Phase transitions and critical phenomena: the (micro)physicist viewpoint

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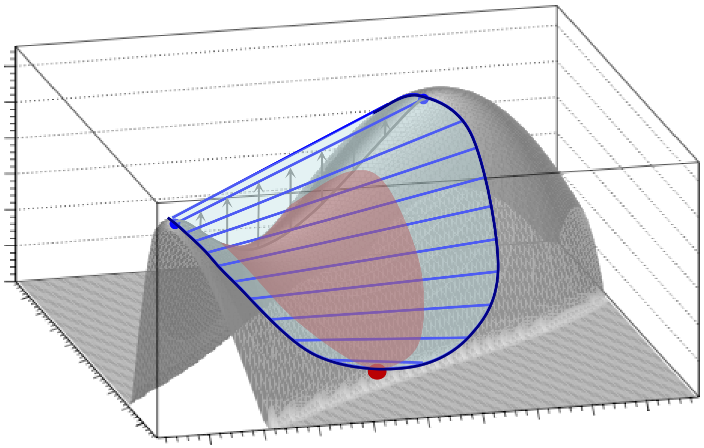


Menu

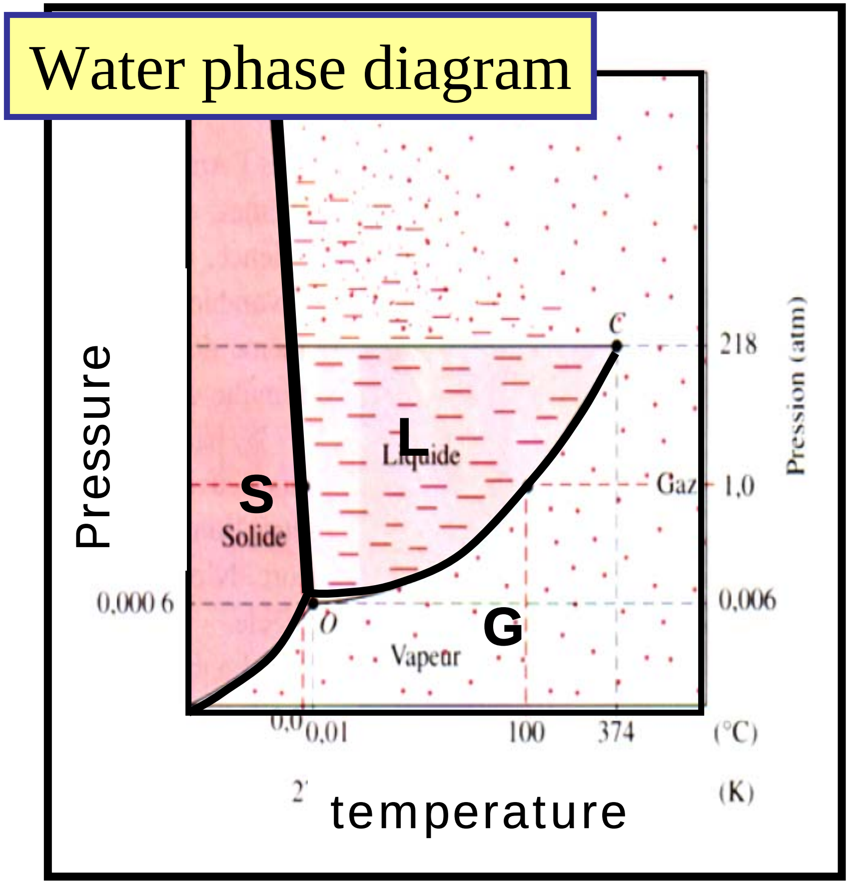
1. Phase transitions in physical systems
   – Concepts and definitions
   – Classical theory: Landau
2. First order transitions
   – Phase coexistence
   – Extension to many densities
3. Second order and critical phenomena
   – Divergence of the correlation length
   – Scale invariance and renormalization
4. Extension to finite systems
   – Precursor of phase transitions
   – Thermodynamic anomalies
   – Frustration and ensemble inequivalence

1 - Phase transitions in physical systems

- Concepts and definitions

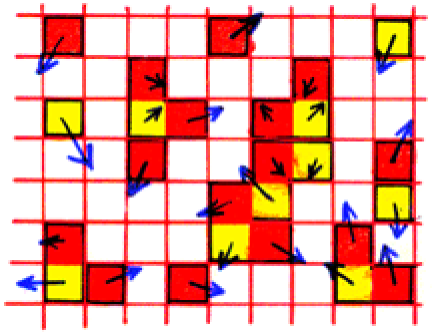
What is a phase transition?

- A small variation of a control parameter induces a dramatic qualitative modification of the system properties.



A paradigm in statistical mechanics: lattice models

- Site variables: n_i, $\mathbf{p}_i$
- Couplings among sites ε_{ij}



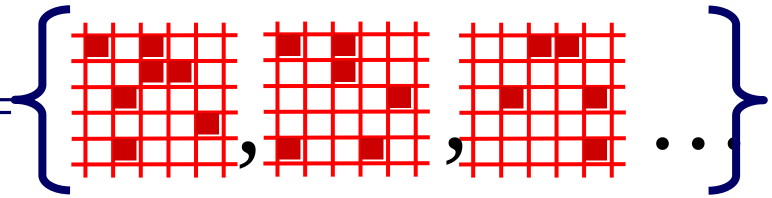
configuration or microstate

Ising , Heisenberg, Lattice Gas, Potts, spherical…...

- solid state physics: ferro-para transition, spin glasses
- molecular systems: liquid-gas transition
- site and bond percolation: critical phenomena
- Lattice QCD: deconfinement phase transition …..

A small dictionary

- Statistical ensemble (**macrostate**): exhaustive ensemble of all microstates satisfying some given conditions (**constraints**)
- Observable (**extensive variable B**) : expectation value of an operator on a microstate
- Conjugated field (**intensive variable β)** : what is shared among microstates
- **Control parameter** : constraint characterizing the ensemble (intensive variable or conservation law)
- **Order parameter** : average extensive variable changing among different phases

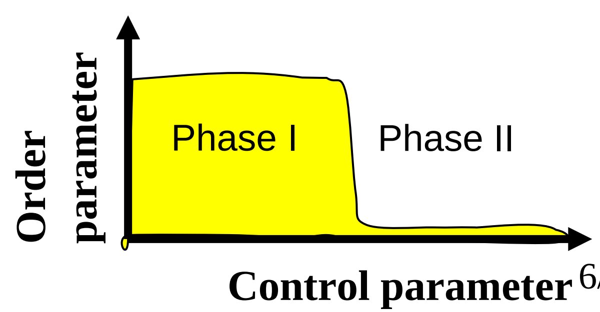
{k}= 

$$H(k)=\sum_{i,j\in k}\varepsilon_{ij}n_i n_j \quad \text{energy}$$

$$N(k)=\sum_{i\in k}n_i \quad \text{particle number}$$

$$T = f(\langle H(k)\rangle_k) \text{ Temperature}$$

$$\mu = f(\langle M(k)\rangle_k) \text{ Chemical potential}$$



Information theory in a nutshell

- Controlled variables: $\beta_\ell : \ell = 1,\ldots,r \quad A_j^0 : j = 1,\ldots,m$
  (constraints)
- Statistical entropy: $S = -\sum_{(n)} p^{(n)} \log\, p^{(n)} \quad I = -S$
  (lack of Information)

An equilibrium is the statistical ensemble of microstates which maximizes the statistical entropy under a given set of constraints

- Microstate probability: $p^{(n)} = Z^{-1} e^{-\sum_\ell \beta_\ell B_\ell^{(n)}} \prod_j \delta\left(A_j^{(n)} - A_j^0\right)$
- Partition sum: $Z(\beta, A) = \sum_{(n:A_j^{(n)}=A_j^0 \forall j)} e^{-\sum_\ell \beta_\ell B_\ell^{(n)}}$
  (sum on the physical realizations of the system)
- Equations of state: $< B_\ell > = -\partial_{\beta_\ell} \log Z(\vec{\beta}) \quad \alpha_\ell = \partial_{\langle A_\ell \rangle} S$
  (relations between extensive and intensive)

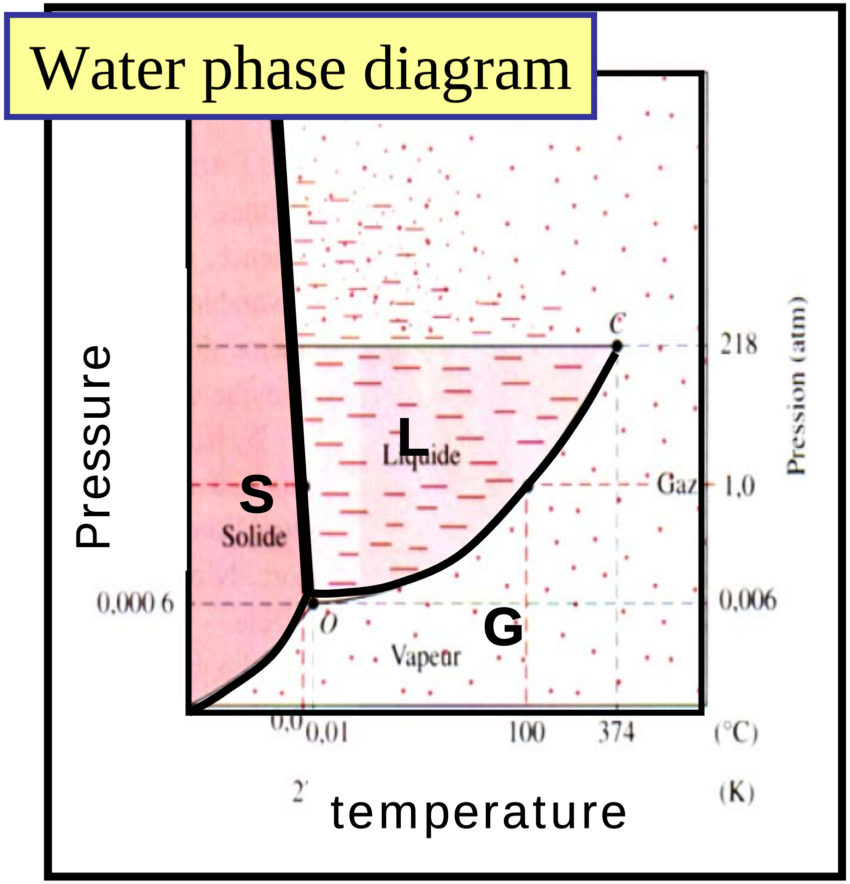
What is a phase transition?

- A small variation of a control parameter **(β)** induces a dramatic qualitative modification of the system properties **(<B>)**

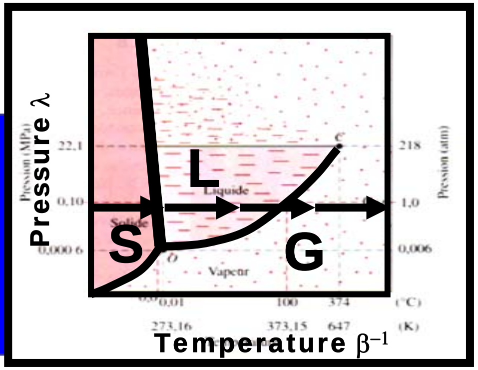
⇒ « accident » in an Equation of State

$$< B > (\beta) = -\partial_\beta \log Z$$

⇒ Non-analitycity of the partition sum



Definition of a phase transition (thermo limit)



- Non-analyticity of the thermodynamical potential

  G=-TlogZ(β) $\quad N \to \infty$

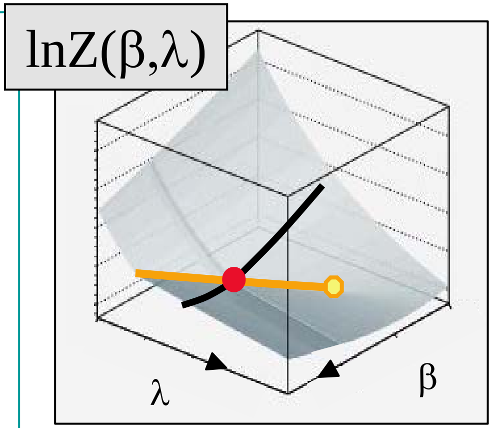
  $$Z(\beta) = \sum_{(n)} e^{-\sum_\ell \beta_\ell B_\ell^{(n)}}$$

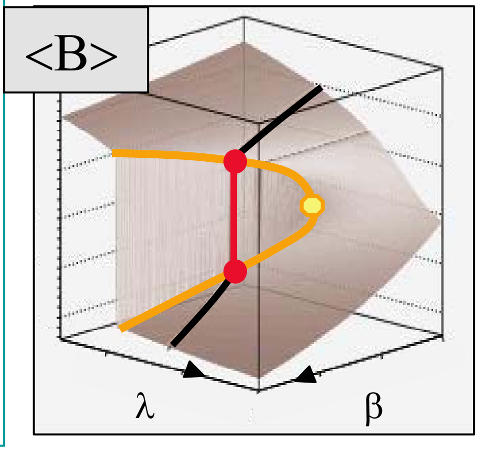
- Order of the transition: discontinuity (or divergence) in

  $$\partial_\beta^n \log Z$$

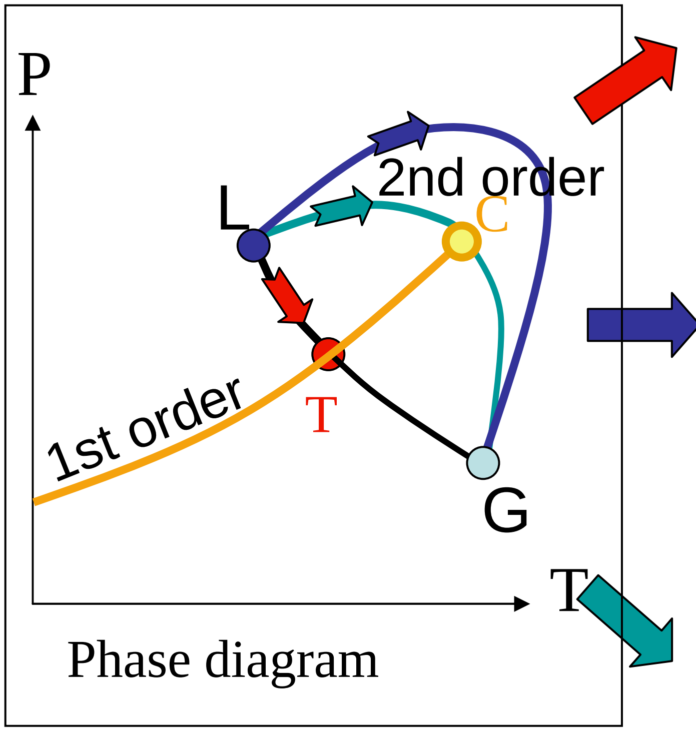
- First order:

