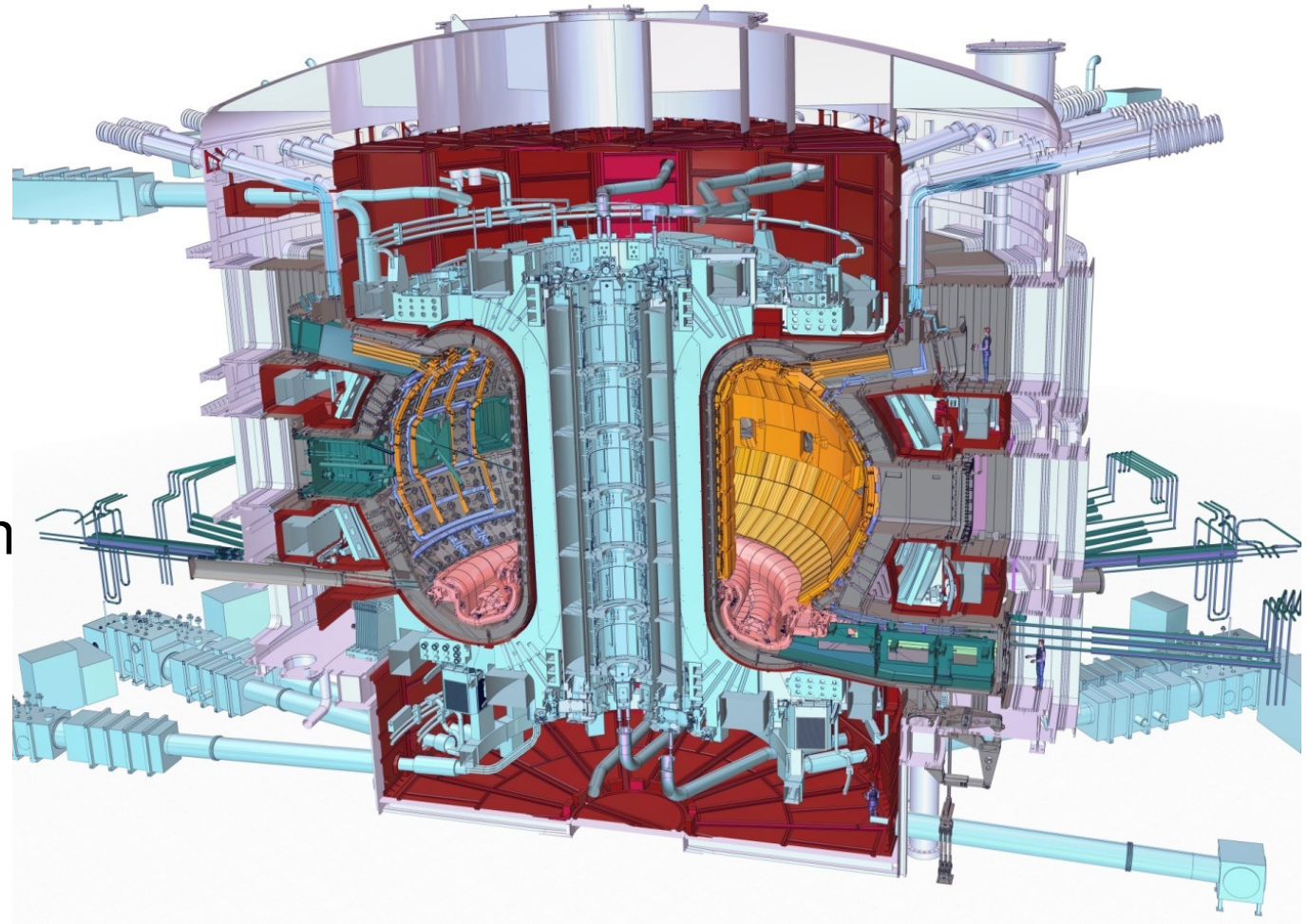

MHD Simulations for ITER

Guido Huijsmans
ITER Organization

Disclaimer: The views and opinions expressed herein do not necessarily reflect those of the ITER Organization

Outline

- ITER
 - Goals
 - Machine
 - Status
- MHD in ITER
 - Disruptions
 - ELMs
- MHD Simulation
 - JOREK
- Applications
 - VDE
 - QH-mode
- Conclusions



ITER Scope - Mission Goals

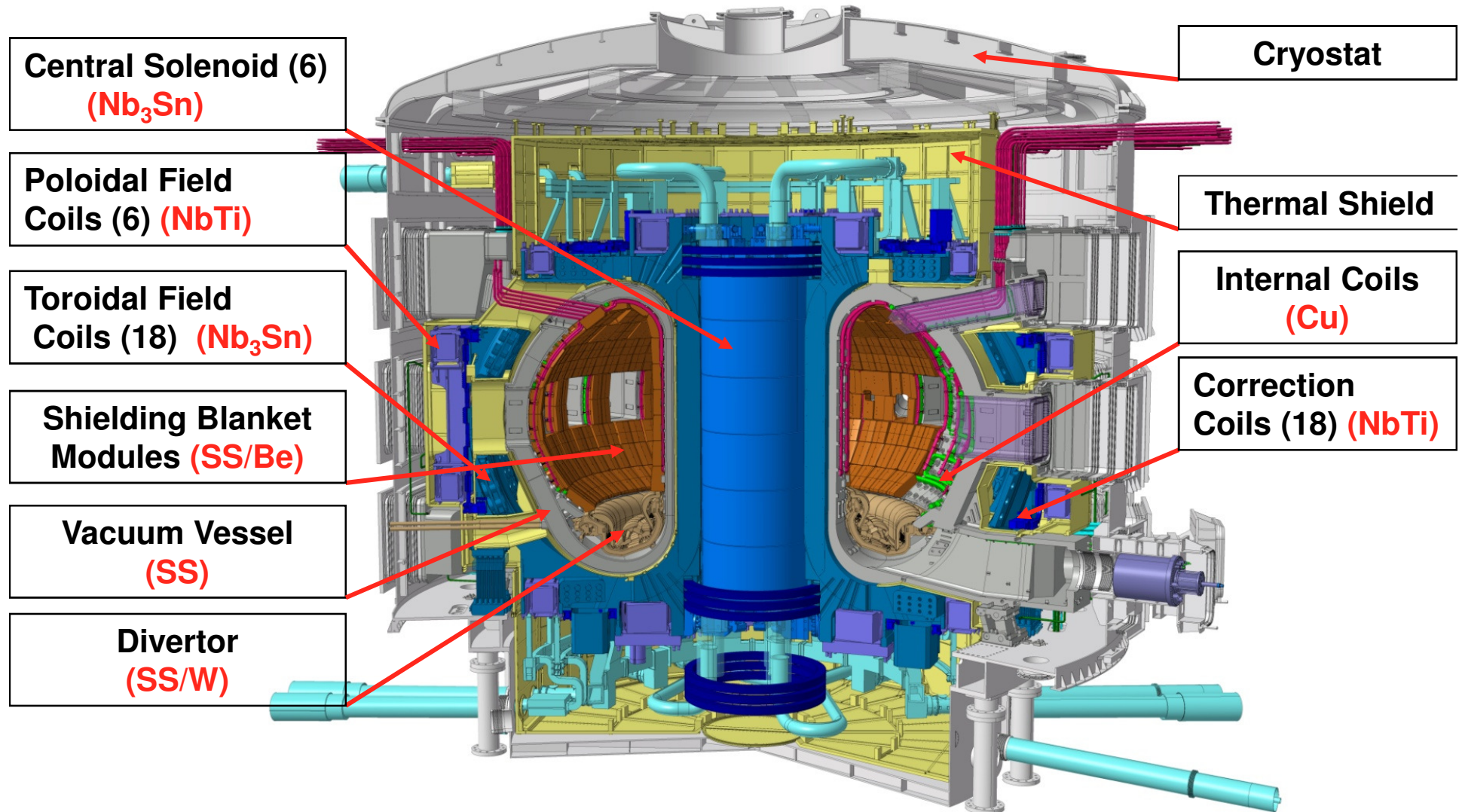
Physics:

- ITER is designed to produce a plasma dominated by α -particle heating
- produce a significant fusion power amplification factor ($Q \geq 10$) in long-pulse operation (300 – 500 s)
- aim to achieve steady-state operation of a tokamak ($Q \geq 5, \leq 3000$ s)
- retain the possibility of exploring 'controlled ignition' ($Q \geq 30$)

Technology:

- demonstrate integrated operation of technologies for a fusion power plant
- test components required for a fusion power plant
- test concepts for a tritium breeding module

