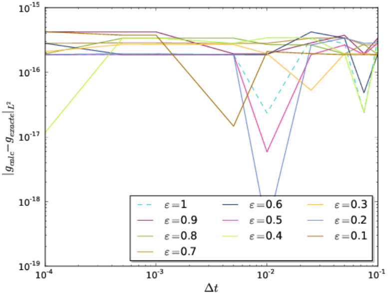
We first consider $E = 0$ and the initial condition $f(x_1, x_2, v_1, v_2) = \frac{1}{2\pi} e^{-|v|^2/2}(1 + v_1)$ such that the solution writes

$$f(t, x_1, x_2, v_1, v_2) = \frac{1}{2\pi} e^{-|v|^2/2}(1 + \cos(t/\varepsilon^2)v_1 - \sin(t/\varepsilon^2)v_2)$$

$N_x = N_y = N_{|v|} = N_\theta = 16$, $x_1 \in [0, 2\pi]$, $x_2 \in [0, 2\pi]$, $r \in [0, 5]$, $\theta \in [0, 2\pi]$.



(a) Error of f as a function of Δt



(b) Error of g as a function of Δt

Validation on Vlasov-Poisson tests

Vlasov equation in guiding-center limit

$$\varepsilon \partial_t f + v \cdot \nabla_x f + \left(E + \frac{B}{\varepsilon}\,{}^{\perp} v \right) \cdot \nabla_v f = 0,$$

Validation on Vlasov-Poisson tests

"Adaptation"

$$\varepsilon_t \partial_t f + v \cdot \nabla_x f + \left(E + \frac{B}{\varepsilon_B} {}^\perp v \right) \cdot \nabla_v f = 0,$$

Validation on Vlasov-Poisson tests

We consider $\varepsilon_t = 1$, thus we have

$$\partial_t f + v \cdot \nabla_x f + \left(E + \frac{B}{\varepsilon_B}{}^\perp v \right) \cdot \nabla_v f = 0,$$

Then, if we consider $\varepsilon_B \gg 1$, we should obtain Vlasov-Poisson 4D results.

We consider E solving Poisson and the initial condition $f_0(x_1, x_2, v_1, v_2) = \frac{1}{2\pi} e^{-|v|^2/2}(1 + 0.001\cos(kx_1))$

$N_x = N_y = N_{|v|} = N_\theta = 64,$

$x_1 \in [0, 2\pi/k], x_2 \in [0, 2\pi], r \in [0, 5], \theta \in [0, 2\pi],\ k = 0.4,$

$\Delta t = 0.005.$

Validation on Vlasov-Poisson tests

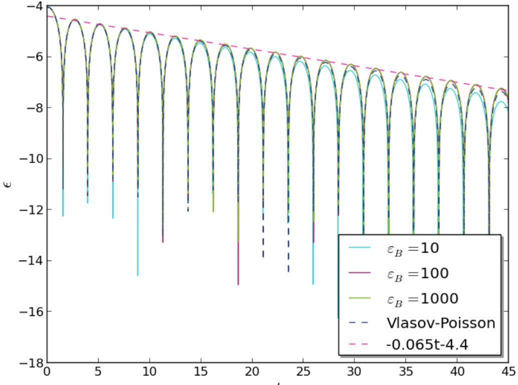
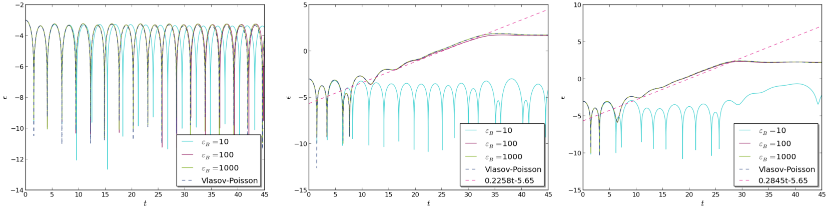


FIGURE: L^2 norm of electric field in logarithmic scale

We considere E solving Poisson and the initial condition
$f_0(x_1, x_2, v_1, v_2) = \frac{1}{2\pi}(e^{-(v-v_0)^2/2} + e^{-(v+v_0)^2/2})(1 + 0.001\cos(kx_1))$
$N_x = N_y = N_{|v|} = N_\theta = 64$, $x_1 \in [0, 2\pi]$, $x_2 \in [0, 2\pi]$, $r \in [0, 7]$,
$\theta \in [0, 2\pi]$, $k_x = 0.2$, $\Delta t = 0.005$.