  $< B > = -\partial_\beta \log Z$ discontinuous



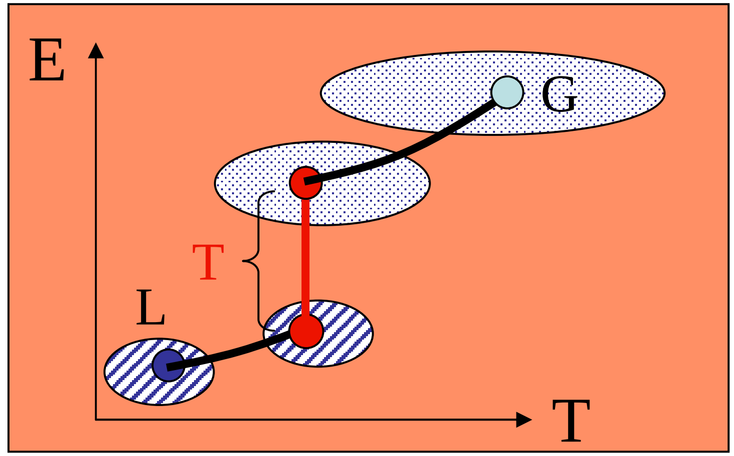


Order of Phase Transitions



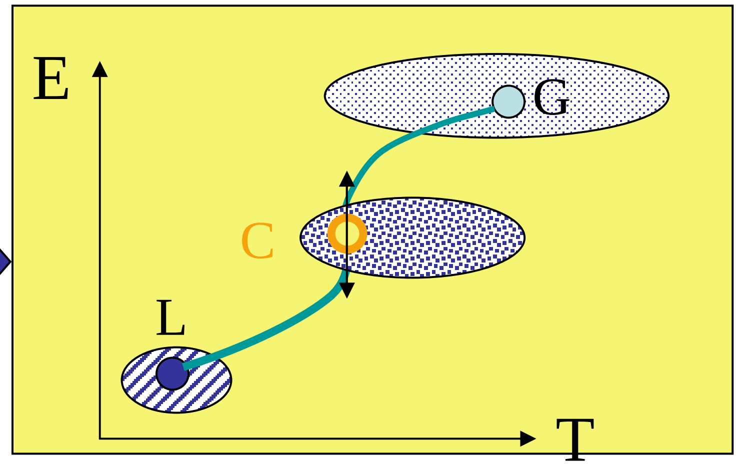
Ex: Liquid-Gas

B = Energy,Volume



Order 1

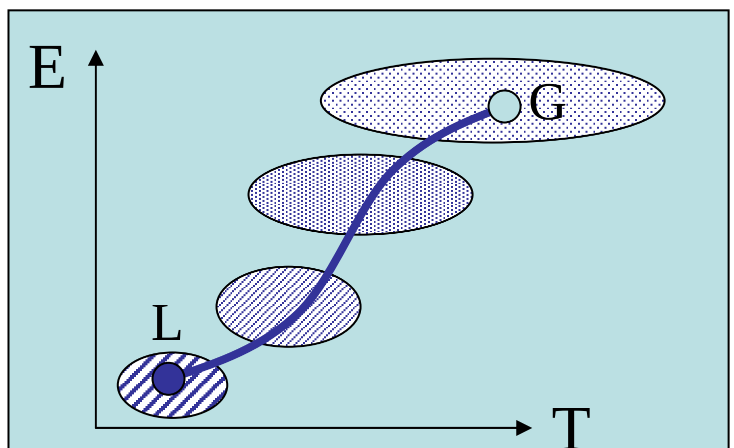
B discontinuous



Order 2

B continuous

δB divergent

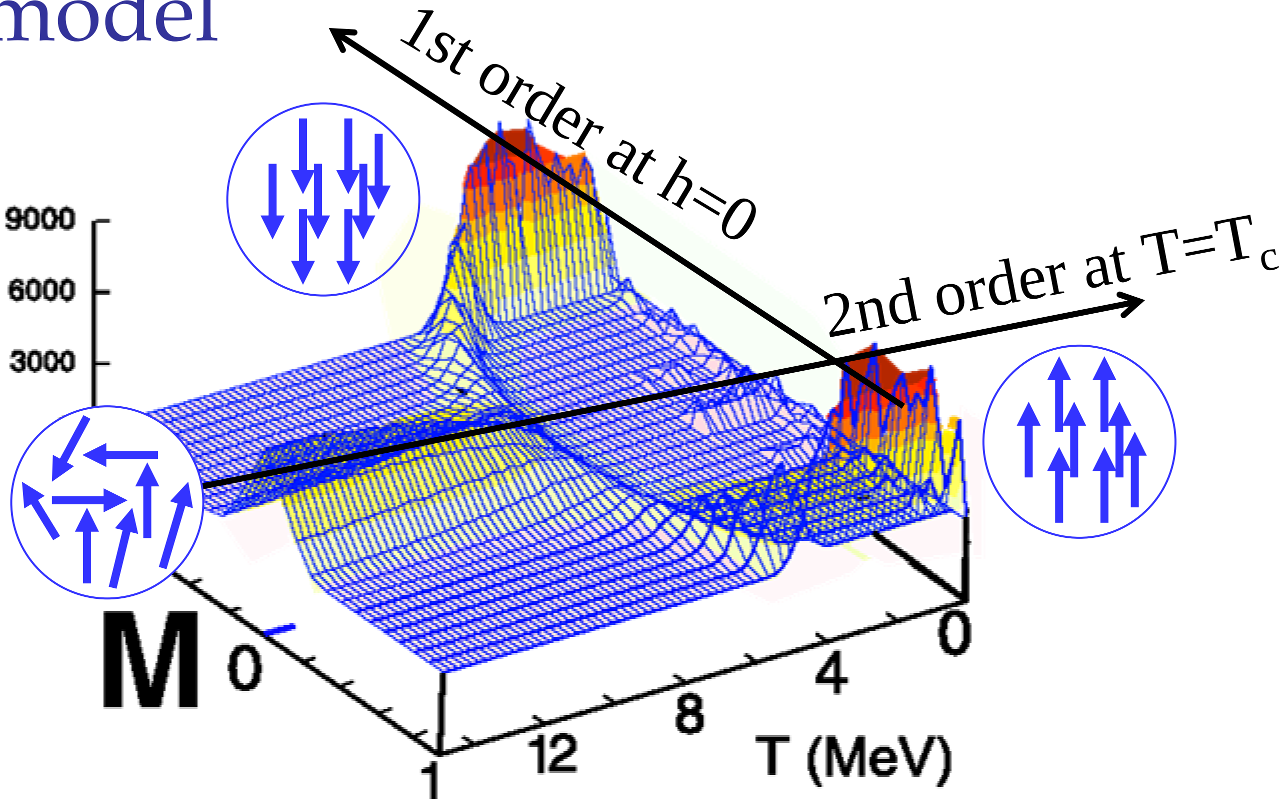


cross-over

Example: the ferro-para phase transition in solid-state physics

- Ising model

$$\vec{M} = \sum_i \vec{s}_i$$



1 - Phase transitions in physical systems

- The classical mean field Landau theory

Landau theory: First order

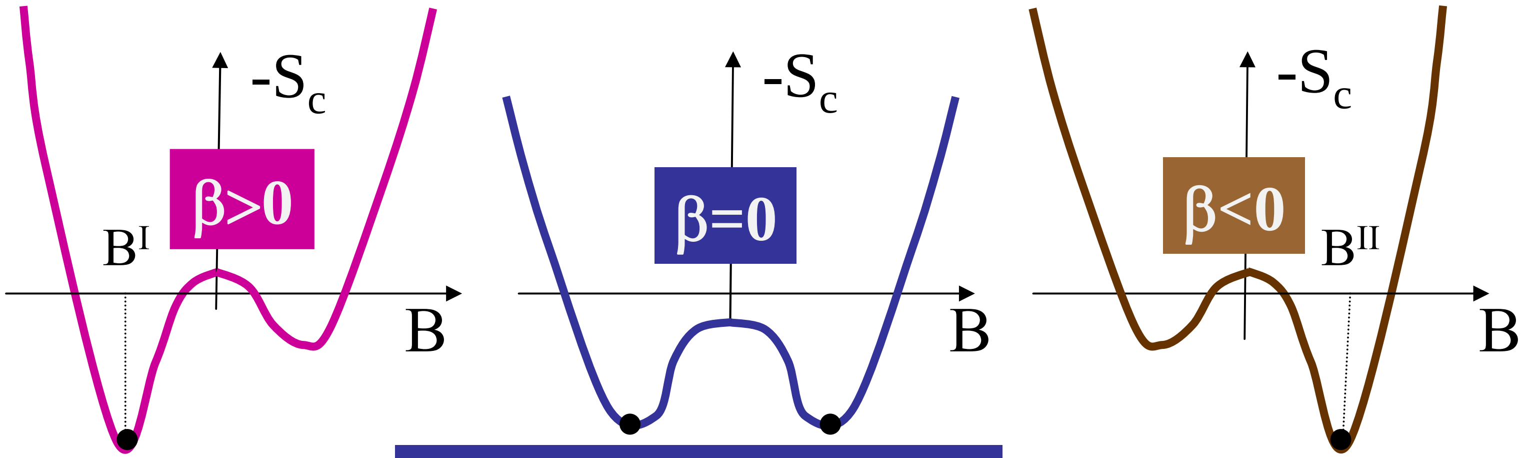
Lev Landau, 1936

- Constrained entropy or thermodynamic potential G/T
- Series developement around the transition point B–B^{II} => B = 0
- two minima of equal depth => a first order transition

$$-\log Z_{\beta\lambda} = -S + \beta B + \lambda L = -S_c^{\beta\lambda}(B) \quad = \text{min}$$

$$\log Z_{\beta\lambda} = -\beta B + \frac{1}{2} a_\lambda B^2 + \frac{1}{3} b_\lambda B^3 + \frac{1}{4} c_\lambda B^4 + \dots$$

$$a = a_0 + a_1 \lambda \rightarrow B = \begin{cases} B^I & \beta < 0 \\ B^{II} & \beta > 0 \end{cases}$$



β,λ control variables
B order parameter

Landau theory: second order

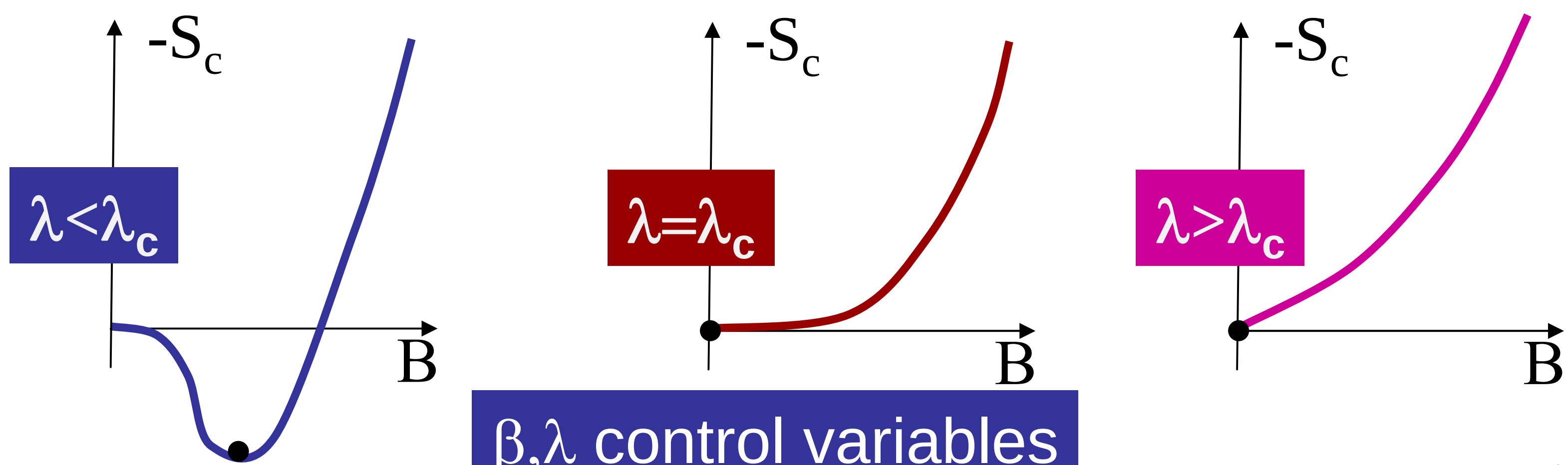
Lev Landau, 1936

- Constrained entropy or thermodynamic potential G/T
- Series developement around the transition point B–B^{II} => B = 0
- Symmetry B⇔–B => a second order transition

$$-\log Z_{\beta\lambda} = -S + \beta B + \lambda L = -S_c^{\beta\lambda}(B) = \min$$