ITER - Major Components

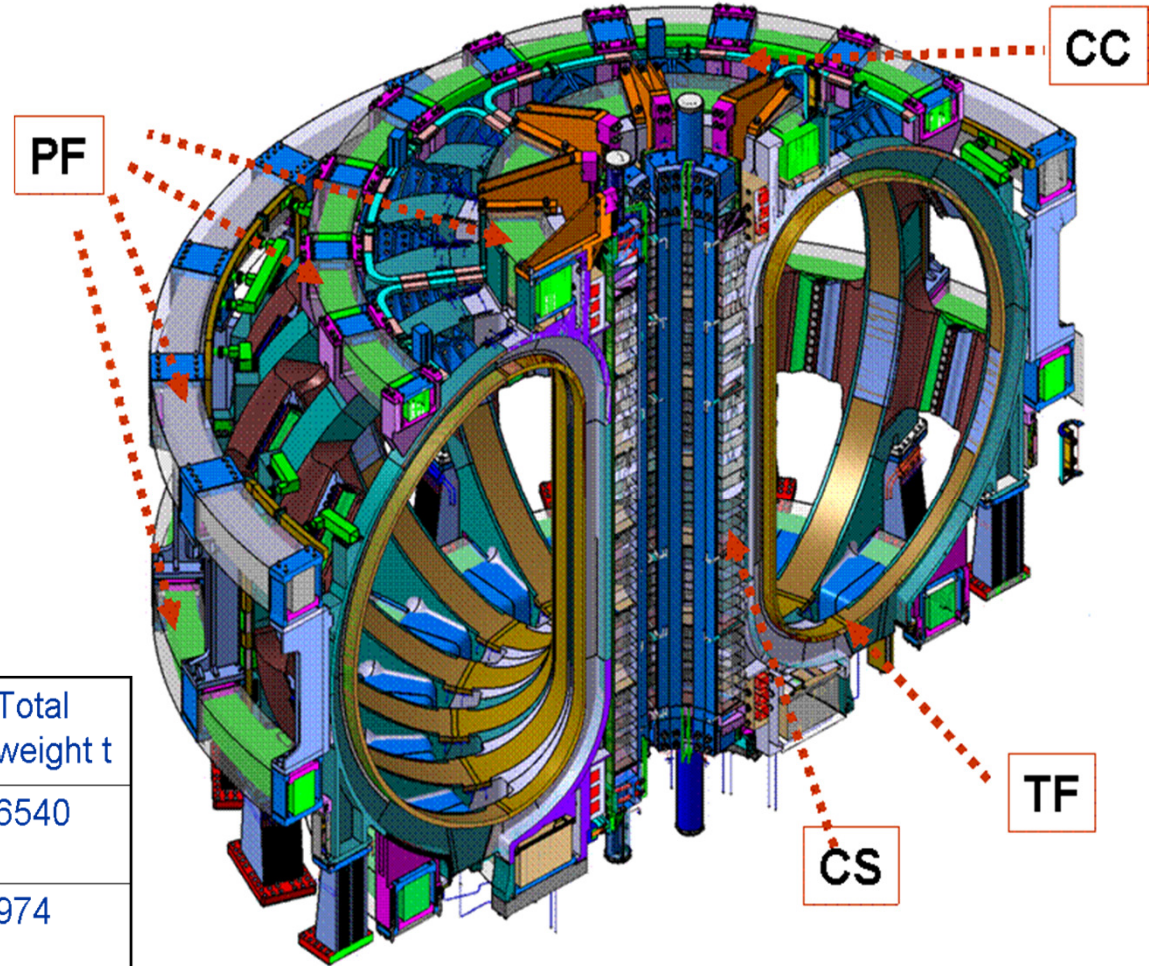


$R=6.2\text{m}$, $a=2\text{m}$, $B=5.3\text{T}$, $I=15\text{MA}$

ITER - Magnet Systems

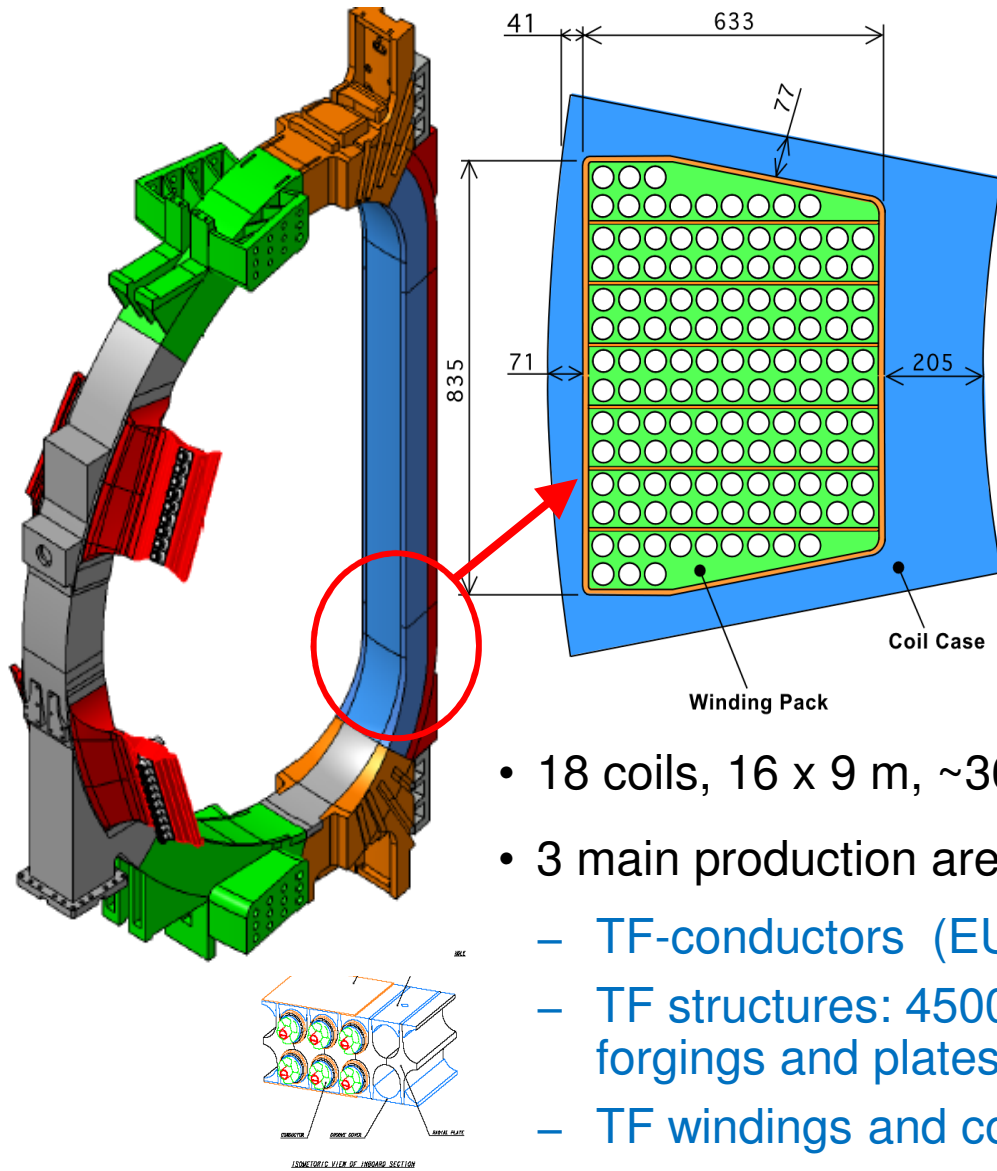
48 superconducting coils:

- 18 Toroidal Field coils
- 6 Central Solenoid modules
- 6 Poloidal Field coils
- 9 pairs of Correction Coils



System	Energy GJ	Peak Field	Total MAT	Cond length km	Total weight t
Toroidal Field TF	41	11.8	164	82.2	6540
Central Solenoid	6.4	13.0	147	35.6	974
Poloidal Field PF	4	6.0	58.2	61.4	2163
Correction Coils CC	-	4.2	3.6	8.2	85

Toroidal Field Coils



- 18 coils, 16 x 9 m, ~360 t each
- 3 main production areas:
 - TF-conductors (EU, JA, RF, CN, KO, US), *95% complete*
 - TF structures: 4500t of high precision stainless steel forgings and plates, assembled by welding
 - TF windings and coils (EU, JA)

ITER Site April 2014



ITER Basemat (July 2014)



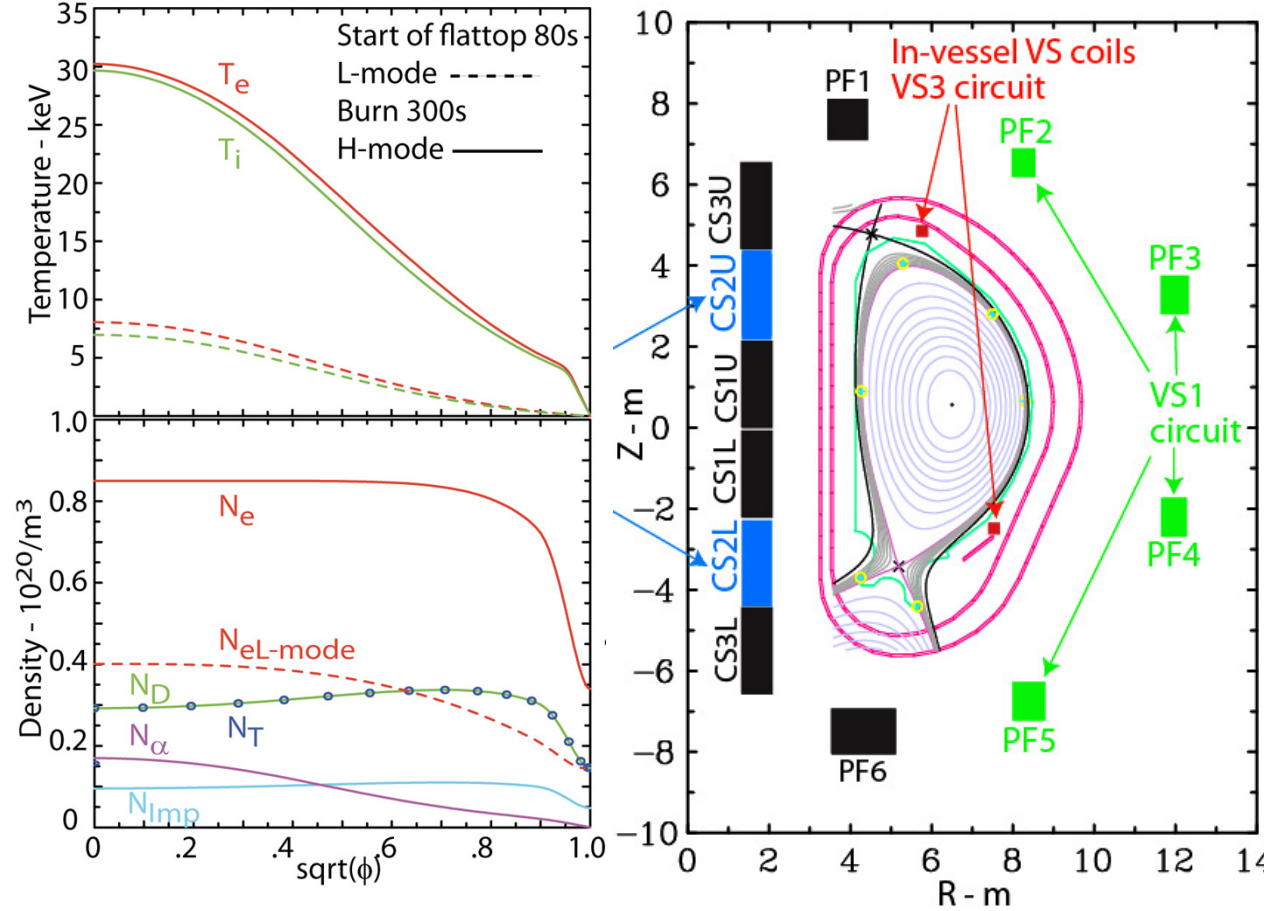
Tokamak Building & Assembly Hall (2020)

- (artist impression)



ITER Plasmas

- Q=10, 15MA, ELMy H-mode
 - 350MJ, 800m³
 - P_{heat} ~ 40-50MW
 - P_{fusion} ~ 400-500MW
 - flattop 300-500s
- High density
 - close to density limit
 - Pellet fuelling
 - Gas fuelling inefficient
- Detached divertor
 - P_{sol}=100MW
 - Impurity seeding (Ne)



- Also: hybrid and advanced scenarios

Magneto-Hydro-Dynamics (MHD)

- Theoretical model (H. Alfvén)
 - description of the plasma as a electrically conducting fluid embedded in a magnetic field
 - Combining Navier-Stokes fluid equations with (pre-) Maxwell's equations
 - Conservation of mass, momentum, energy and flux
 - 19th century physics model
 - applications: plasmas, solar physics, astrophysics, magnetosphere, dynamos, ...

- MHD in tokamak plasmas
 - refers to large scale, magnetic, instabilities in the plasma
 - Driven unstable by pressure gradients and current (gradients)
 - Limits global pressure and current and local pressure gradients
 - accurately described by the MHD model

MHD Instabilities in ITER

- Disruptions, Vertical Displacement Events (VDEs)
 - Sudden termination of plasma, leading to high heat loads and electro-mechanical forces
- Edge Localised Modes (ELMs)
 - Repetitive MHD instabilities at the plasma edge leading to enhanced erosion of the plasma facing components (divertor)
- Neoclassical Tearing Modes (NTMs)
 - Magnetic islands driven by plasma pressure, leading to a reduction in the energy confinement (reduced plasma performance)
- Sawteeth
- Resistive wall modes
- Fast particle driven modes (Alfven eigenmodes)