(a) Case $v_0 = 1.3$ (b) Case $v_0 = 2.4$ (c) Case $v_0 = 3$

FIGURE: L^2 norm of electric field in logarithmic case for two-stream instability

Validation on Vlasov-Poisson tests

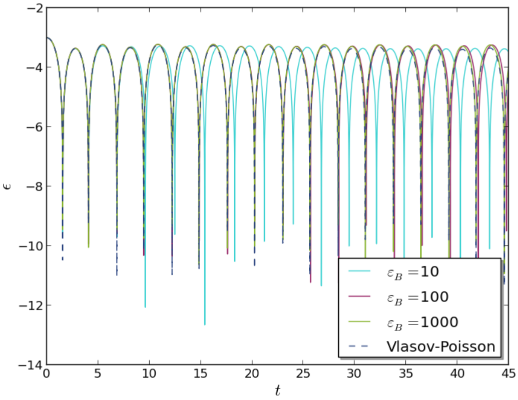


FIGURE: Case $v_0 = 1.3$

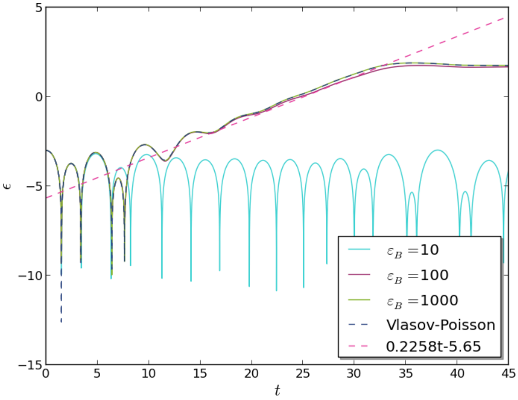


FIGURE: Case $v_0 = 2.4$

Validation on Vlasov-Poisson tests

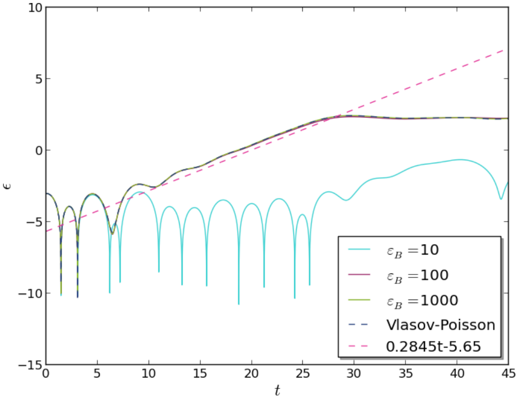


FIGURE: Case $v_0 = 3$

We considere E solving Poisson and the initial condition

$$f_0(x,y,|v|,\theta) = \frac{1}{4\pi}\left(e^{-\frac{(v-v_0)^2}{2}} + e^{-\frac{(v+v_0)^2}{2}}\right)(1+\alpha\cos(k_x x))\,.$$

$N_x = N_y = N_{|v|} = N_\theta = 16$, $x_1 \in [0, 2\pi]$, $x_2 \in [0, 2\pi]$, $r \in [0, 7]$, $\theta \in [0, 2\pi]$, $k_x = 0.2$, $\alpha = 0.001$ $\Delta t = 0.01$.

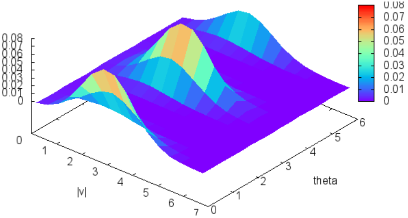
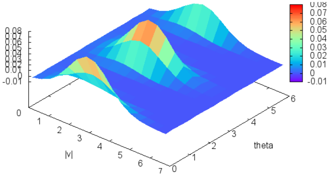
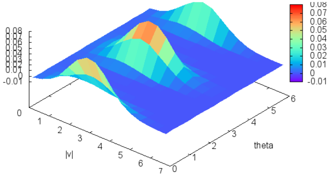