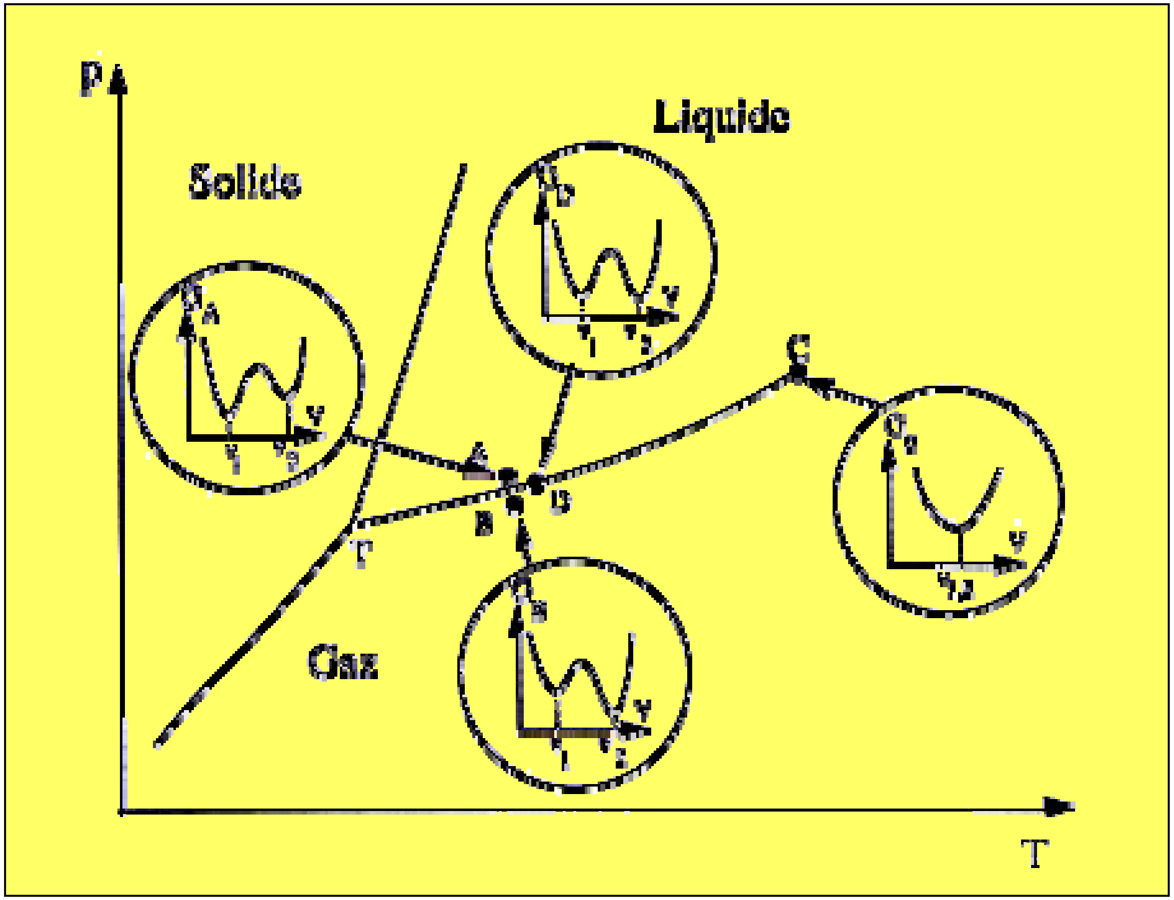
$$\log Z_{\beta\lambda} = \cancel{-\beta B} + \frac{1}{2} a_\lambda B^2 + \cancel{\frac{1}{3} b_\lambda B^3} + \frac{1}{4} c_\lambda B^4 + \ldots$$

$$a = a_0(\lambda - \lambda_c) \rightarrow B = \pm\sqrt{\frac{a_0}{c}}(\lambda_c - \lambda)^{1/2}$$



β,λ control variables
B order parameter

Fluid phase diagram in the Landau picture



2 – First order transitions

- Phase coexistence

Equilibrium and instabilities

$$-S_c^{\tilde{\beta}}(B) = -S + \tilde{\beta} B$$ Thermo potential G/T

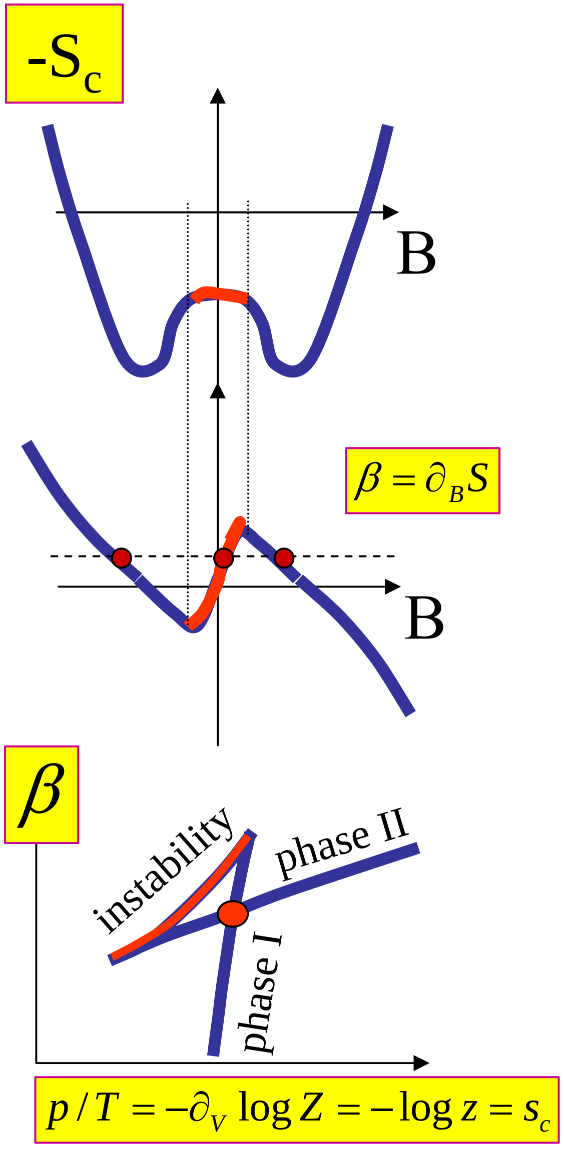
$$\delta S_c = 0 \Leftrightarrow \tilde{\beta} = \beta(B)$$

All B are stationary points but a convex entropy gives a multi-valued equation of state!
=> Which one is the equilibrium state?

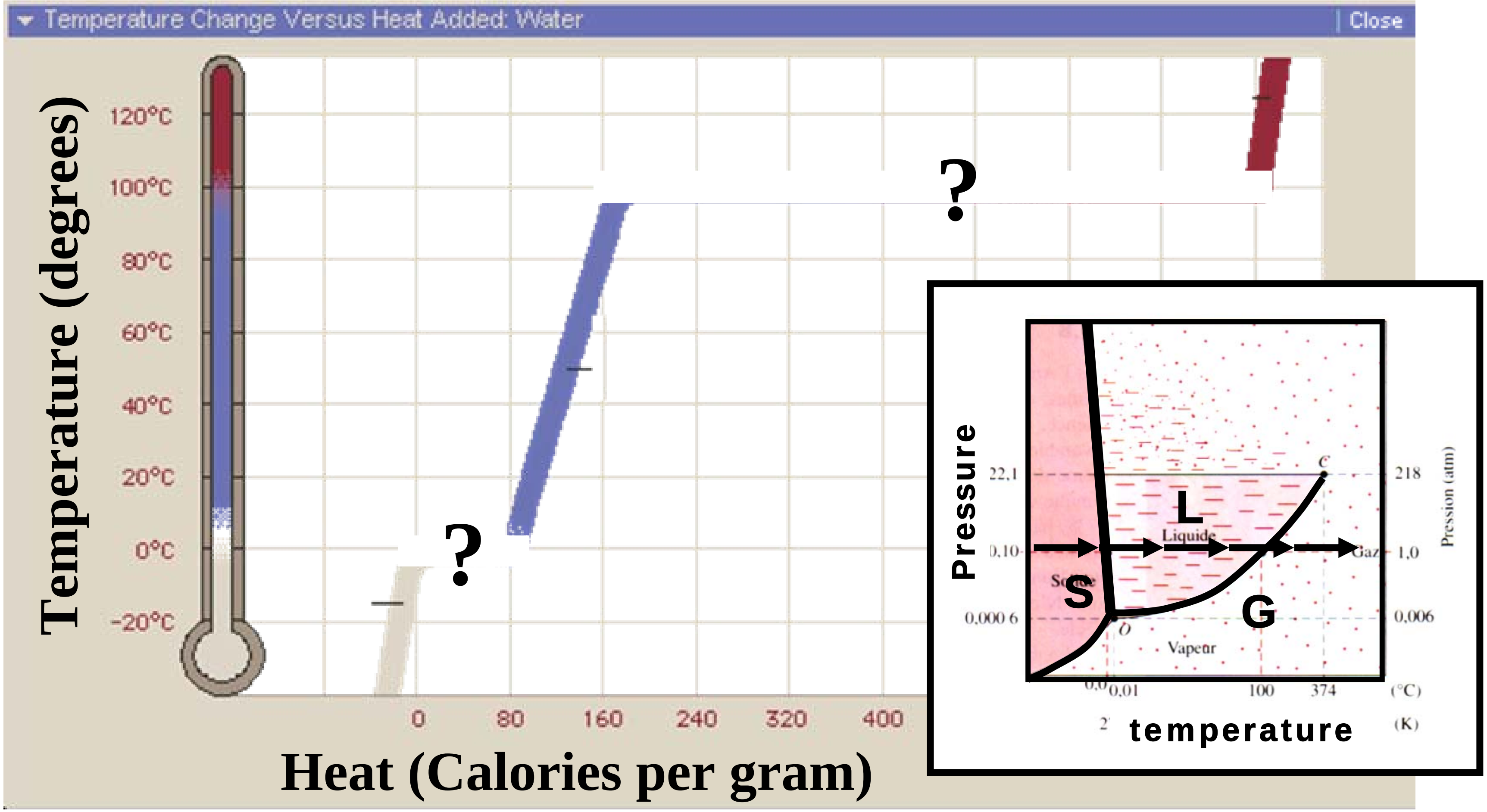
$\delta^2 S_c < 0$ Stability condition

$$\frac{\partial^2 S_c}{\partial B^2} = \frac{\partial \beta}{\partial B} > 0$$ Instable region (**spinodal**)

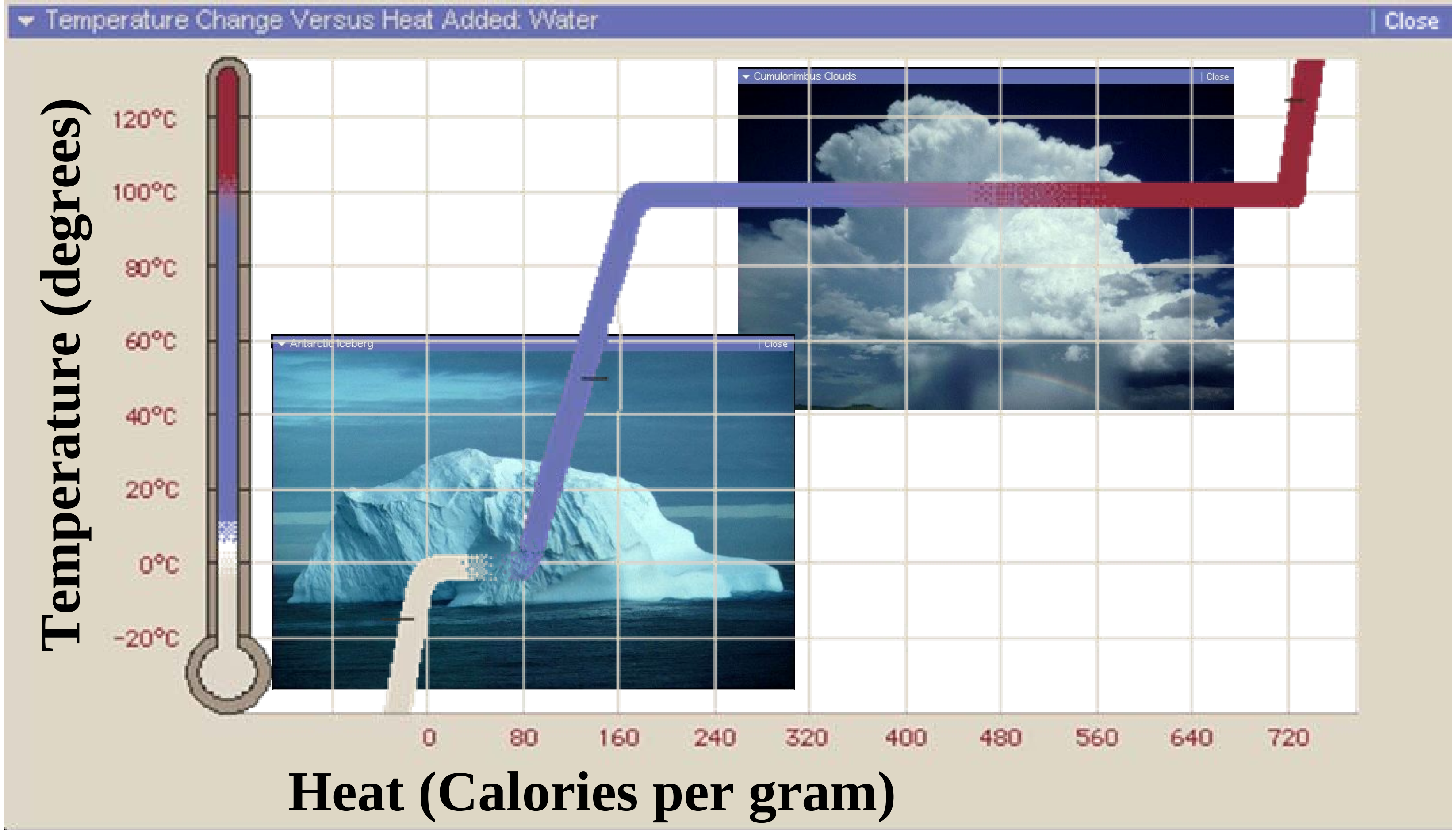
$$\frac{\partial B}{\partial t} = -\nu \nabla^2 \frac{\delta S_c}{\delta B}$$ Diffusion equation (Ginzburg-Landau)



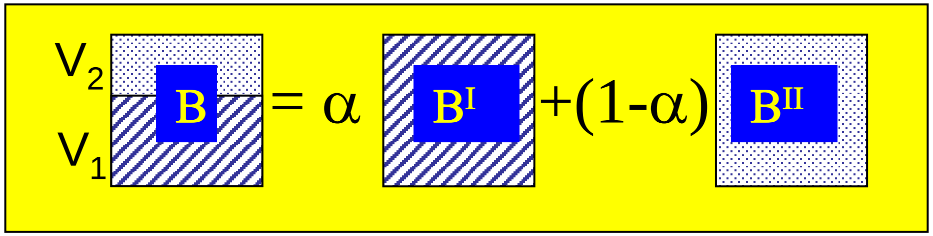
What happens in the spinodal region ?



What happens in the spinodal region ?