MHD Control in ITER

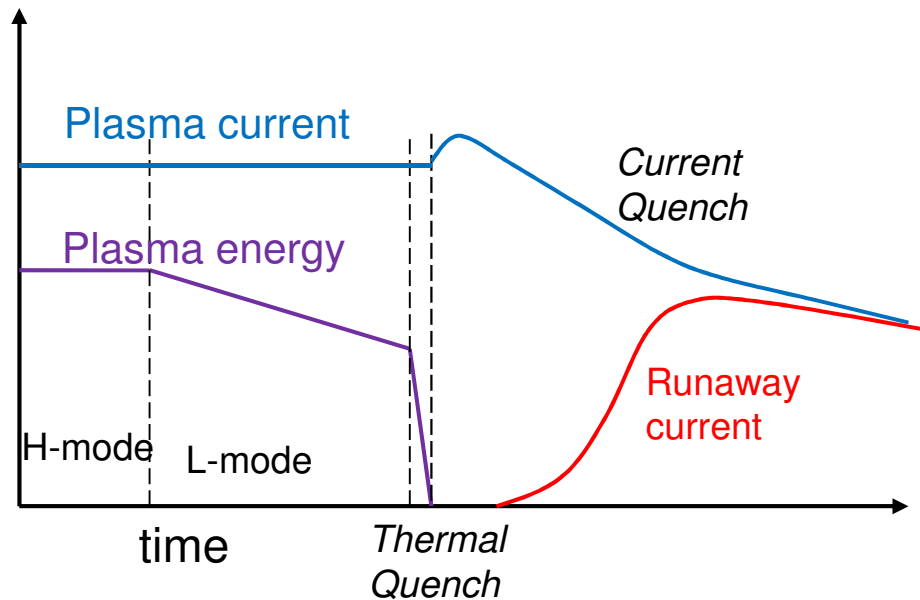
- Control of MHD instabilities in ITER is essential:
 - For machine protection:
 - Transient heat and mechanical loads due to **disruptions** and **ELMs** are acceptable in current tokamaks but need to be controlled / avoided / mitigated in ITER
 - For performance optimization:
 - In the baseline scenario, to obtain $Q=10$ (at $\beta_N \sim 1.8$ with large sawteeth) **Neoclassical Tearing modes** (NTM) need to be suppressed. **Sawteeth** may also need to be controlled.
 - In advanced steady state scenario at $Q=5$ (at $\beta_N \sim 3.0$), **Resistive Wall Modes** (RWM) will need to be stabilized to operate above the no-wall limit.
 - Fast particle driven **Alfven modes** may also need to be controlled

MHD Control Methods in ITER

- Disruptions
 - Massive gas injection, MGI (or massive material injection, MMI)
 - Valves, shattered pellet injector
- ELMs
 - Resonant Magnetic Perturbations (27 in-vessel coils)
 - Pellet injection (~40Hz)
 - Vertical kicks, <10MA
- Neoclassical tearing modes
 - Electron cyclotron Heating/current drive (ECRH/ECCD)
- Sawteeth
 - Ion cyclotron heating (ICRH), ECRH/ECCD
- Resistive wall modes
 - In-vessel coils

ITER Disruptions

- Sequence of events:



- Major Disruptions (MD)
 - locked modes
 - density limit
 - beta limit
- Vertical Displacement Event (VDE, up/down)
 - loss of vertical position control

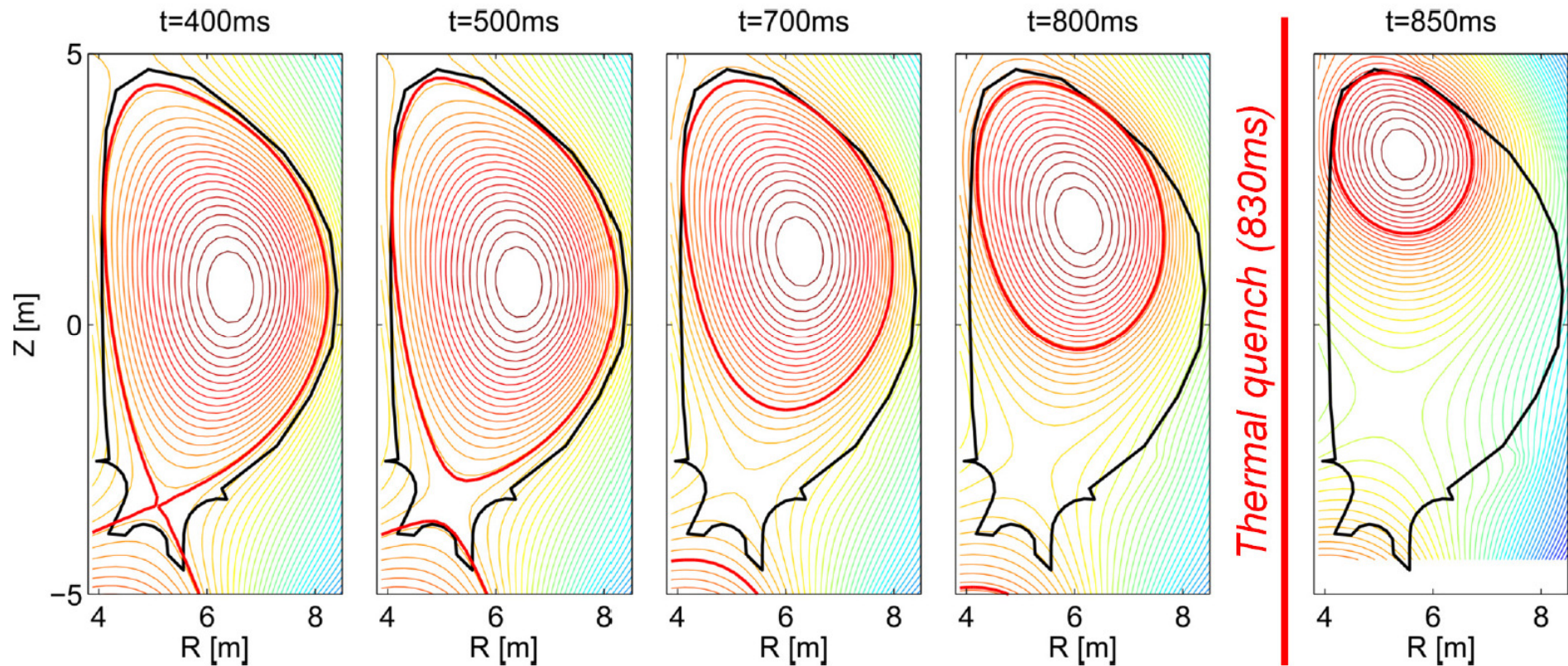
- The largest thermal loads occur Thermal Quench (TQ, 1-3ms)
- Major mechanical forces act on plasma facing components during Current Quench (CQ, 50-150ms)
- Runaway electrons will be generated during Current Quench

ITER Disruptions

- Heat loads:
 - Plasma thermal energy 350MJ, spread over 10-30m² , in 1-3 ms
 - Heat load \sim energy / (area x time^{0.5}) \sim 100-2000 MJ/m²s^{0.5}
 - Melting of Tungsten at 50 MJ/m²s^{0.5} (2700° C)
- Mitigation requires >90% radiation to spread power over walls
 - Injection of impurities (Ne, He, D₂, <1.8x10²⁴ particles, gas and/or shattered pellets)
 - How to obtain an homogeneous distribution of radiation?
 - Number and position of injection points, amounts, time delay
 - Peaking factor of radiation influenced by MHD activity

Vertical Displacement Events (VDE)

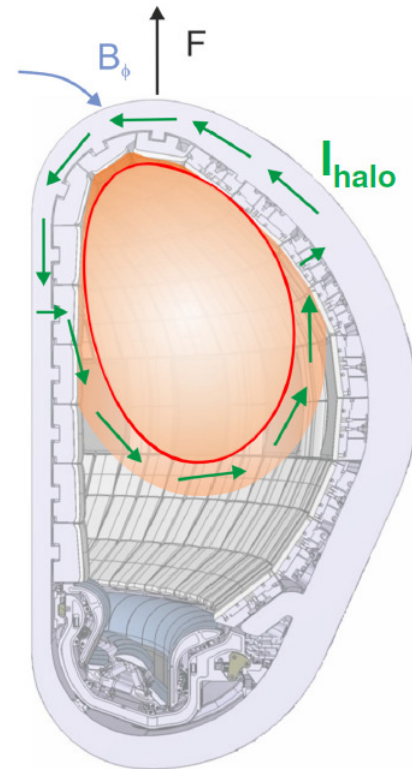
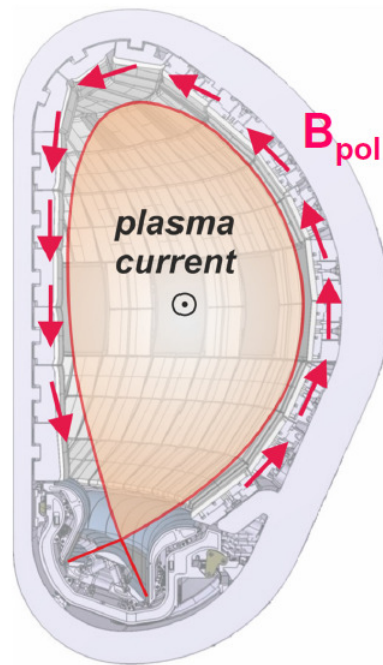
- Elongated plasmas are vertically unstable
 - feedback control keeps the plasma in place
 - In ITER, vertical displacements up to 16 cm can be controlled
- **Failure of control leads to a vertical displacement event (VDE)**
 - time scale determined by magnetic field diffusion through resistive wall



Wall Currents

- Movement of the plasma and decay of the plasma current cause a time variation of the magnetic field in the walls
 - Leading to induced currents in the metallic structures : **eddy currents**
- Direct contact of the plasma with the wall leads to a “current sharing”
 - Plasma current (partially) flows into the wall and back: **halo currents**
- Axi-symmetric VDEs lead to large vertical forces on the vessel
 - ~1000 tons in ITER

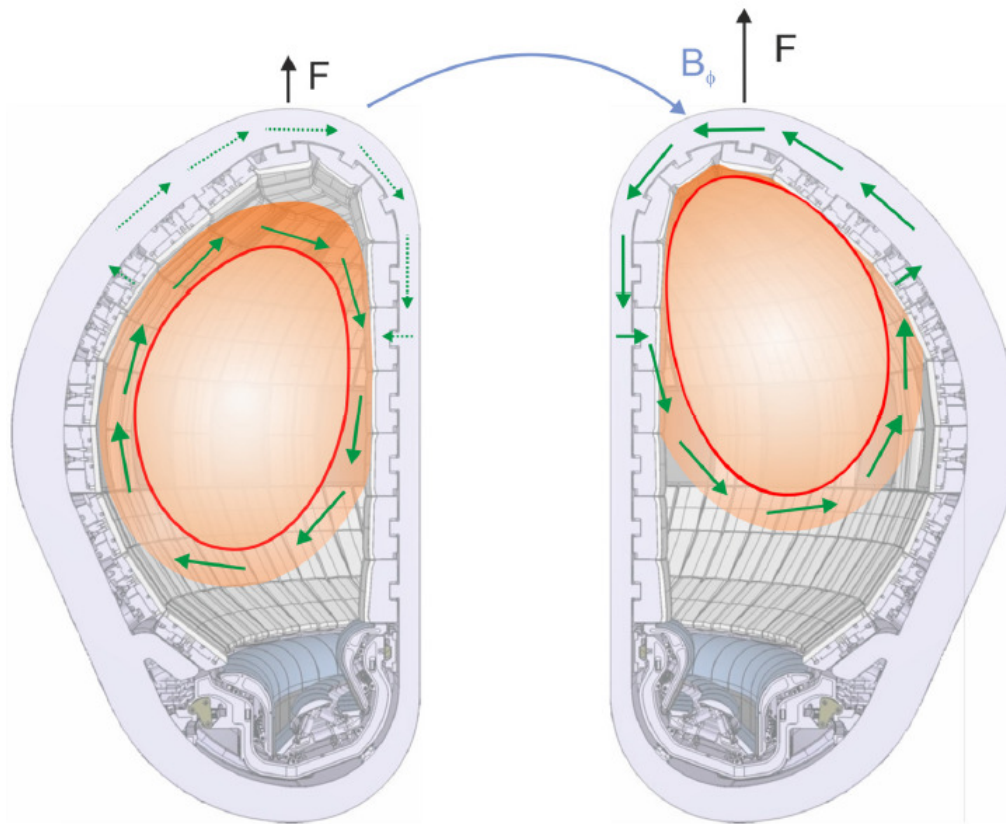
eddy currents
forces on the
in-vessel
structures



halo currents
vertical forces
on the vacuum
vessel

Asymmetric VDEs

- The shrinking of the plasma during a VDE can destabilize additional instabilities (kink modes)
 - Leading to asymmetric VDEs, vertical and sideways forces
 - Mode rotation may lead to resonant amplification, increasing the forces
- **Physics basis for expected behavior in ITER is high priority**