FIGURE: Initial function in phase space

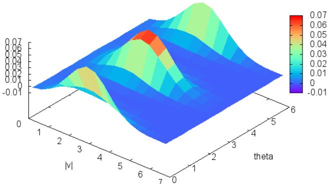
Validation on guiding-center limit



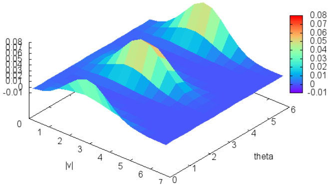
(a) $\varepsilon = 1,\ t = 0.04$



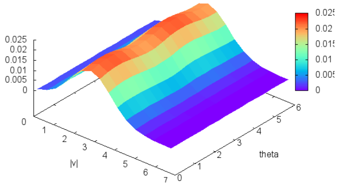
(b) $\varepsilon = 1,\ t = 0.07$



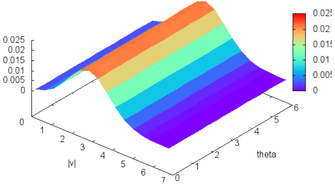
(c) $\varepsilon = 0.5,\ t = 0.04$



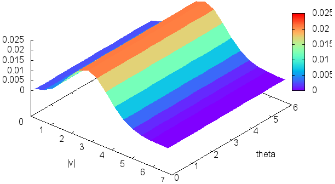
(d) $\varepsilon = 0.5,\ t = 0.07$



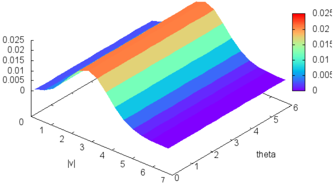
(e) $\varepsilon = 0.1$, $t = 0.04$



(f) $\varepsilon = 0.1$, $t = 0.07$



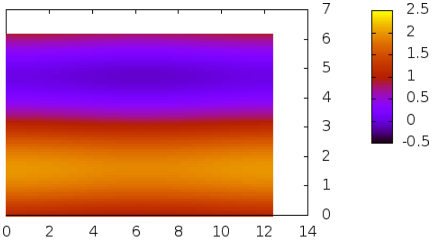
(g) $\varepsilon = 10^{-5}$, $t = 0.04$



(h) $\varepsilon = 10^{-5}$, $t = 0.07$

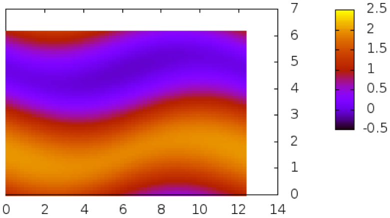
Validation on guiding-center model[3]

We consider E solving Poisson and the initial condition
$f_0(x_1, x_2, v_1, v_2) =$
$\frac{1}{2\pi}(e^{-(v-v_0)^2/2} + e^{-|v|^2/2})(1 + \sin(x_2) + 0.05\cos(kx_1))$ with
$x_1 \in [0, 2\pi/k], x_2 \in [0, 2\pi], r \in [0, 5], \theta \in [0, 2\pi]$,
$N_x = N_y = N_{|v|} = N_\theta = 64$, $\varepsilon = 0.01$, $\Delta t = 0.005$.

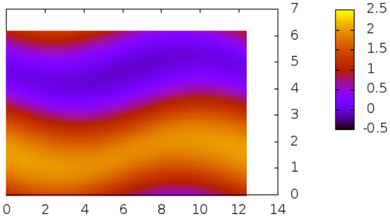


(i) Shape of $f^{\rm in}$ in space(x_1, x_2)

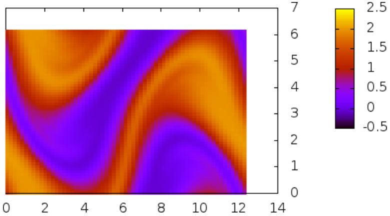
3. Shoucri



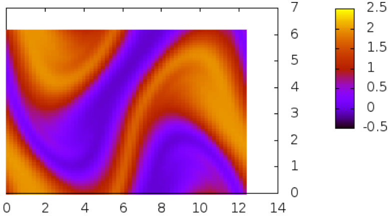
(j) micro-macro scheme, $t = 5$



(k) guiding-center scheme, $t = 5$

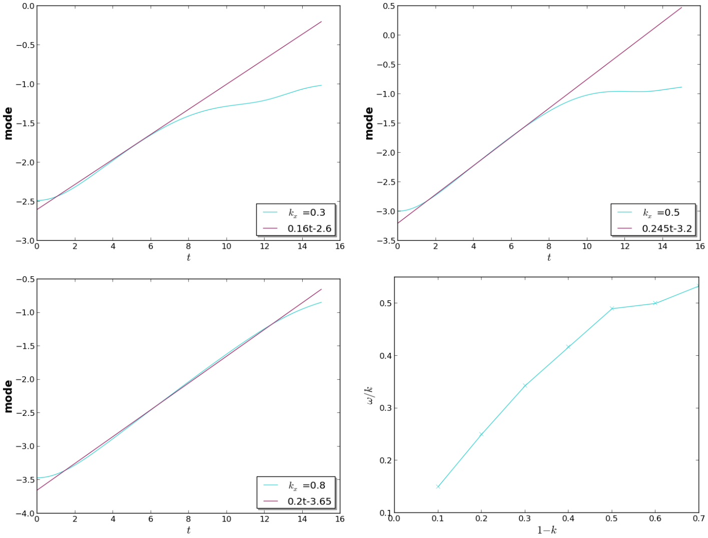


(l) micro-macro scheme, $t = 12.5$



(m) guiding-center scheme, $t = 12.5$

We consider the time evolution of Fourier mode (1,1) of electric potential $\phi(x_1, x_2)$



- AP scheme in strong magnetic field limit
- First academic validation

Possible improvement

- numerical scheme in space phase
- non homogenic magnetic field

Perspectives

- other gyrokinetic limit ?

Thank you for your attention !