Phase Separation: Maxwell construction



tangent construction

$$S^c = V_1 s^c\left(b^I\right) + V_2 s^c\left(b^{II}\right) \qquad V = V_1 + V_2$$

$$s^c = \alpha s_1 + \left(1-\alpha\right) s_2 \quad \alpha = V_1 / V$$

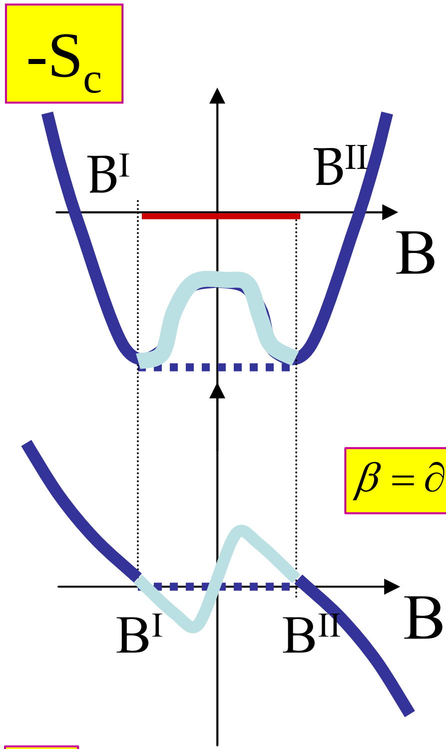
same ordinate

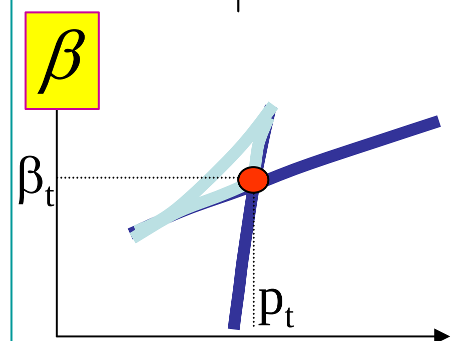
$$\beta_t\left(b^I - b^{II}\right) = \int_{b^{II}}^{b^I} s' db$$

same derivative

$$\left.\frac{\partial s}{\partial b}\right|_{b^I} = \left.\frac{\partial s}{\partial b}\right|_{b^{II}} = \beta_t$$

→ (B^I, B^{II}) coexistence zone





$$p/T = -\partial_V \log Z = -\log z = s_c$$

2 – First order transitions

- extension to many densities
  $b=B/V$

Extension to many densities: from Maxwell to Gibbs

- Tangent hypersurface: equality of all fields

$$\left.\frac{\partial s}{\partial b_\ell}\right|_{b_\ell^I} = \left.\frac{\partial s}{\partial b_\ell}\right|_{b_\ell^{II}} = \bar{\beta}_\ell \quad \forall \ell$$

- Multi-dimensional spinodal

$$C = \begin{pmatrix} \frac{\partial^2 s}{\partial b_1^2} & \cdots & \frac{\partial^2 s}{\partial b_1 \partial b_k} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 s}{\partial b_k \partial b_1} & \cdots & \frac{\partial^2 s}{\partial b_k^2} \end{pmatrix}$$

Illustration in 2D

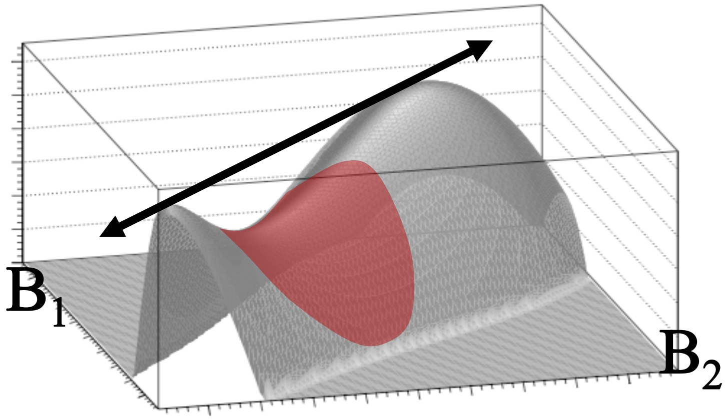
- the saddle: one single order parameter

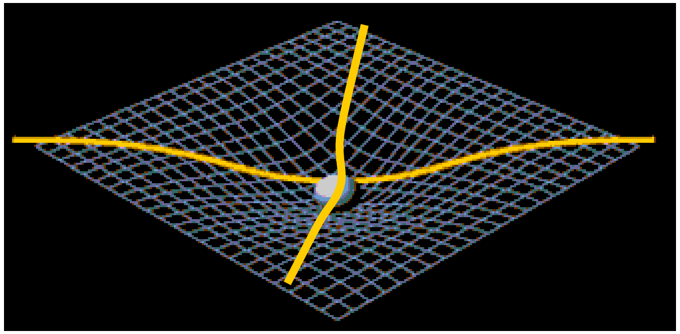
$\Rightarrow$ direction of phase separation

(LG: B_1=V B_2=E)

- The well: two order parameters

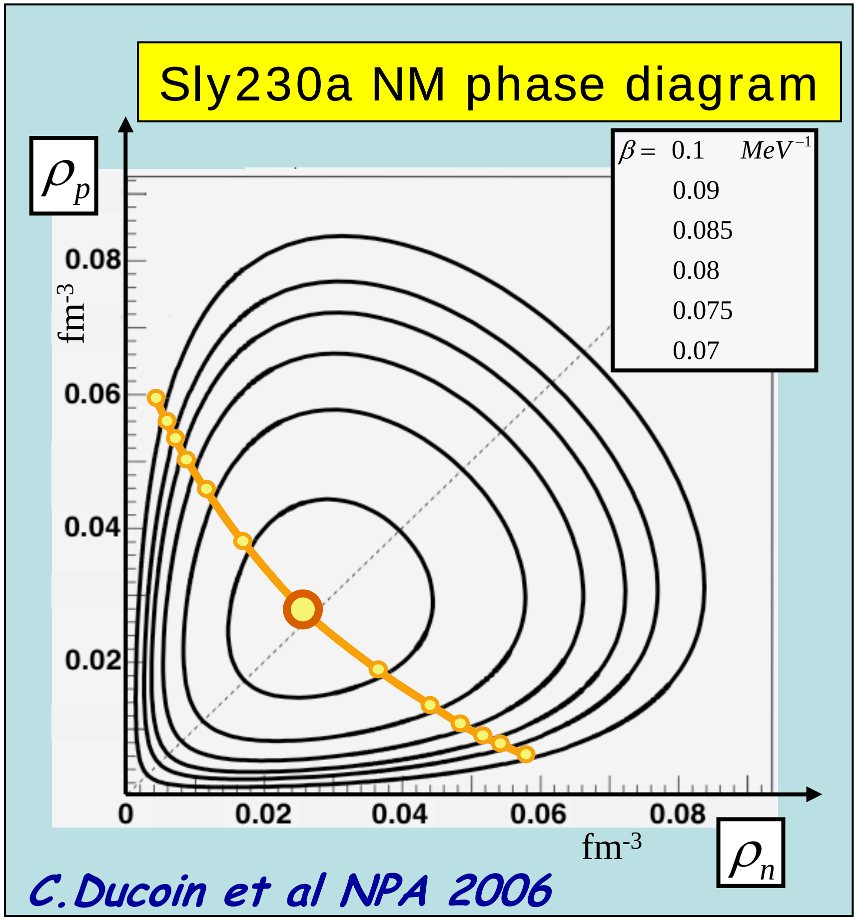
(He3-He4: $B_1=\Delta$ $B_2=X_3/X_4$)



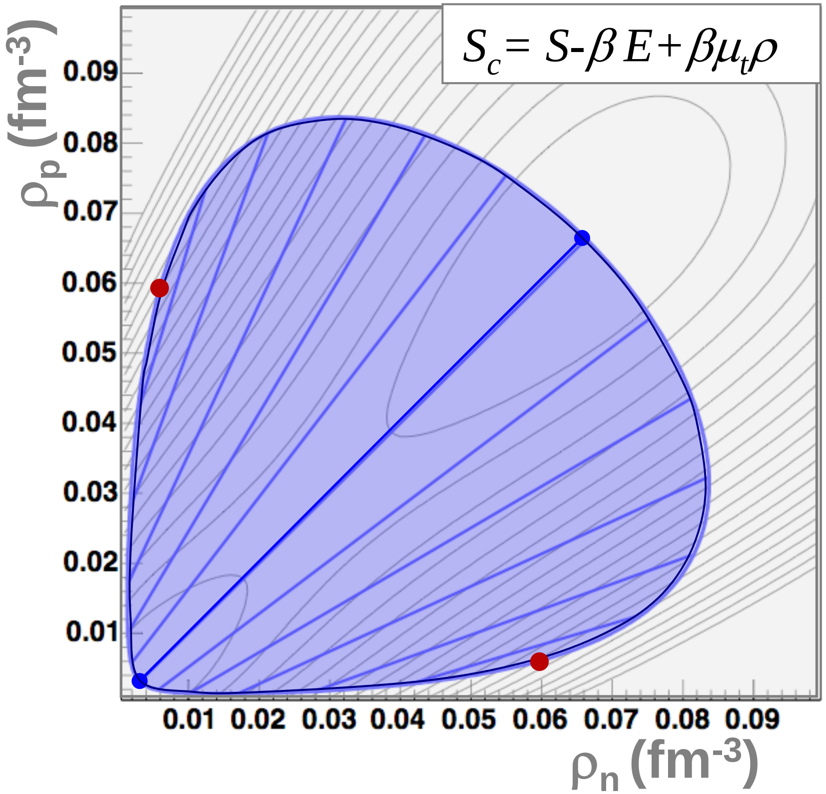


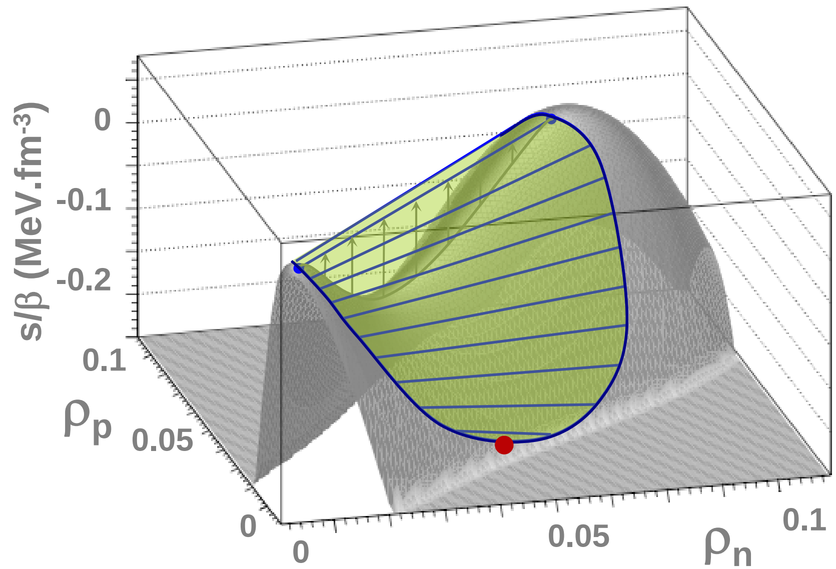
Example: nuclear matter

- Two possible phases L+G
- A region of **first order** phase transitions
- A line of **second order** phase transitions



Phase coexistence in nuclear matter



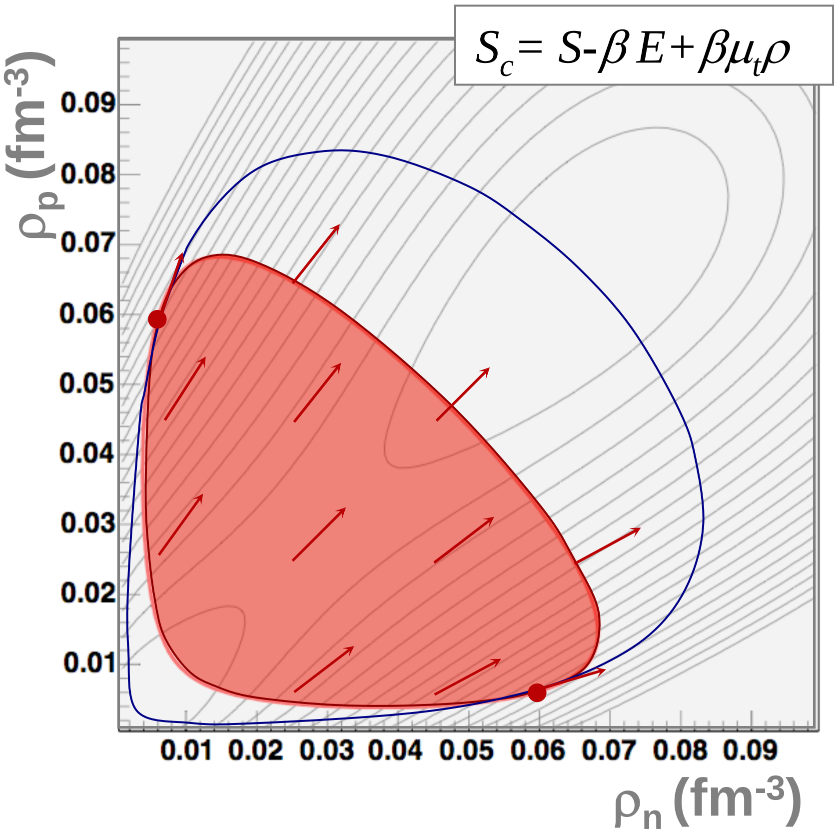


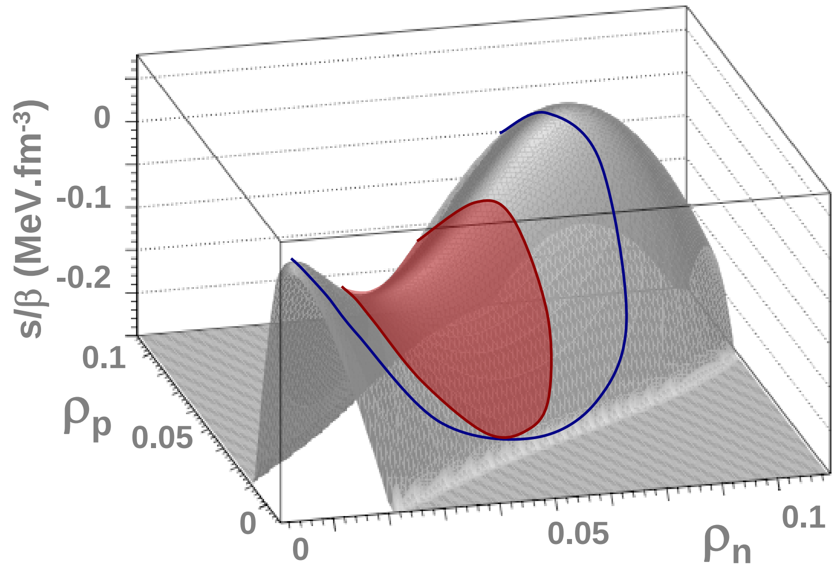
Gibbs construction

B/V=(ρ_p ,ρ_n)

$$\left.\frac{\partial s}{\partial \rho_\ell}\right|_{\rho_\ell^I} = \left.\frac{\partial s}{\partial \rho_\ell}\right|_{\rho_\ell^{II}} = \beta\overline{\mu}_\ell \qquad \ell = n, p$$