Runaway Electrons

- During the Thermal Quench (1-3ms) the electron temperature drops from ~ 10 keV to ~ 10 eV
 - Increase in plasma resistivity ($\sim T^{3/2}$) leads to a large electric field
 - Electric field \sim resistivity \times current density ~ 20 V/m
- Friction forces decrease with increasing electron energy
 - Electric field accelerates electrons to relativistic speeds
 - Runaway electrons : high energy electron beam
- ITER parameters: $I_{RE} < 10$ MA, electron energy ~ 20 MeV
 - Beam energy ~ 10 MJ (kinetic), 200MJ (magnetic)
 - Can lead to deep melting of Be first wall
 - What fraction of magnetic energy is lost?
 - Runaway electrons must be mitigated ($I_{RE} < 2$ MA)
 - Massive gas injection

MHD Equations

- Resistive MHD equations:
 - Evolution of density, velocity, temperature and magnetic field

$$\partial_t \rho = -\nabla \cdot (\rho \mathbf{v}) + \nabla \cdot (D \nabla \rho) + S_\rho,$$

$$\rho \partial_t \mathbf{v} = -\rho \mathbf{v} \cdot \nabla \mathbf{v} - \nabla(\rho T) + \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{v},$$

$$\partial_t T = -\mathbf{v} \cdot \nabla T - (\gamma - 1) T \nabla \cdot \mathbf{v} + \nabla \cdot (\bar{K} \nabla T) + S_T,$$

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{J}),$$

$$\nabla \cdot \mathbf{B} = 0$$

- Using vector potential:

$$\partial_t \mathbf{A} = \mathbf{v} \times (\nabla \times \mathbf{A}) - \eta (\nabla \times \nabla \times \mathbf{A})$$

Reduced MHD

- Formulation using electric potential (u) and magnetic flux (ψ):

$$\vec{v} = -R\vec{\nabla}u(t) \times \vec{e}_\phi + v_\parallel(t) \vec{B} \qquad \vec{B} = \frac{F_0}{R} \vec{e}_\phi + \frac{1}{R} \vec{\nabla} \psi(t) \times \vec{e}_\phi$$

Poloidal flux $\frac{1}{R^2} \frac{\partial \psi}{\partial t} = +\eta \nabla \cdot \left(\frac{1}{R^2} \nabla_\perp \psi \right) - \frac{1}{R} [u, \psi] - \frac{F_0}{R^2} \partial_\phi u$

parallel momentum $\vec{B} \cdot \left(\rho \frac{\partial \vec{v}}{\partial t} = -\rho (\vec{v} \cdot \vec{\nabla}) \vec{v} - \vec{\nabla}(\rho T) + \vec{J} \times \vec{B} + \mu \Delta \vec{v} \right)$

poloidal momentum $\vec{e}_\phi \cdot \nabla \times \left(\rho \frac{\partial \vec{v}}{\partial t} = -\rho (\vec{v} \cdot \vec{\nabla}) \vec{v} - \vec{\nabla}(\rho T) + \vec{J} \times \vec{B} + \mu \Delta \vec{v} \right)$

Temperature $\rho \frac{\partial T}{\partial t} = -\rho v \cdot \nabla T - (\gamma - 1) \rho T \nabla \cdot v + \nabla \cdot (K_\perp \nabla_\perp T + K_\parallel \nabla_\parallel T) + S_T$

Density $\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v}) + \nabla \cdot (D_\perp \nabla_\perp \rho) + S$

Boundary Conditions

- Plasma-wall interaction:
 - Wall is a strong pump for plasma
 - Fluid boundary conditions at the sheath entrance:

- Parallel velocity:

$$\frac{\vec{v} \cdot \vec{B}}{|B|} \geq c_s \qquad \vec{v} \cdot \vec{n} \geq \frac{\vec{B} \cdot \vec{n}}{|B|} c_s$$

- Parallel energy flux:

$$nT v_{\parallel} + K_{\parallel} \nabla_{\parallel} T = \gamma_{sh} K_{\parallel} \nabla_{\parallel} T$$

- Potential:

$$e\phi = -T_e \ln \left(\frac{m_i}{2\pi m_e} \right)^{\frac{1}{2}}$$

- Magnetic field:

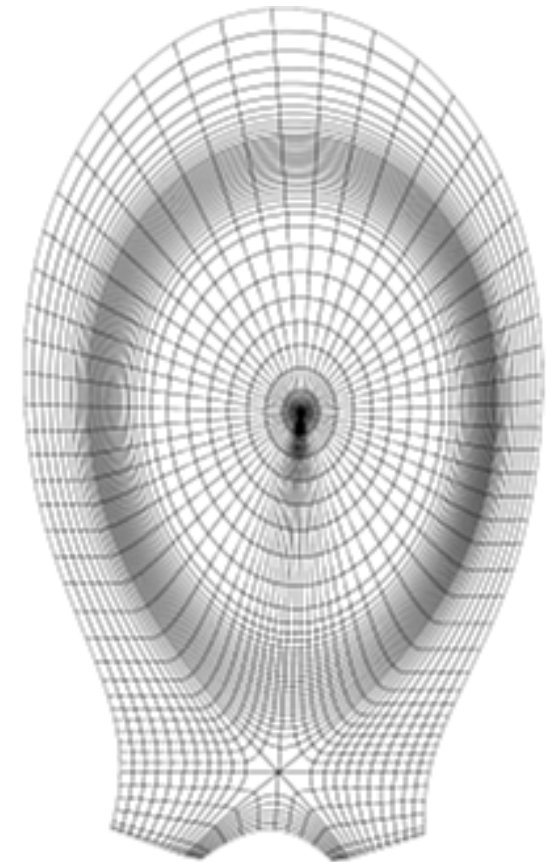
- Fixed boundary (ideal wall) : $\delta \vec{B} \cdot \vec{n} \Big|_{wall} = 0$

- Free boundary (resistive wall, vacuum, coils)

- Continuity of total magnetic (electric) field

Non-linear MHD code JOREK

- Initial motivation: non-linear MHD simulations of Edge Localised Modes
 - Reduced MHD in toroidal geometry
 - Whole domain inside vacuum vessel, including open and closed field lines, x-point(s)
 - Divertor boundary conditions
 - Long time scales
- Evolving towards general MHD simulation code
 - Reduced and full (extended) MHD models
 - Including interaction with resistive walls, coils
 - JOREK team
- Characteristics:
 - C^1 iso-parametric Bezier finite elements (refinement)
 - real Fourier series in toroidal direction
 - Fully implicit time evolution
 - PastiX sparse matrix solver
 - Parallelisation MPI-OPENMP
 - 256 - 2048 cpus



Bezier Curves (1D)

- Bezier curves were defined by Pierre Bézier (1910–1999) at Renault in 1960s to describe parametrised curved surfaces
 - widely used in CAD-CAM, font definitions etc., OpenGL

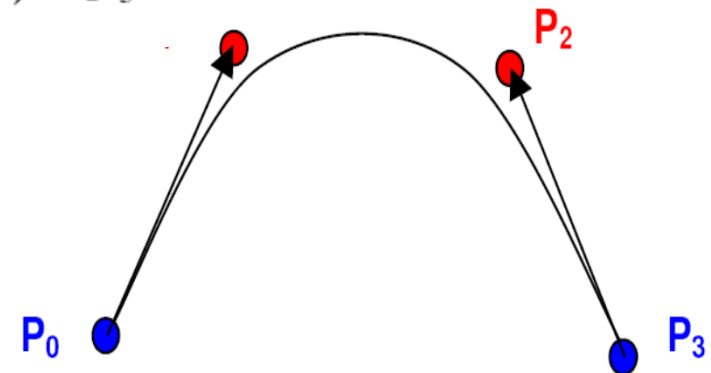
$$\vec{B}(t) = \sum_{k=0}^N \vec{p}_k \frac{N!}{k!(N-k)!} t^k (1-t)^{N-k} \quad 0 < t < 1$$

- Cubic Bezier curve defined by 4 control points:

$$\vec{B}(t) = \vec{p}_0 (1-t)^3 + 3\vec{p}_1 t (1-t)^2 + 3\vec{p}_2 t^2 (1-t) + \vec{p}_3 t^3$$

- (naturally) Isoparametric:

$$\vec{p}_k = (x, y, \psi)_k$$

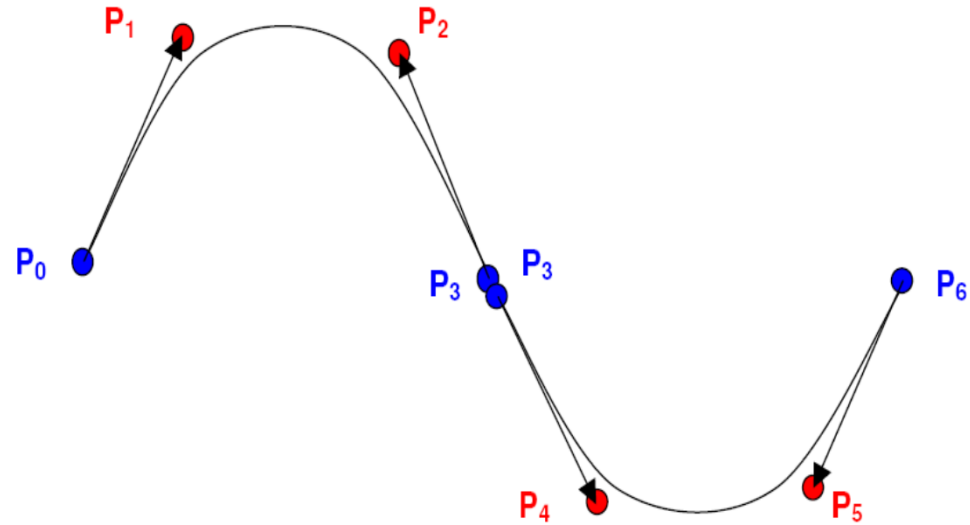


C1 Continuity

- Continuity requirement for a 1D Bezier curve (in 2D or 3D space)

– control points lie on a line:

$$(\vec{p}_2 - \vec{p}_3) = \alpha(\vec{p}_4 - \vec{p}_3)$$



- Physical variables and their first derivative are continuous in real space but not in the local coordinate

– as opposed to cubic Hermite finite elements $\alpha = 1$

- Additional freedom allows local mesh refinement

Bezier Elements

- Redefine Bezier curves in terms of quantities defined at the nodes:

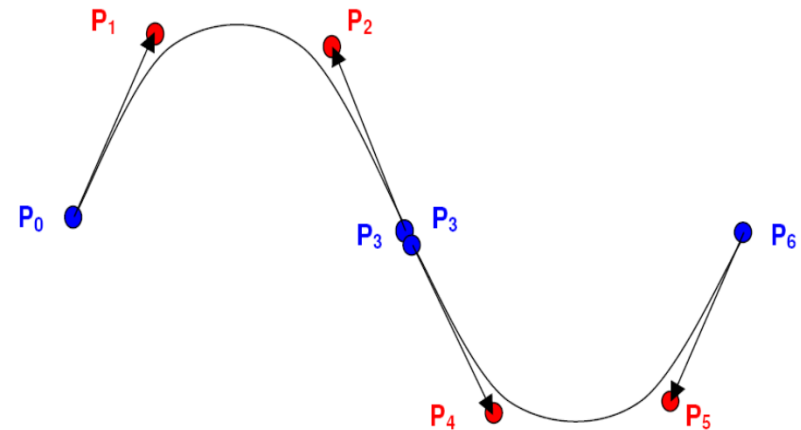
- Scale factors (property of an element)

$$h_{32} = \|\vec{p}_2 - \vec{p}_3\| \quad h_{34} = \|\vec{p}_4 - \vec{p}_3\|$$

- Unit vectors u_i (property of a node)

$$\vec{u}_3 = \frac{(\vec{p}_2 - \vec{p}_3)}{h_{32}} = -\frac{(\vec{p}_4 - \vec{p}_3)}{h_{43}}$$