Spinodal of nuclear matter



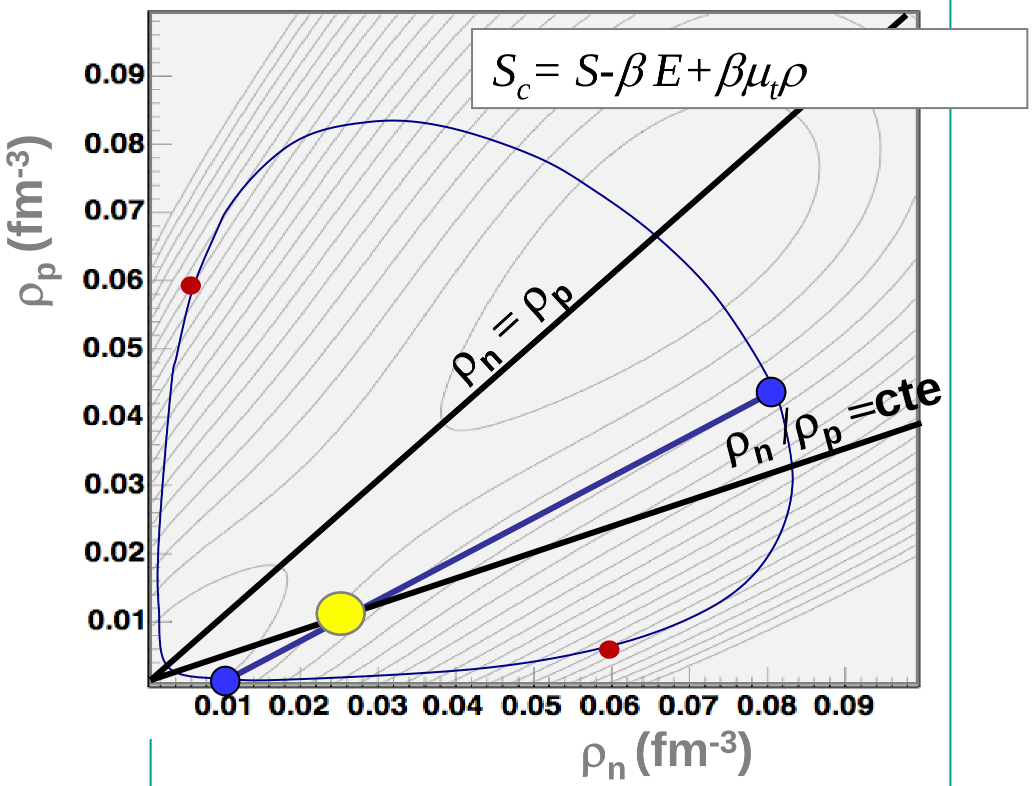


- negative curvature => instability
- 1 eigen-value <0 => 1 spinodal

$$C = \begin{pmatrix} \dfrac{\partial^2 s}{\partial \rho_n^2} & \dfrac{\partial^2 s}{\partial \rho_n \partial \rho_p} \\ \dfrac{\partial^2 s}{\partial \rho_p \partial \rho_n} & \dfrac{\partial^2 s}{\partial \rho_p^2} \end{pmatrix}$$

Distillation

- Distillation: generic phenomenon of phase separation with >1 densities
  - ordered phase more symmetric
  - desordered phase more asymmetric



(3) Second order transitions and critical phenomena

- Divergence of the correlation length

Back to Landau : $\Phi^2\!-\!\Phi^4$

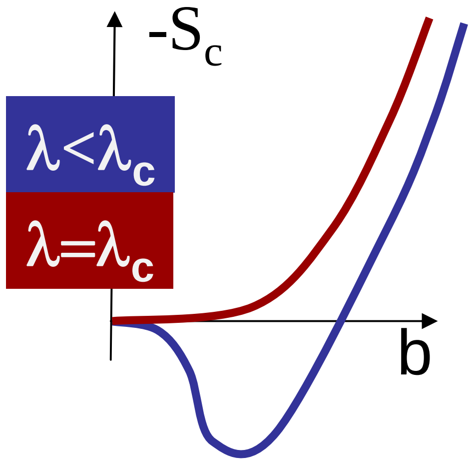
- Constrained entropy or thermo potential (Symmetry B⇔-B)
- series developement around the transition point B = 0 $\lambda=\lambda_c$
- Second order transition => divergent susceptibility

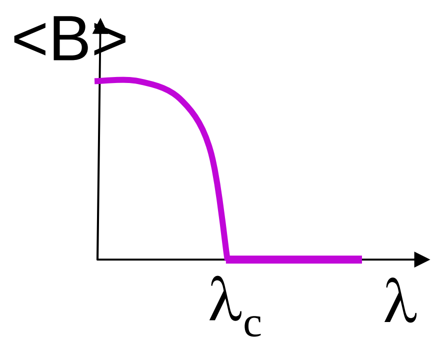
$$-\log Z = -S_c^{\beta\lambda} = \frac{1}{2}a_\lambda B^2 + \frac{1}{4}cB^4 + \ldots = \min$$

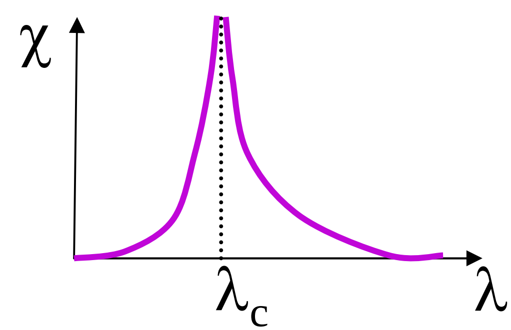
$$a_\lambda = a_0\left(\lambda - \lambda_c\right)$$

$$\langle B\rangle = \frac{a_0}{c}\left(\lambda_c - \lambda\right)^{1/2}$$

$$\chi = \frac{\partial\langle B\rangle}{\partial\beta} = \begin{cases} \frac{1}{2a_0}\left(\lambda_c - \lambda\right)^{-1} & \lambda < \lambda_c \\ \frac{1}{a_0}\left(\lambda - \lambda_c\right)^{-1} & \lambda > \lambda_c \end{cases}$$







β,λ control variables
B order parameter

Divergences and Fluctuations

Ornstein-Zernike-Fisher 1964

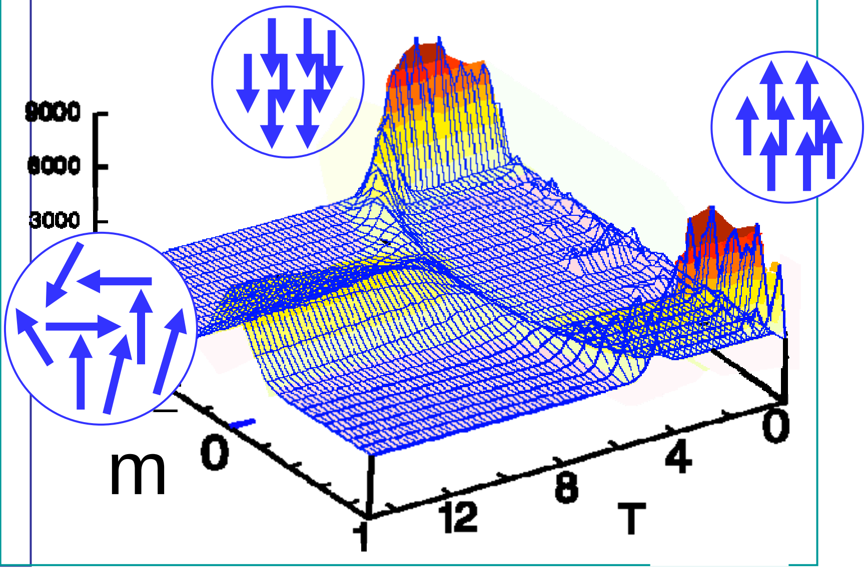
- **Distribution of the order parameter B**
- **B Fluctuation**

  diverges at the critical point

$$p^{(n)} = Z_\beta^{-1} e^{-\beta B_n} \; ; \; p_\beta(B) = \sum_n p^{(n)} \delta(B - B_n)$$

$$\chi = \left(\frac{\partial^2 S_c}{\partial B^2}\right)^{-1} = \frac{\partial \langle B \rangle}{\partial \beta} = \frac{\partial^2 \log Z_\beta}{\partial \beta^2} = \sigma_B^2$$

Ising h=0



Fluctuations and Correlations

Ornstein-Zernike-Fisher 1964

- **Distribution of the order parameter B**

  B Fluctuation

  diverges at the critical point
- **Correlation fonction**

  ex: B = N => b=ρ(r)
- **Correlation length ξ**

  correlations over all length scales at the critical point
- **Critical opalescence:**

  Fourier response : radiated intensity. q=2ksinθ/2 momentum transfer

$$p_n = Z_\beta^{-1} e^{-\beta B_n} \; ; \; Z_\beta = \sum_n e^{-\beta B_n}$$

$$\chi = \left(\frac{\partial^2 S_c}{\partial B^2}\right)^{-1} = \frac{\partial \langle B \rangle}{\partial \beta} = \frac{\partial^2 \log Z_\beta}{\partial \beta^2} = \sigma_B^2$$

$$= \int d\vec{r}\, d\vec{r}\,'\left(b(\vec{r}) - <b>\right)\left(b(\vec{r}\,') - <b>\right)$$

$$= \int d\vec{r}\, d\vec{r}\,' G(\vec{r}, \vec{r}\,')$$

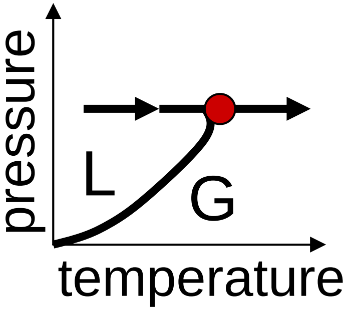
$$G(\vec{s}) \propto e^{-s/\xi} s^{-(d-2+\eta)}$$

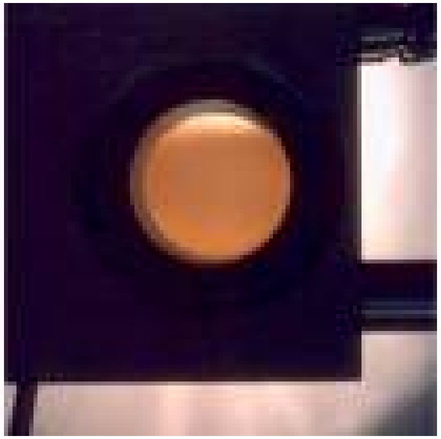
$$I(q) = \int d\vec{s}\, e^{i\vec{q}\cdot\vec{s}}\, G(\vec{s})$$

Liquid sodium hexafluoride

p=37 atm

T=42 ^oC





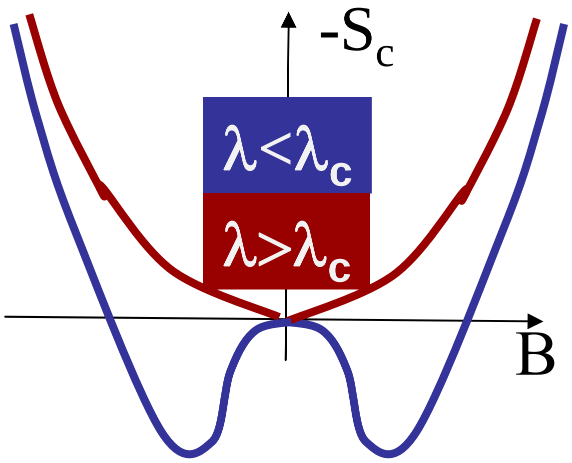
T=43.9 ^oC

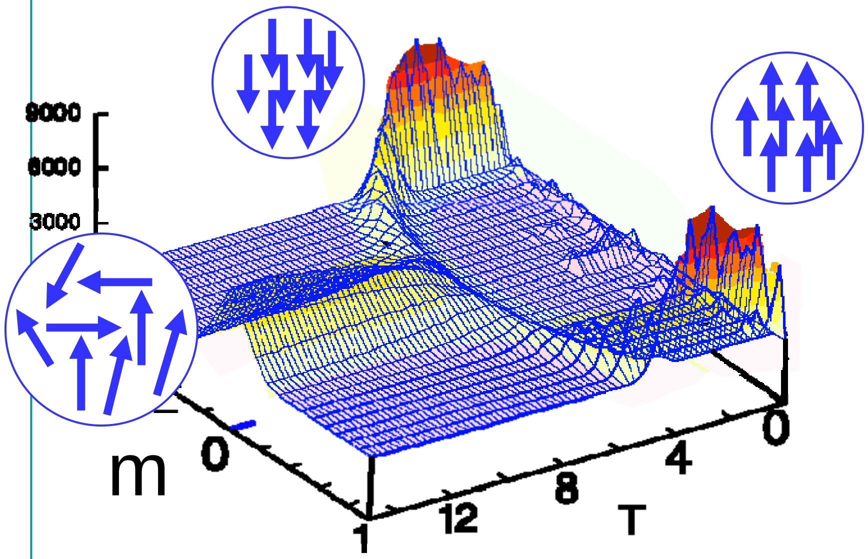
T=44 ^oC

Divergent fluctuations and spontaneous symmetry breaking

- Critical point: bifurcation
- B≠0 with β=0 (=> logZ preserving B reflection symmetry):

⇒ spontaneous symmetry breaking



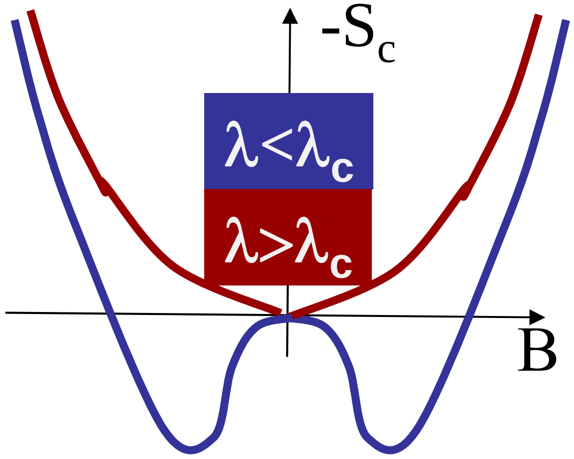


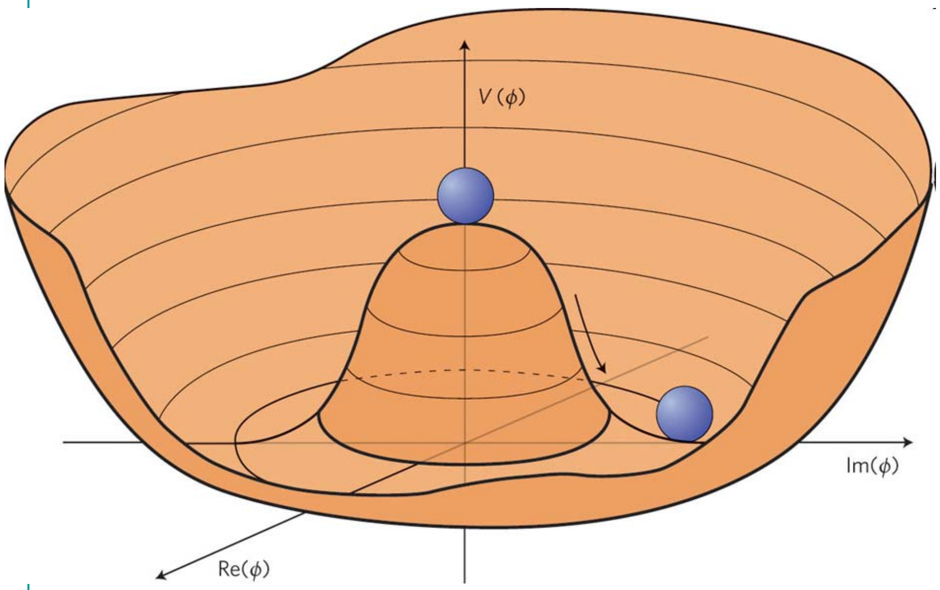
Distribution of the order parameter for the Ising model without external field

Spontaneous symmetry breaking

- Critical point: bifurcation
- B≠0 with β=0 (=> logZ preserving B reflection symmetry):

=> spontaneous symmetry breaking





- **ferro/para phase transition**
- **superconductivity below T_c**
- **Higgs mechanism in the Standard Model**

(3) Second order transitions and critical phenomena

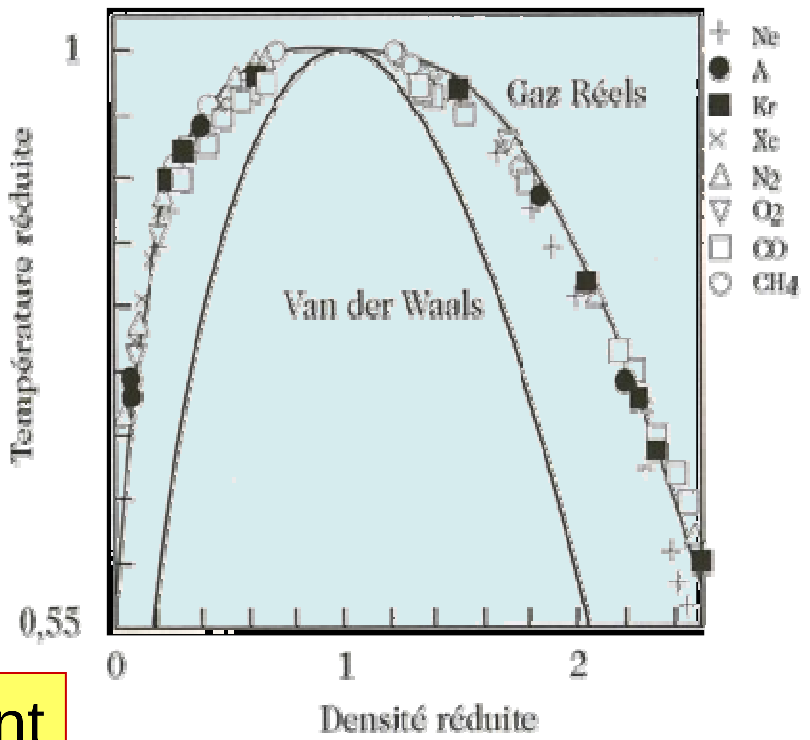
- Scale invariance and renormalization

Universality and critical exponents

- 2 « universal » critical exponents

$$B \propto (T_c - T)^{\beta} \qquad \chi \propto |T_c - T|^{-\gamma}$$

exponent	*System property*	*Expression*
α	Specific heat	$C \propto t^{-\alpha}$
β	Order parameter	$m \propto t^{\beta}$
γ	Susceptibility	$\chi \propto t^{-\gamma}$
δ	Order parameter	$m \propto h^{-1/\delta}$
η	Correlation function	$G(r) \propto r^{-\lvert d-2+\eta \rvert}$
ν	Correlation length	$\xi \propto t^{-\nu}$



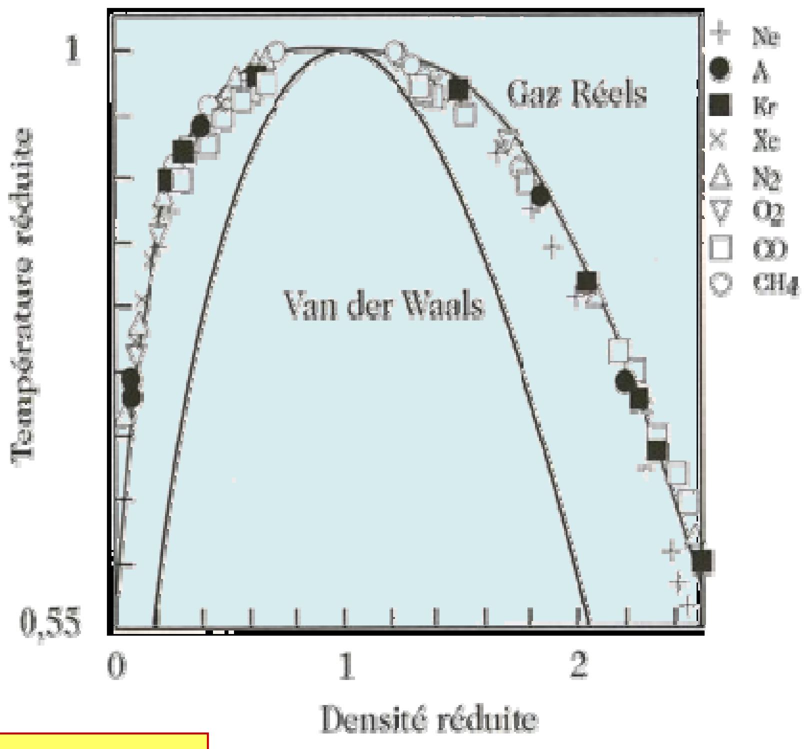
Only 2 exponents are independent

Universality and critical exponents

- 2 « universal » critical exponents

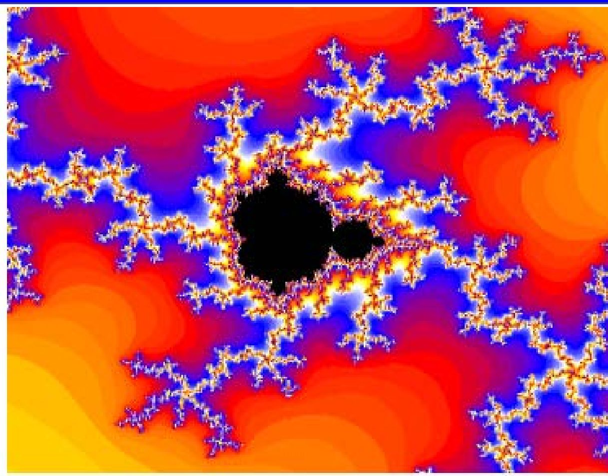
$$B \propto (T_c - T)^{\beta} \qquad \chi \propto |T_c - T|^{-\gamma}$$

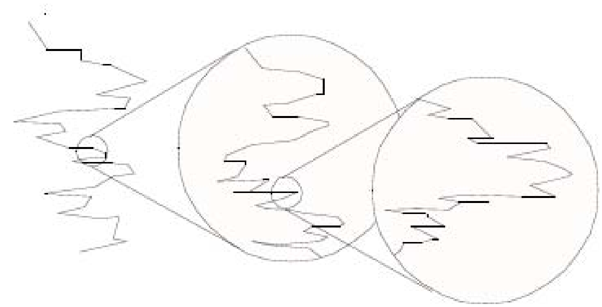
Ising *n=1* *Liquid/gas*	**XY** *n=2* *superfluidity*	**Heisenberg** *n=3* *ferromagnetism*
$\beta = 0.325$	0.346	0.369
$\gamma = 1.231$	1.315	1.392



Only depend on the dimensionality of B

Critical exponents and scale invariance



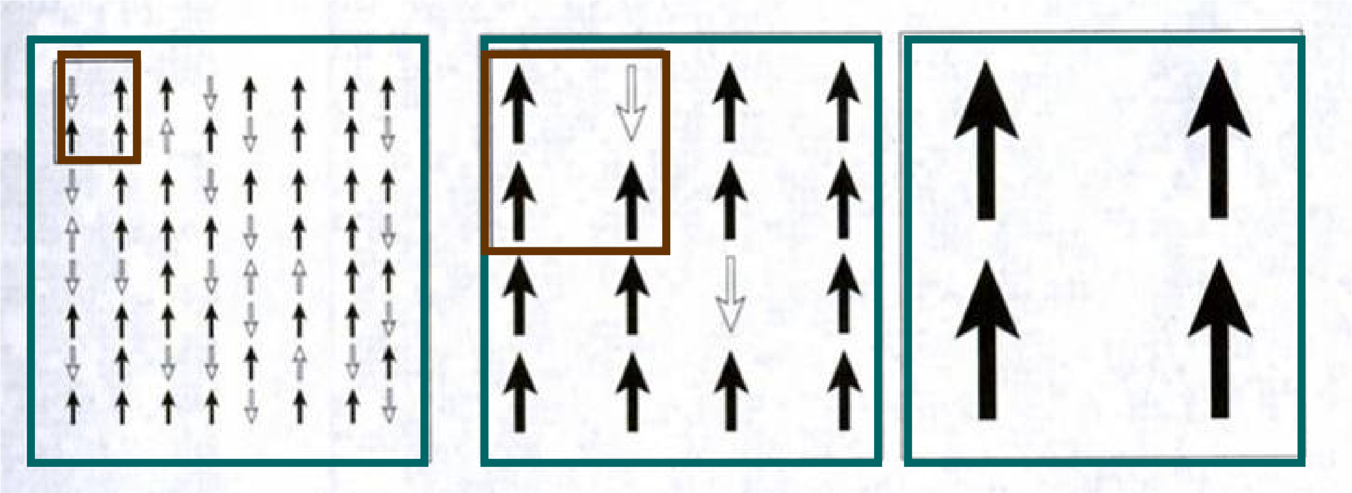


- *x,y control parameters*
- *scaling hypothesis (Widom 1965)* $\ln Z(\lambda^a x, \lambda^b y) = \lambda \ln Z(x,y)$
- *=> only two independent critical exponents*
- *definition of* β,γ $\beta=(1-b)/a \quad \gamma=(2b-1)/a$

Scaling hypothesis ⟺ ***scale invariance*** ⟺ ***self-similarity***

Scale invariance and renormalization

Wilson,1975



Renormalization: at the critical point the configuration is self-similar: it will stay invariant. This is due to the divergence of the correlation length.