- Automatic C1 continuity



- Physical variables (unknowns):

$$\vec{p} = (R, Z, \psi)$$

$$h_{23} = \sqrt{(R_3 - R_2)^2 + (Z_3 - Z_2)^2}$$

$$\vec{u}_i = \begin{pmatrix} \delta R_i \\ \delta Z_i \\ \delta \psi_i \end{pmatrix}$$

JOREK : time stepping

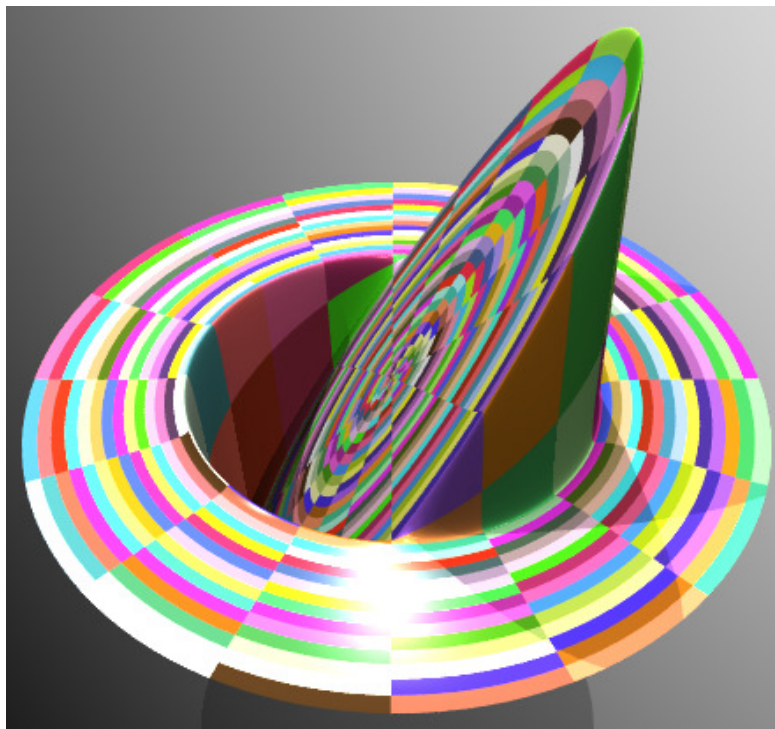
- Fully implicit time evolution allows large time steps:
 - All variables implicitly updated in one step
 - time step independent of grid size
 - 0.5 - 5 Alfven times for ELM simulations to 10.000 Alfven times for slow growing tearing modes
- Linearised Crank-Nicholson scheme (or Gear's scheme):

$$\frac{\partial A(\vec{y})}{\partial t} = B(\vec{y}) \quad \Rightarrow \quad \left(\frac{\partial A(\vec{y}_n)}{\partial y} - \frac{1}{2} \delta t \frac{\partial B(\vec{y}_n)}{\partial y} \right) \delta \vec{y} = B(\vec{y}_n) \delta t$$

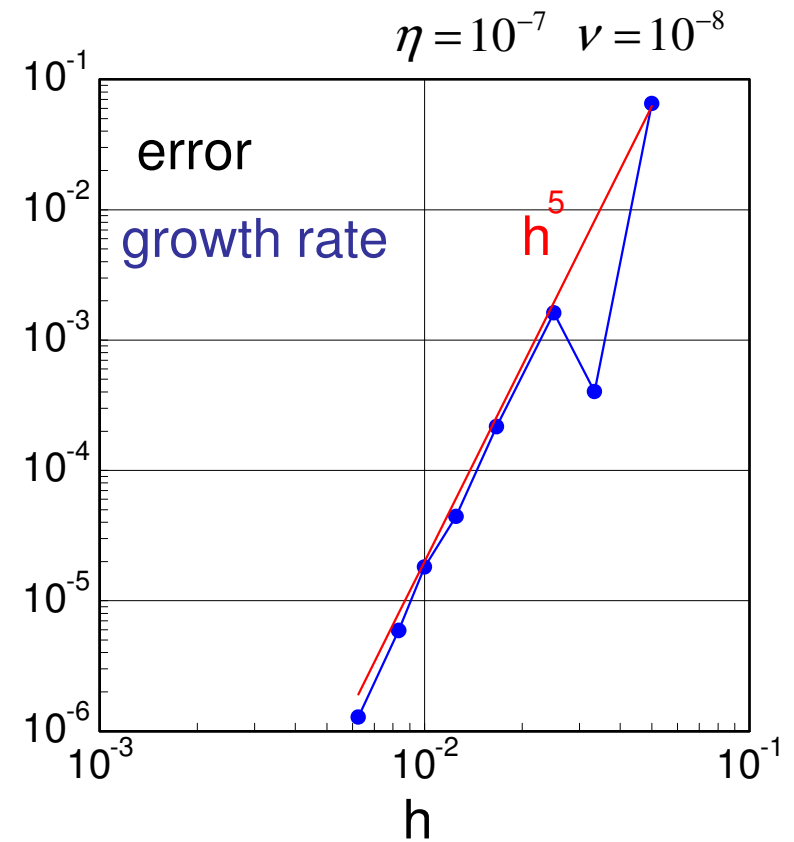
- Leads to very large systems of equations to be solved at every time step
 - sparse matrix solved using iterative method (GMRES)
 - Preconditioning matrices: one for each toroidal harmonic
 - solved using **PastiX** parallel sparse matrix solver
 - recalculated when GMRES iterations too large
 - DOFs $\sim 1 \times 10^7$

Convergence

- Verification on resistive MHD instabilities:
 - Linear growth rate $n=1$ resistive internal kink mode :
 - correct scaling error $\sim h^5$

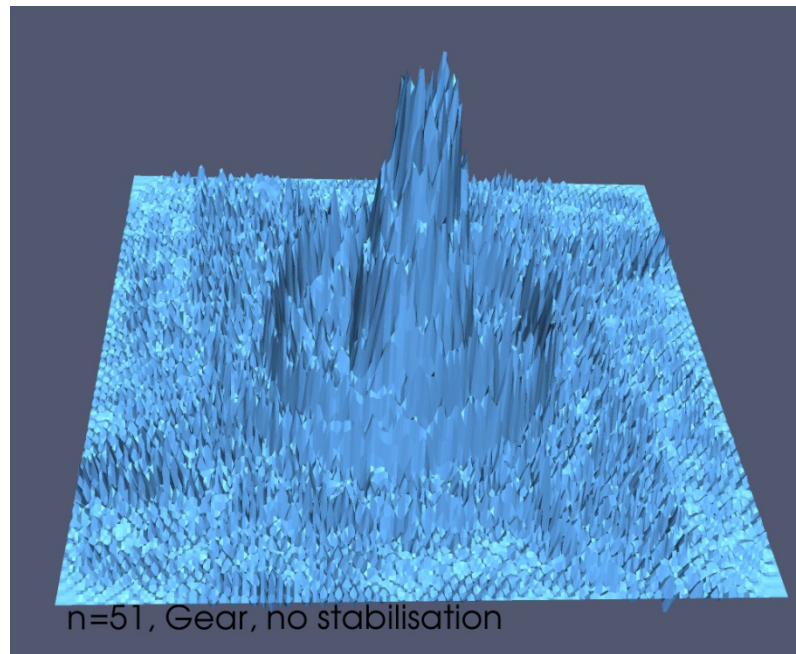


m=1 perturbation internal kink mode



Stabilisation

- Large (and non resolved) flows may lead to spurious oscillations
 - test case vortex mixing, vorticity equation (model001)



$$\frac{\partial w}{\partial t} = -[w, u] + \nu \nabla^2 w$$

- Use finite element stabilisation techniques
 - **Taylor-Galerkin (TG2, TG3)**
 - Galerkin Least Square
 - SUPG (Stream-upwind Petrov-Galerkin)
 - ...

Taylor-Galerkin Stabilisation

- Use higher order time derivatives:

$$\frac{\partial w}{\partial t} = -[w, u] + \nu \nabla^2 w$$

$$w^{n+1} = w^n + \delta t \frac{\partial w^n}{\partial t} + \frac{1}{2} \delta t^2 \frac{\partial}{\partial t} \frac{\partial w^n}{\partial t} + \frac{1}{6} \delta t^3 \frac{\partial^2}{\partial t^2} \frac{\partial w^n}{\partial t}$$

$$\frac{w^{n+1} - w^n}{\delta t} = -[w^n, u^n] + \nu \nabla^2 w^n + \frac{1}{2} (\delta t) \frac{\partial}{\partial t} (-[w^n, u^n])$$

$$\frac{\partial}{\partial t} (-[w^n, u^n]) \approx [-[w^n, u^n], u^n]$$

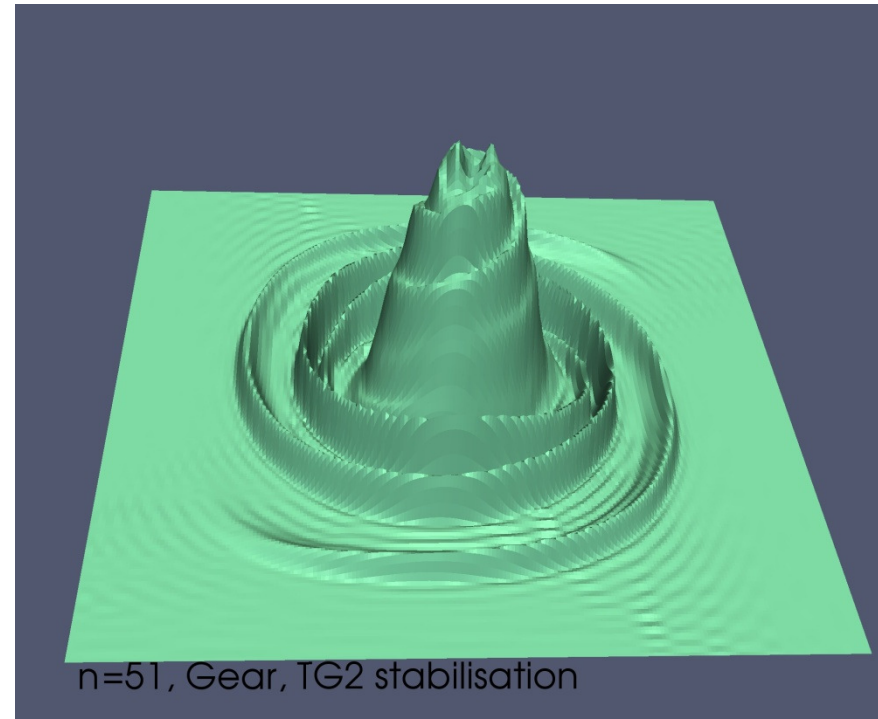
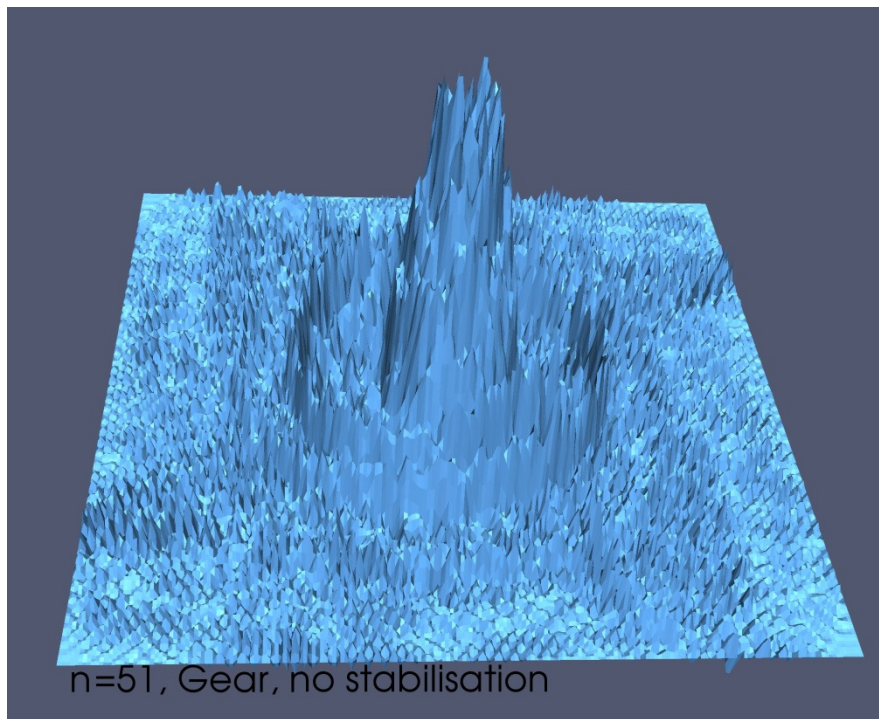
- Weak form (implicit TG2):