- **Observable set**
  K_i intensive variables (couplings)
  $\{s\}=s_1 \ldots s_N$ degrees of freedom

- **Scale change:**
  re-iterated ad libitum

- **Partition sum:**
  preserved structure

- **Fixed point and its attraction domain:**

- **Correlation length:**
  infinite if at the fixed point $\xi\left(\vec{K}^*\right) \neq 0$

$$H = -\beta\overline{H} = K_1 f_1(\{s\}) + K_2 f_2(\{s\})$$

$$H^{(1)} = R_x\left[H\right] = H(\vec{K}^{(1)})$$

$$\ell \to \ell / x \quad N \to N / x^d$$

$$H^{(i+1)} = R_x\left[H^{(i)}\right] \quad K^{(i+1)} = R_x\left[K^{(i)}\right]$$

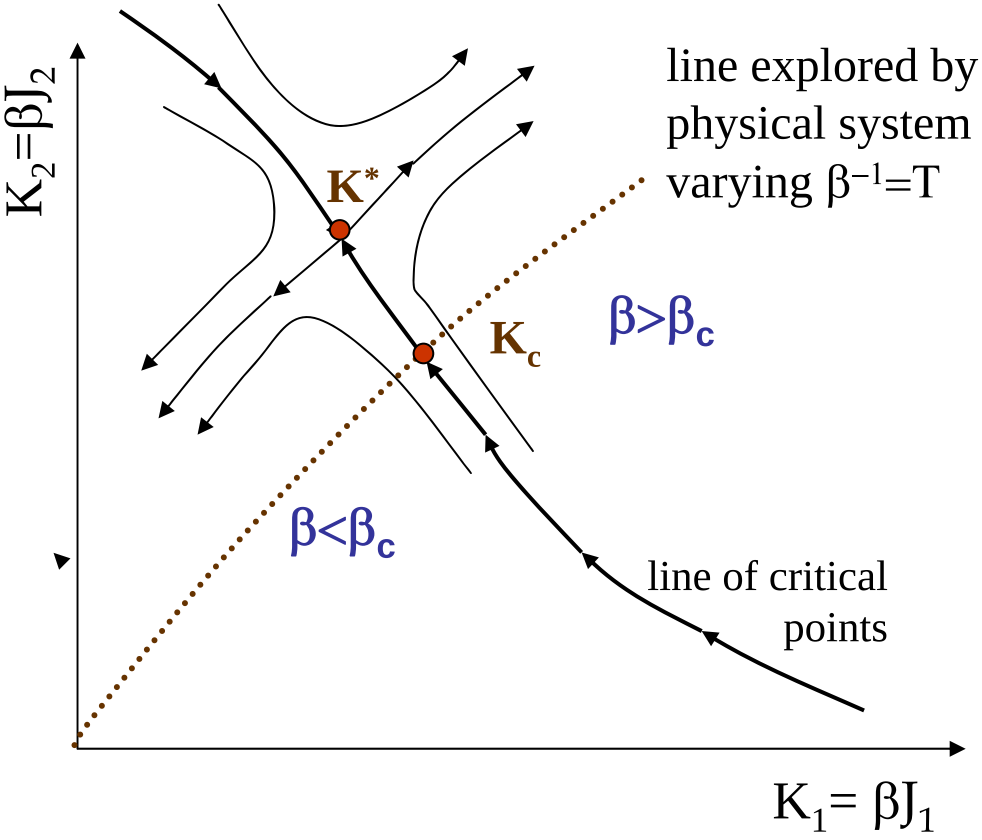
$$\log Z\left(N,\vec{K}\right) = \log Z\left(N/x^d, \vec{K}^{(1)}\right) + Ng(\vec{K})$$

$$\vec{K}^* = R_x\left[\vec{K}^*\right] \quad R_{nx}\left[\vec{K}_c\right] = \vec{K}^*$$

$$\xi\left(\vec{K}_c\right) = x^n \xi\left(R_{nx}\left[\vec{K}_c\right]\right)$$

$$= x^n \xi\left(\vec{K}^*\right) = \infty$$

Scale invariance and renormalization



If the flow of coupling constants converges to a non trivial fixed point:

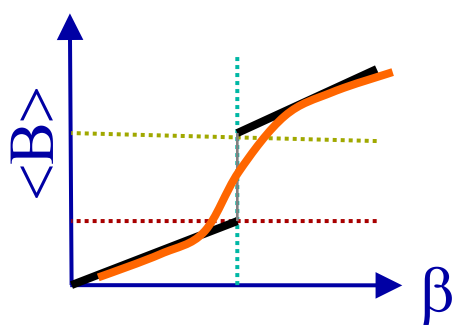
- the initial system is critical
- the partition sum is scale invariant on the critical line
- all systems (observations) converging towards the same fixed point have the same critical behavior => the same exponents
- this only depends on the observable set => on the order parameter

(4) Extension to finite systems

Phase transitions in finite systems

- Z analytic: transition rounded

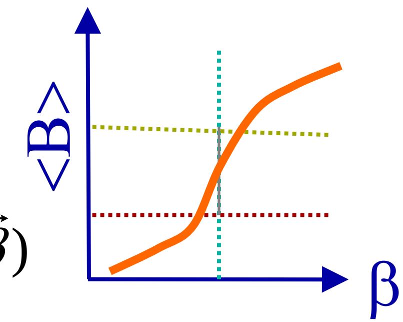
$$Z(\beta) = \sum_{n=1}^{N} e^{-\sum_\ell \beta_\ell B_\ell^{(n)}}$$



- Ensemble inequivalence

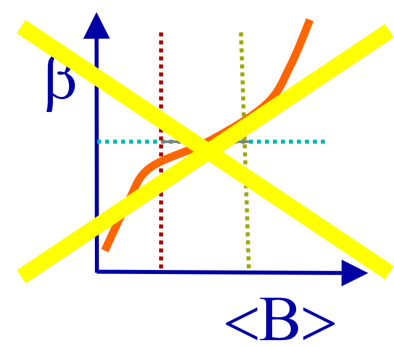
=>Intensive β controlled

$$< B_\ell >= -\partial_{\beta_\ell} \log Z(\vec{\beta})$$



=>Extensive B controlled

$$\beta_\ell = \partial_{\langle B_\ell \rangle} S$$



(4) Extension to finite systems

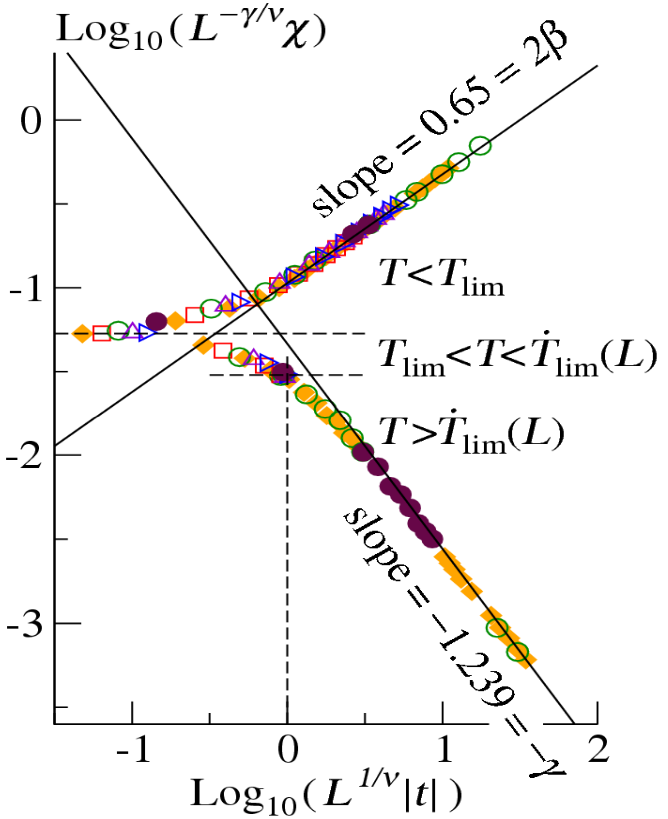
- Precursor of phase transition

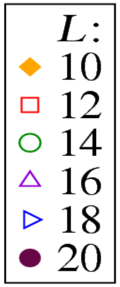
Finite size scaling

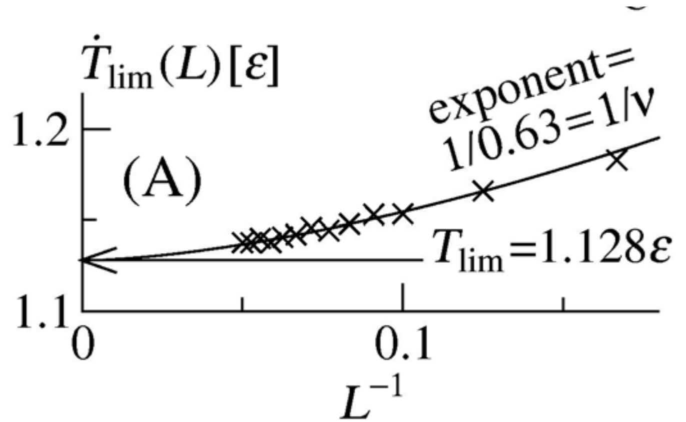
• Scale invariance at the critical point $L=\infty$	$Y \propto \lvert\varepsilon\rvert^{-x} \quad \varepsilon = \dfrac{\lambda - \lambda_c^{\infty}}{\lambda_c^{\infty}}$
• Scaling Hypothesis (Widom 1965)	$Y = f\left(L/\xi\right)\lvert\lambda - \lambda_c(L)\rvert^{-x}$ $\begin{cases} \xi \ll L \Rightarrow f = cte \\ \xi \sim L \Rightarrow Y = cte \end{cases}$
$\Rightarrow$ Finite size scaling * Critical temperature * susceptibility	$\lvert T_c(L) - T_c\rvert \propto L^{-1/\nu}$ $L^{2\beta/\nu}\chi \begin{cases} \to \text{const.} & \xi \sim L \\ \propto \left(L^{1/\nu}t\right)^{-\gamma} & \xi \ll L, T > T_c(L) \\ \propto \left(L^{1/\nu}t\right)^{2\beta} & \xi \ll L, T < T_c(L) \end{cases}$

Finite size scaling

F.Gulminelli, et al. PRL 2008







The Yang-Lee theorem

C.N. Yang, T.D.Lee Phys.Rev.1952

Phase transition: thermo potential $-\log Z(\beta)$ $\left(Z(\beta) = \sum_{n=1}^{N} e^{-\sum_\ell \beta_\ell B_\ell^{(n)}} \right)$

non analytic for $N \to \infty$

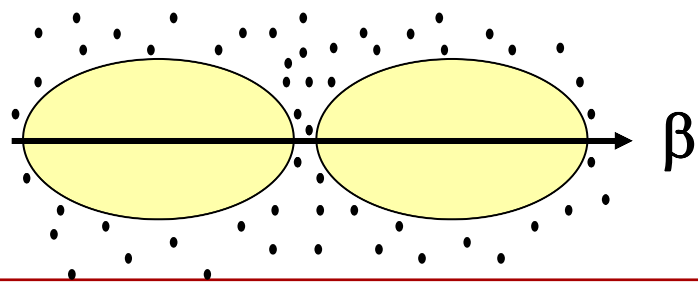
Origin of non-analyticities

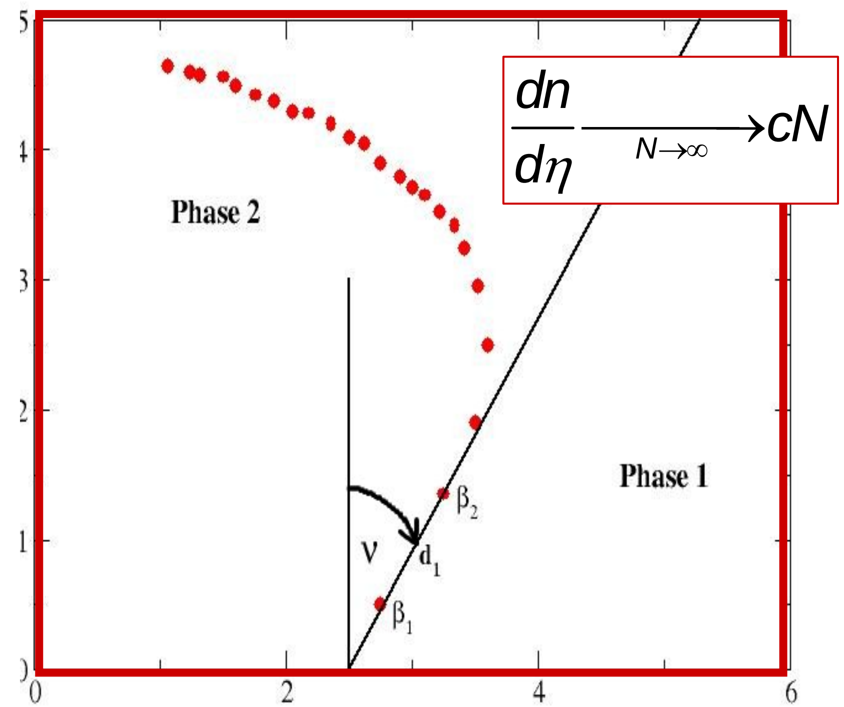
$$\gamma = \beta + i\eta \quad Z(\gamma) = 0$$

$$\Re : Z(\gamma) \neq 0$$

$$\log Z / N \quad N \to \infty$$

analytic





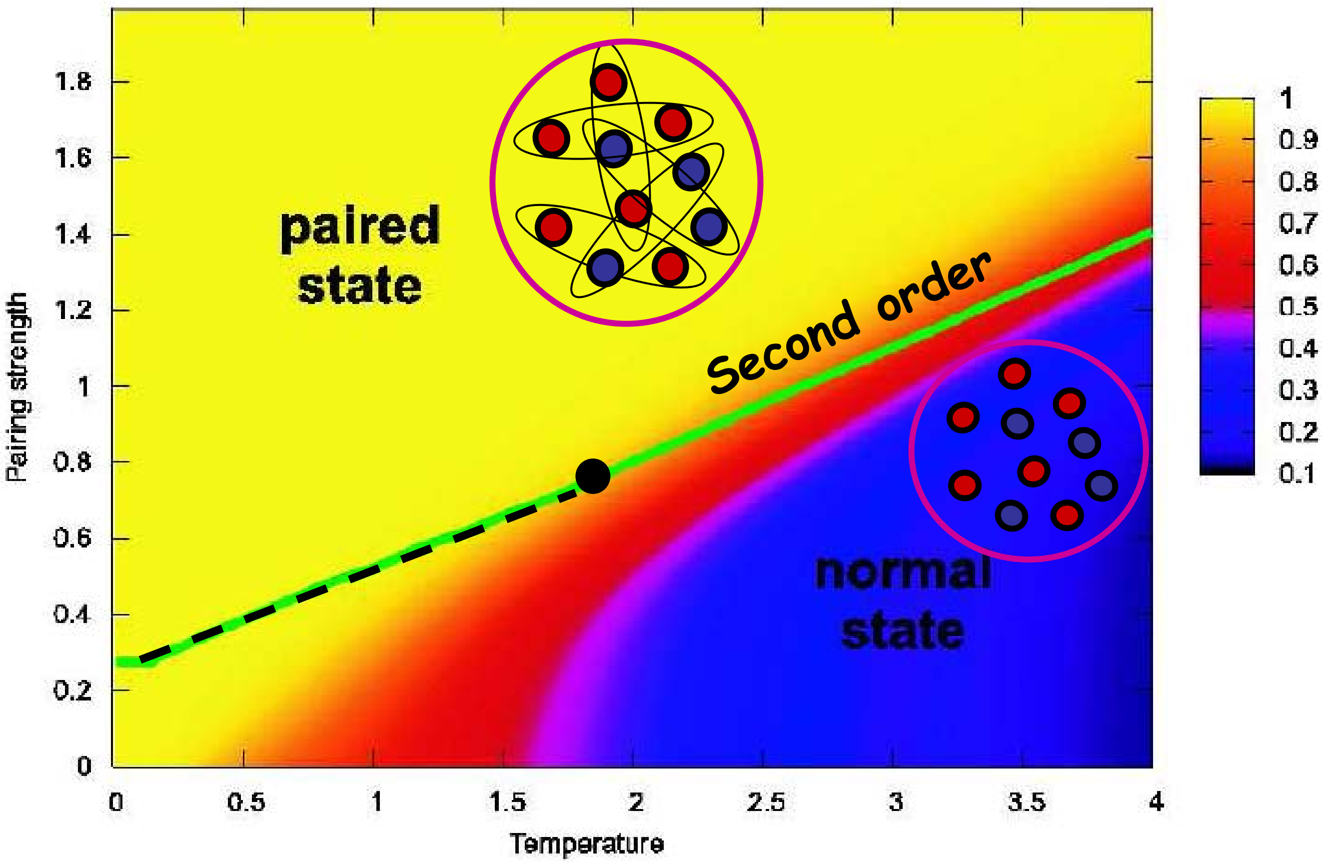
P.Borrmann, O.Mulken, J.Harting, Phys.Rev.Lett 84 (2000) 3511-3514
A. Schiller et al Phys.Rev.C 66 (2002) 024322