$$v^* \frac{\delta w}{\delta t} = -[w^n, u^n] + \nu \nabla^2 w^n - \frac{1}{2} [\delta w, u^n] - \frac{1}{2} [w^n, \delta u] + \frac{1}{2} \nu \nabla^2 \delta w$$

$$+ \frac{1}{4} \delta t [w^n, u^n] [v^*, u^n] + \frac{1}{8} \delta t [w^n, u^n] [v^*, \delta u] + \frac{1}{8} \delta t [\delta w, u^n] [v^*, u^n] + \frac{1}{8} \delta t [w^n, \delta u] [v^*, u^n]$$

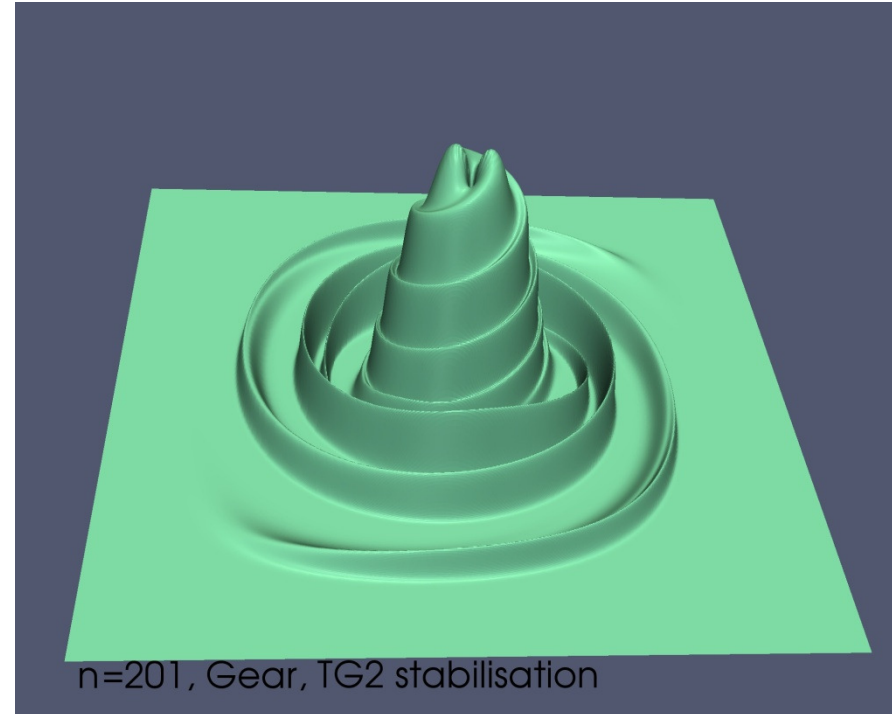
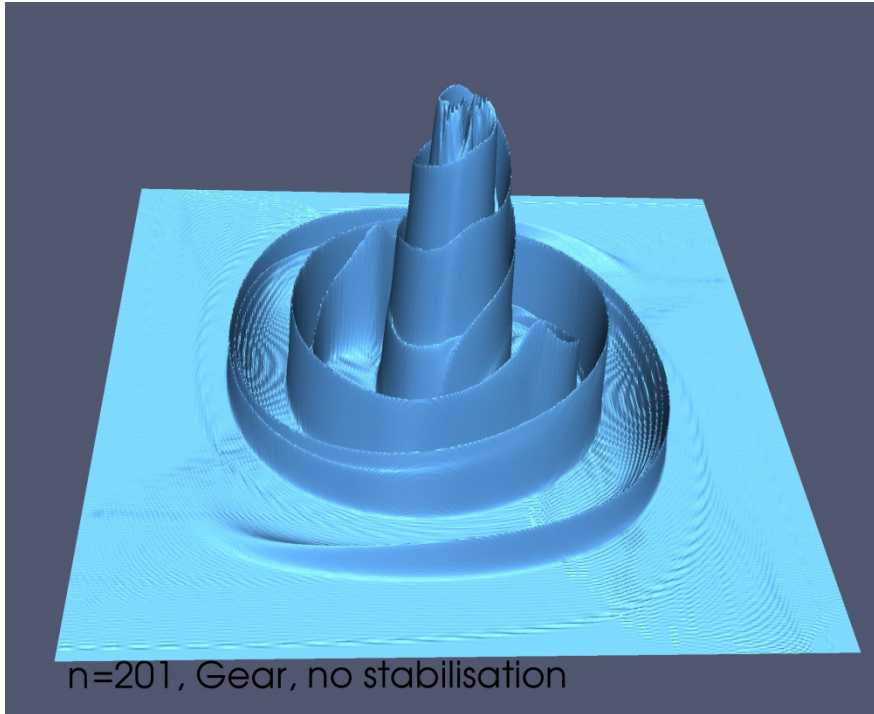
TG2 Stabilisation

- Stabilisation gives large improvement for this test case



TG2 Stabilisation

- At high resolution TG2 stabilisation may be too strong



- TG2 stabilisation implemented (and used) in JOEREK
- Also hyper-diffusion terms

JOREK Activities

- Disruption simulation, massive gas injection
 - Alexandre Fil (CEA), Eric Nardon (CEA)
- ELM simulations, ELM control RMPs
 - Francois Orain (CEA), Marina Becoulet (CEA), Jorge Morales (CEA), Stanislas Pamela (UKAEA), Matthias Holzl (IPP), Guido Huijsmans (ITER)
- ELM control, pellets
 - Shimpei Futatani (Barcelona)
- ELM control, QH-mode
 - Feng Liu (ITER)
- Tearing mode control, current drive
 - Egbert Westerhof (FOM), Jane Pratt (UK)
- VDEs
 - Matthias Holzl (IPP), Eric Nardon (CEA), Ksenia Aleynikova (Moskou)
- Numerical schemes
 - Boniface N’Konga (Nice), Emmanuel Franck (IPP), Ahmed Ratnani (IPP)
- Equations
 - ...

Plasma-Wall-Vacuum

- Plasma
 - Reduced or full MHD

$$E = -\nabla\varphi - \partial A/\partial t$$

- Conducting structures (coils)
 - Some in contact with plasma

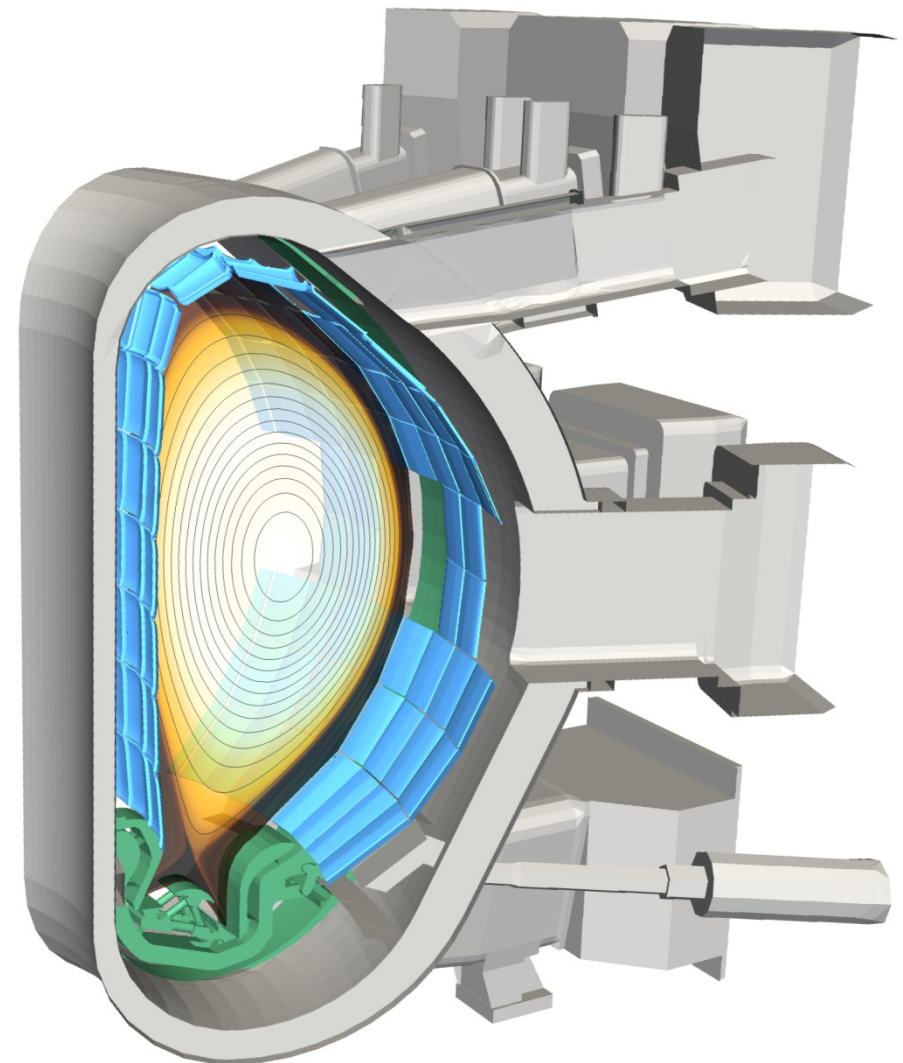
$$J = \sigma E \quad E = -\nabla\varphi - \partial A/\partial t$$

$$\nabla \times \nabla \times A = -\sigma(\nabla\varphi + \partial A/\partial t)$$

- Vacuum

$$\nabla \cdot B = 0$$

$$\nabla \times B = \nabla \times (\nabla \times A) = 0$$



Plasma-Wall-Vacuum

- JOREK-STARWALL

- Plasma: Ohms Law: $\frac{\partial \psi}{\partial t} = \eta R^2 \nabla \cdot \left(\frac{1}{R^2} \right) \nabla_{\perp} \psi - \vec{B} \cdot \nabla \Phi$

- Weak form:

$$\begin{aligned} \int \frac{1}{R^2} \psi^* \frac{\partial \psi}{\partial t} dV &= \int \psi^* \eta \nabla \cdot \left(\frac{1}{R^2} \nabla_{\perp} \psi \right) - \frac{1}{R^2} \psi^* \vec{B} \cdot \nabla \Phi dV \\ &= \int \frac{1}{R^2} \nabla (\eta \psi^*) \cdot \nabla_{\perp} \psi dV - \oint \psi^* \eta \frac{1}{R^2} (\nabla_{\perp} \psi \cdot \vec{n}) dS - \int \frac{1}{R^2} \psi^* \vec{B} \cdot \nabla \Phi dV \end{aligned}$$

- Vacuum solution (STARWALL) yields relation tangential to normal magnetic field at the computational boundary:

$$\nabla \psi \cdot \vec{n} = M (\nabla \psi \times \vec{n} \cdot \vec{e}_{\phi})$$

- Insert in weak form:

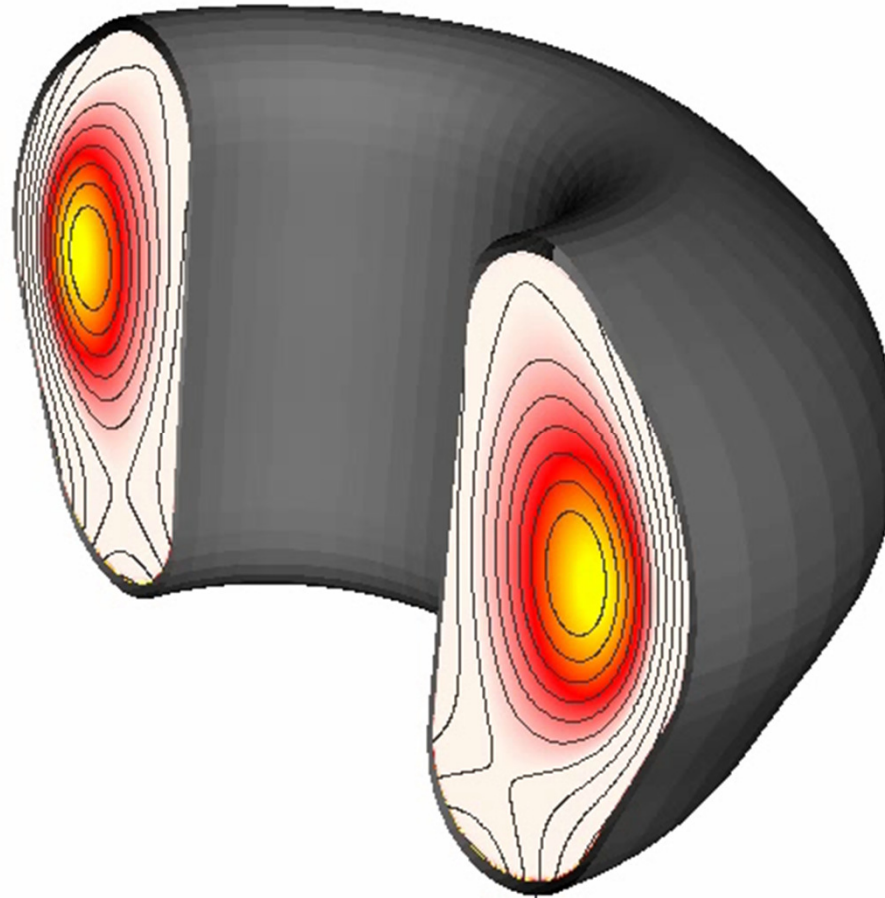
- Natural (Neumann) boundary condition will automatically be satisfied

$$\int \frac{1}{R^2} \psi^* \frac{\partial \psi}{\partial t} dV = \int \frac{1}{R^2} \nabla (\eta \psi^*) \cdot \nabla_{\perp} \psi dV - \oint \psi^* \eta \frac{1}{R^2} M (\nabla \psi \times \vec{n} \cdot \vec{e}_{\phi}) dS - \int \frac{1}{R^2} \psi^* \vec{B} \cdot \nabla \Phi dV$$

- Eddy currents only, new scheme for halo currents is required

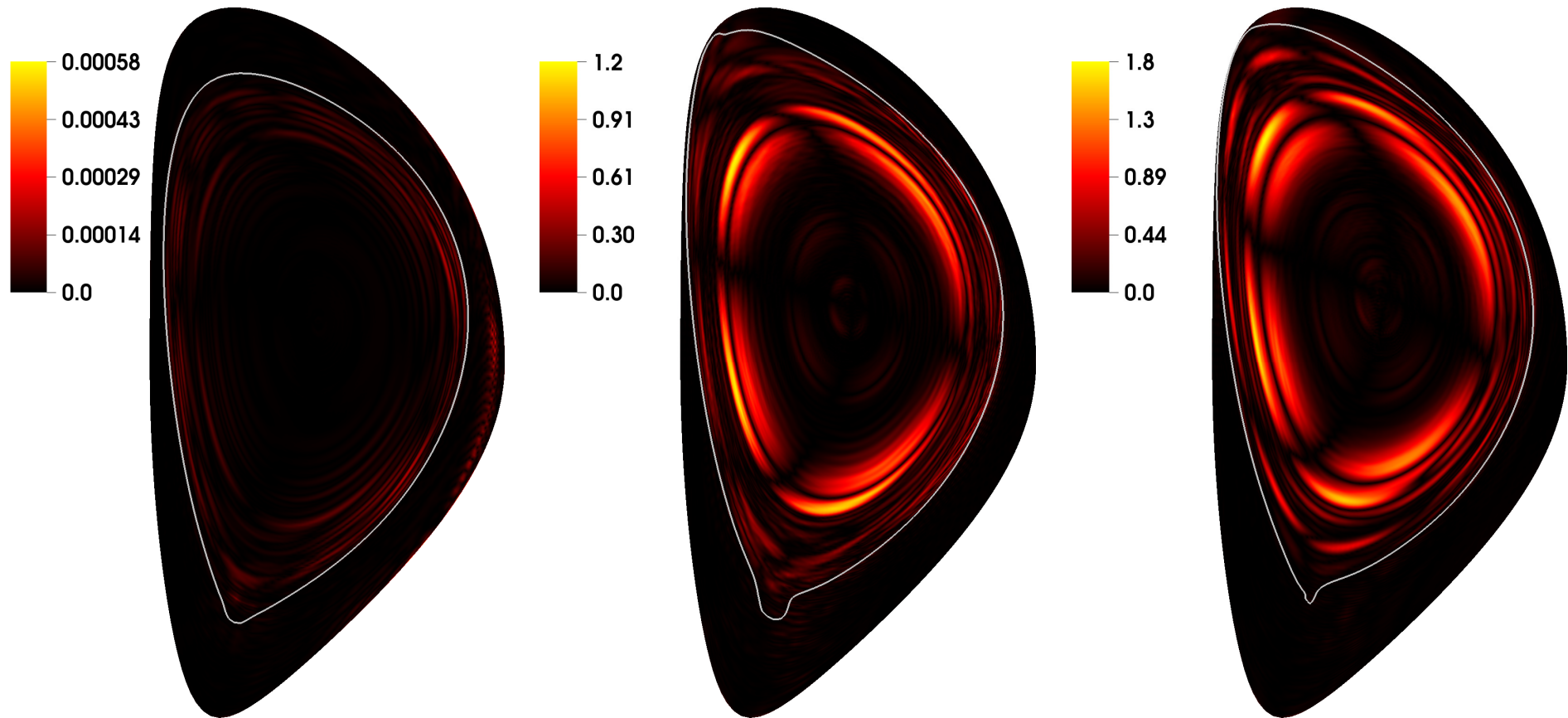
Simulation of VDEs (*Matthias Holzl, IPP*)

- VDE simulation in ITER (JOEAK)



3D VDE (*Ksenia Aleynikova, MIPT*)

- First 3D VDE Simulation in ITER geometry:
 - time scale ~5 ms



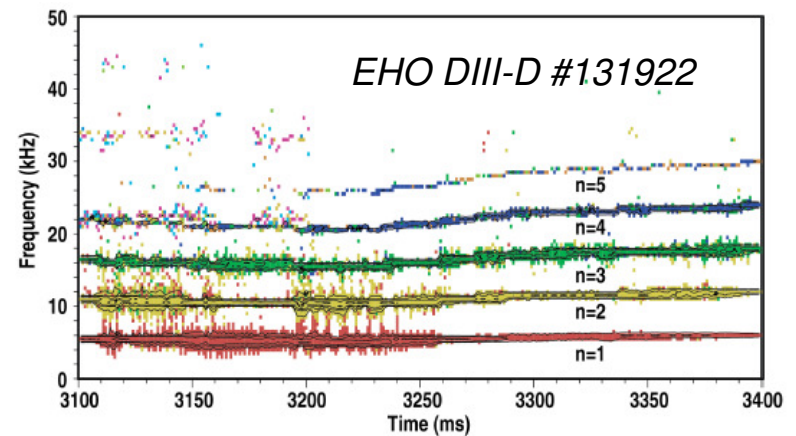
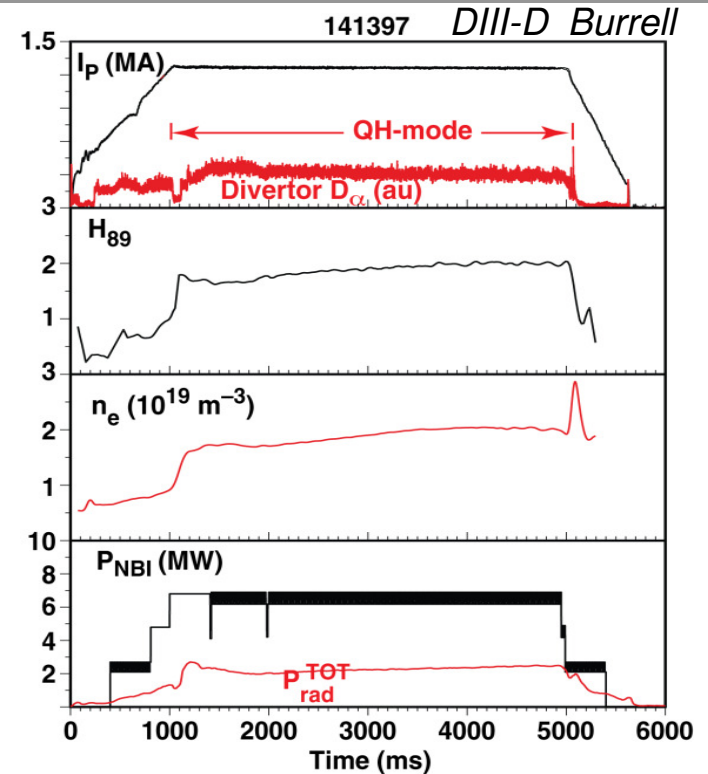
$n=1$ current perturbation

QH mode Plasmas

- ITER H-mode scenario is expected have have large Edge Localised Modes (ELMs) leading to large transient heat loads
- ELMs will be controlled using magnetic perturbations (RMPs) or D₂ pellet injection
- Possible alternative : QH-mode plasma
 - H-mode confinement
 - ELM-free (no transient divertor heat loads)
 - Edge Harmonic Oscillation (EHO) causes density loss and steady state H-mode

Is QH-mode be a viable option for ITER?

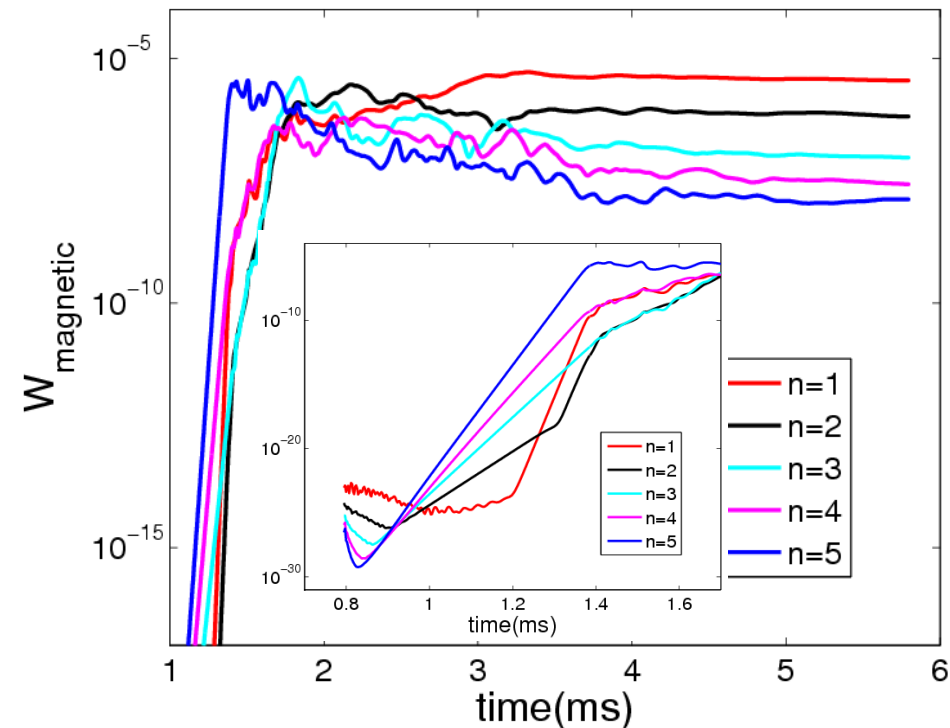
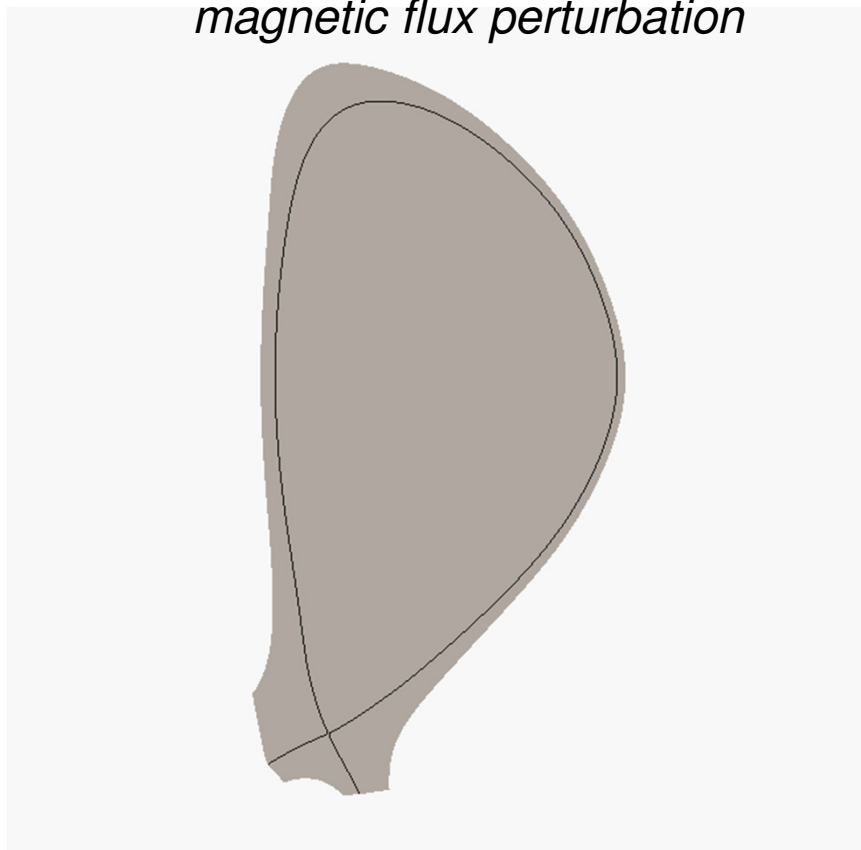
- Validation of Non-linear MHD simulations on DIII-D tokamak
- Extrapolation to ITER



MHD Simulations of QH-mode (*Feng Liu, ITER*)

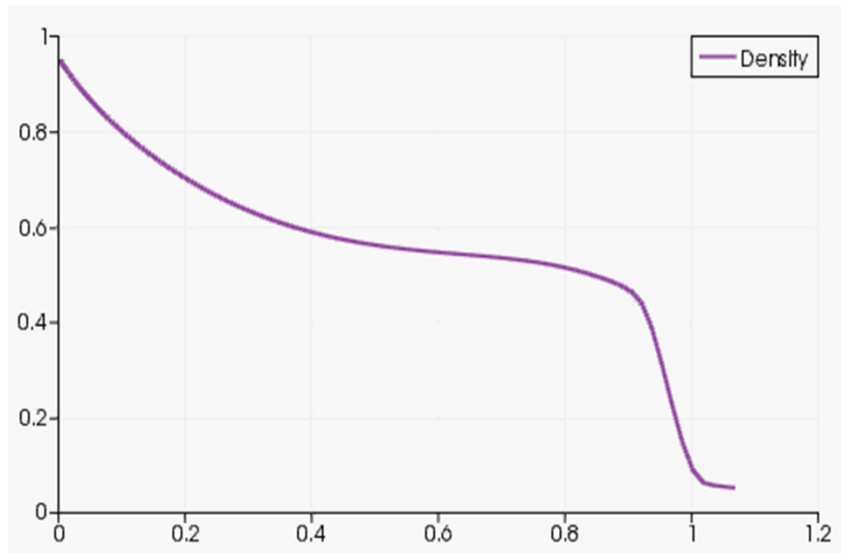
- Simulation starting from initial state from DIII-D QH-mode plasma shows a growing external kink instability
 - External kink mode saturates non-linearly into a new quasi-stationary 3D state

magnetic flux perturbation

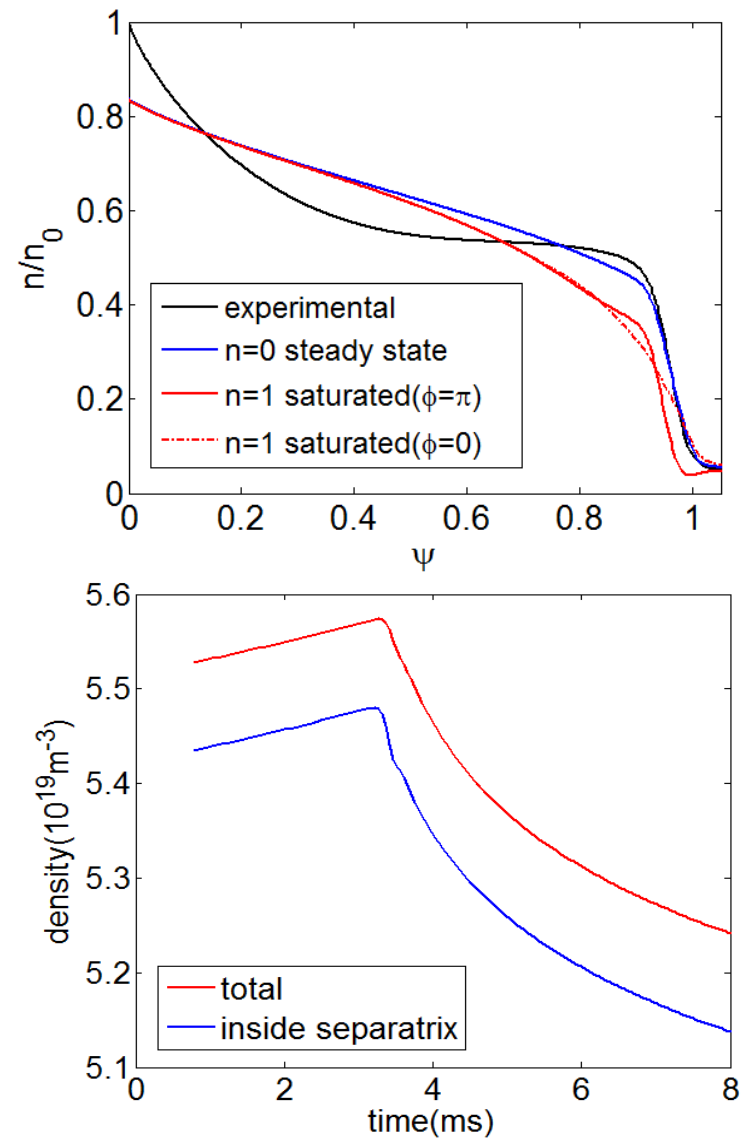


Density losses due to external kink mode

density profile time evolution:



- Saturated Kink mode leads to increased density losses:
 - Pedestal density reduced by 25%
 - Total density by ~10%
- Temperature not affected
- Qualitative agreement with experiment

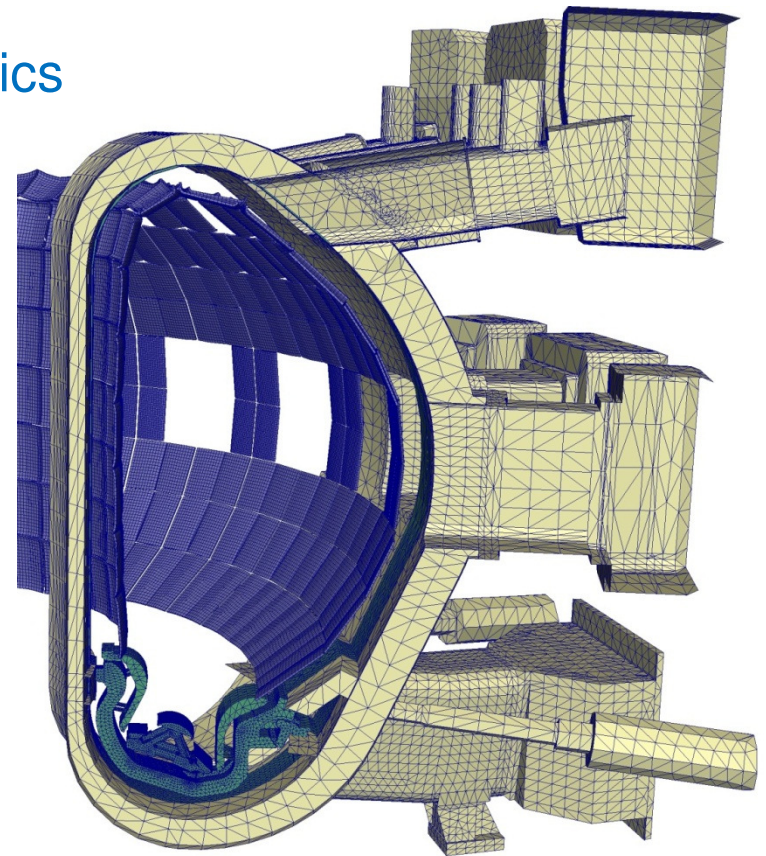


Future directions

- Aim: ITER simulation in realistic geometry/plasma parameters
 - Disruption (VDE) simulations and control methods
 - ELM and ELM control in ITER plasmas
- Extended MHD models
 - Reduced or full MHD (gyrofluid)
 - Comparison of models
- Including radiating impurities
 - Fluid or discrete particles
- Including halo currents
- Interaction with particles
 - Runaway electrons
 - Fast ions (fusion alphas, heating)
 - impurities
- Numerics
 - 3D FEM, splines
 - Solvers, scalability

Conclusions

- High priority ITER issues are related to MHD instabilities (Disruptions, ELMs and their control)
 - Extrapolation from current experiments to ITER requires validation of MHD simulations (i.e. comparison of simulations with experimental observations)
 - MHD simulations also important for physics
- MHD simulations need to be more and more realistic
 - Extended MHD models
 - Exact geometry (use CAD models) interaction with conducting structures
 - Description of detached divertor



MGI Disruption (A. FIL, CEA)

- Thermal Quench simulation with Massive Gas Injection (JOREK)