The pairing transition in finite nuclei

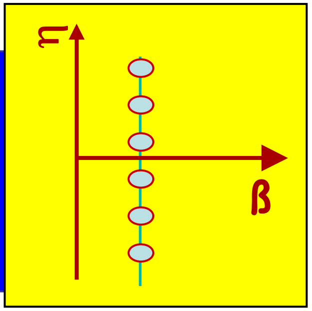
T.Sumaryada, A.Volya 2007

$$\hat{H} = 2\sum_{i>0} \varepsilon_i \hat{n}_i - \sum_{i,j>0} G_{ij} \hat{p}_i^+ \hat{p}_j$$

B=fraction of paired particles

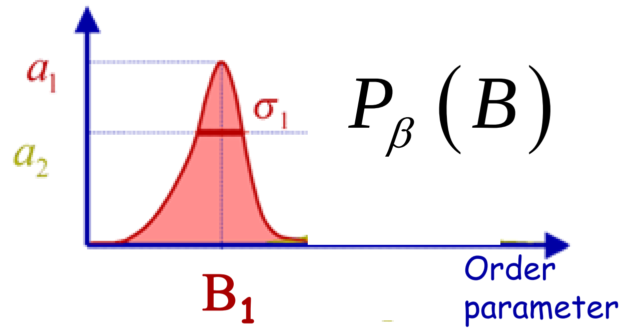


Yang-Lee theorem and Bimodalities



Partition sum and probability distribution

$$Z_\gamma = Z_\beta \int dB P_\beta(B) e^{-i\eta B}$$

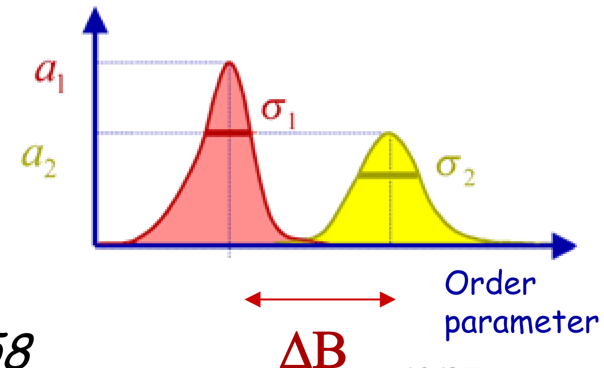


Normal distribution: no zeros

Bimodal distribution $P = P_1 + P_2$: double saddle point approximation

$$\eta_k = \frac{i(2k+1)\pi}{\Delta B}$$

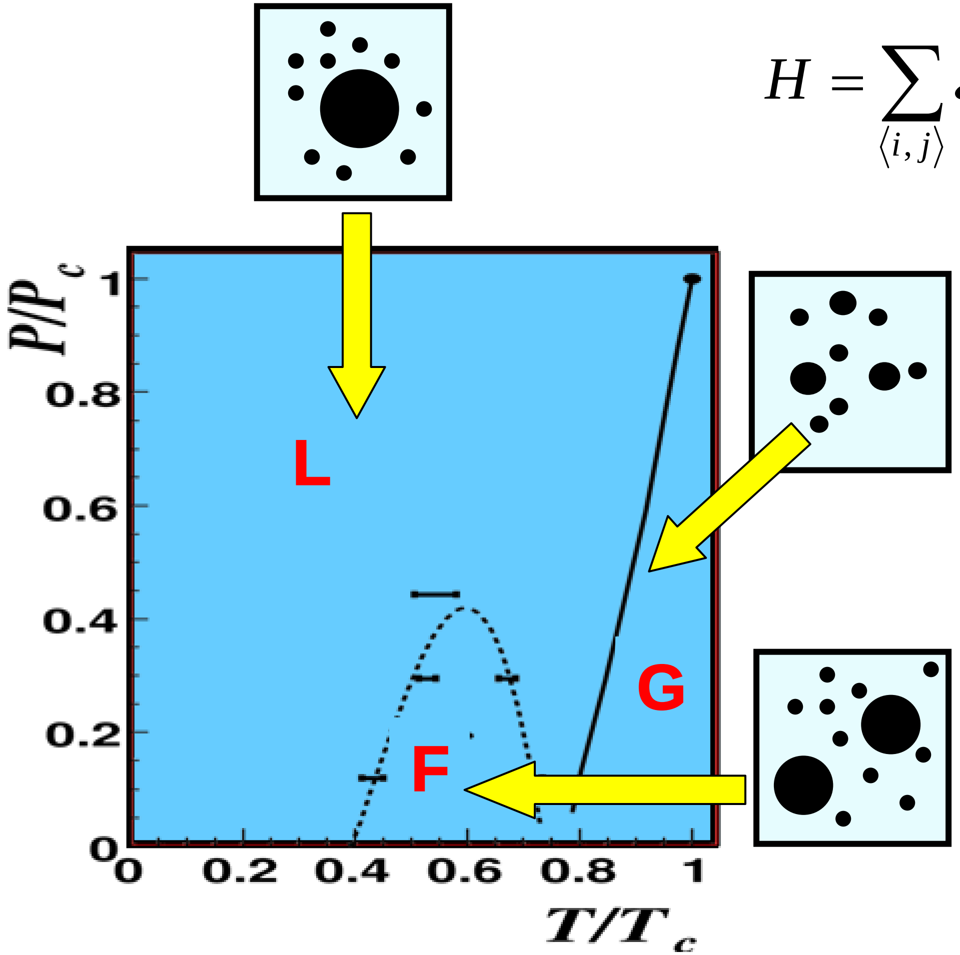
$$\Delta B \xrightarrow{N\to\infty} N\Delta b$$



K.C. Lee Phys Rev E 53 (1996) 6558
Ph.Chomaz, F.G. Physica A (2002)

The fragmentation transition of finite nuclei

$$H = \sum_{\langle i,j \rangle} \varepsilon_{n_i n_j} n_i n_j + \alpha_c \sum_{i,j} \frac{q_i q_j}{r_{ij}} + \sum_i \frac{\vec{p}_i^{\,2}}{2m} n_i$$

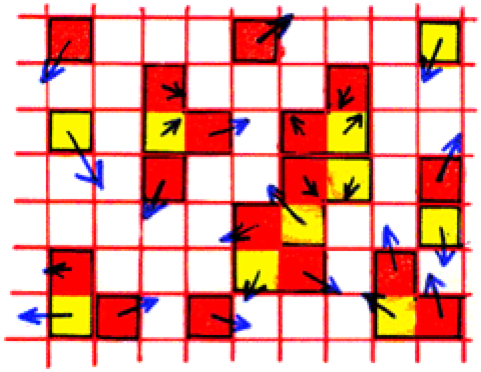


G.Lehaut, F.G., O.Lopez,PRL 2009

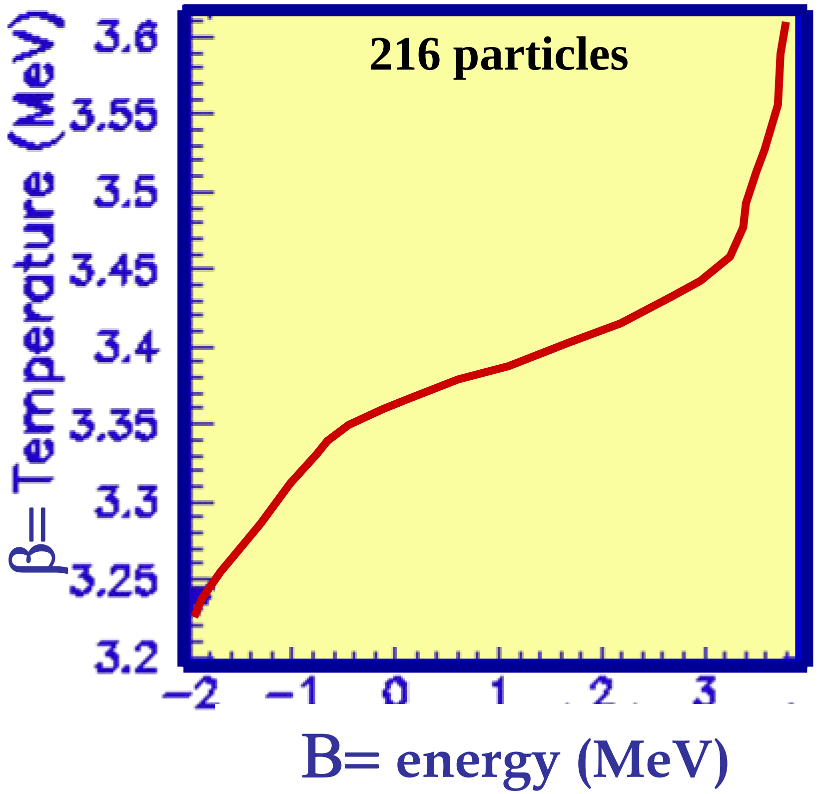
(4) Extension to finite systems

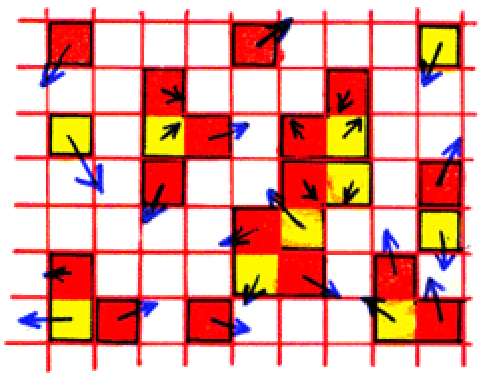
- Thermodynamic anomalies

Illustration with the LG model

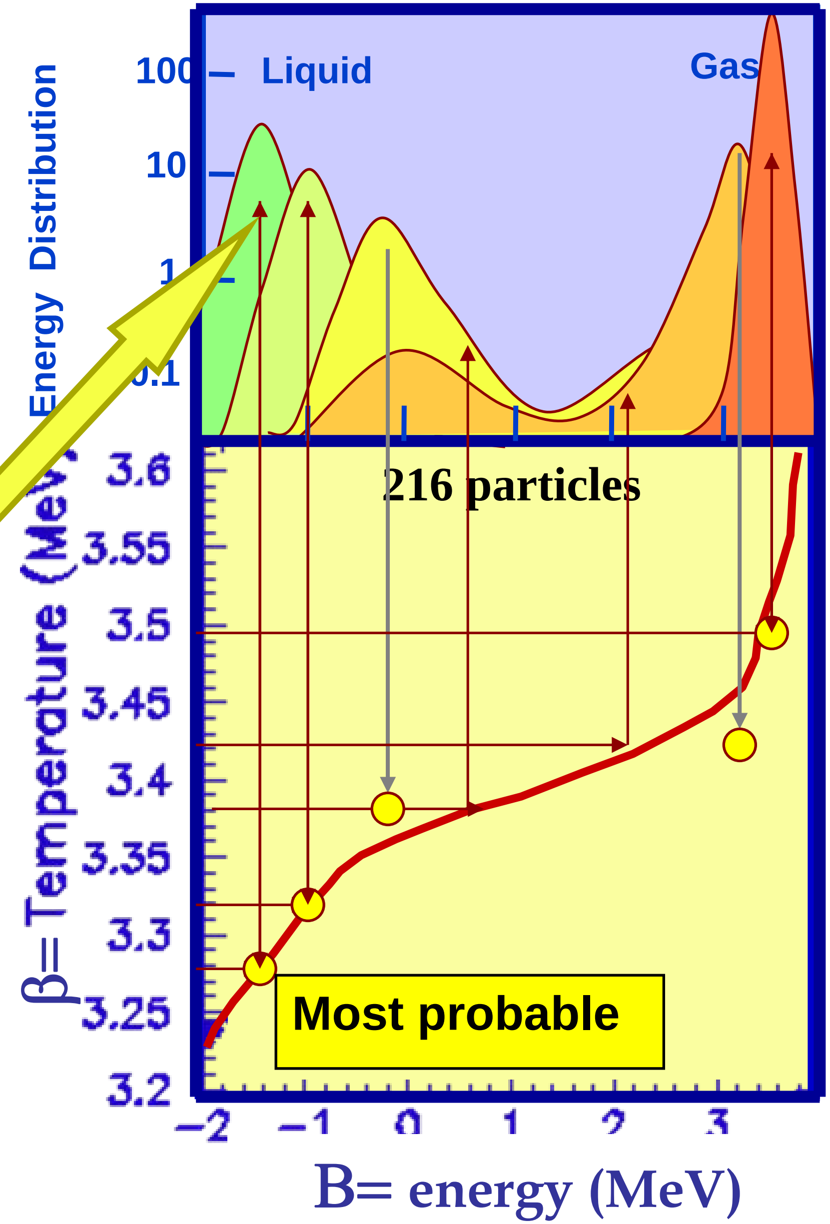


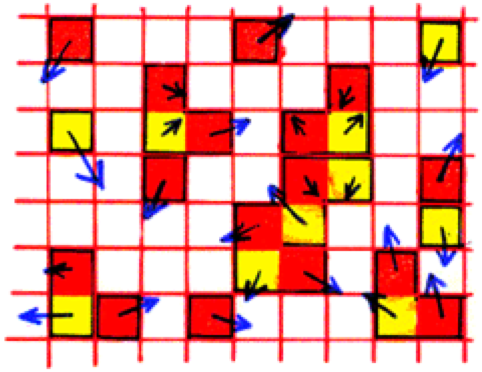
- **Closest neighbors attractive coupling**
- **Belongs to the LG universality class**
- **Temperature (β) controlled: smooth equation of state β(B)**





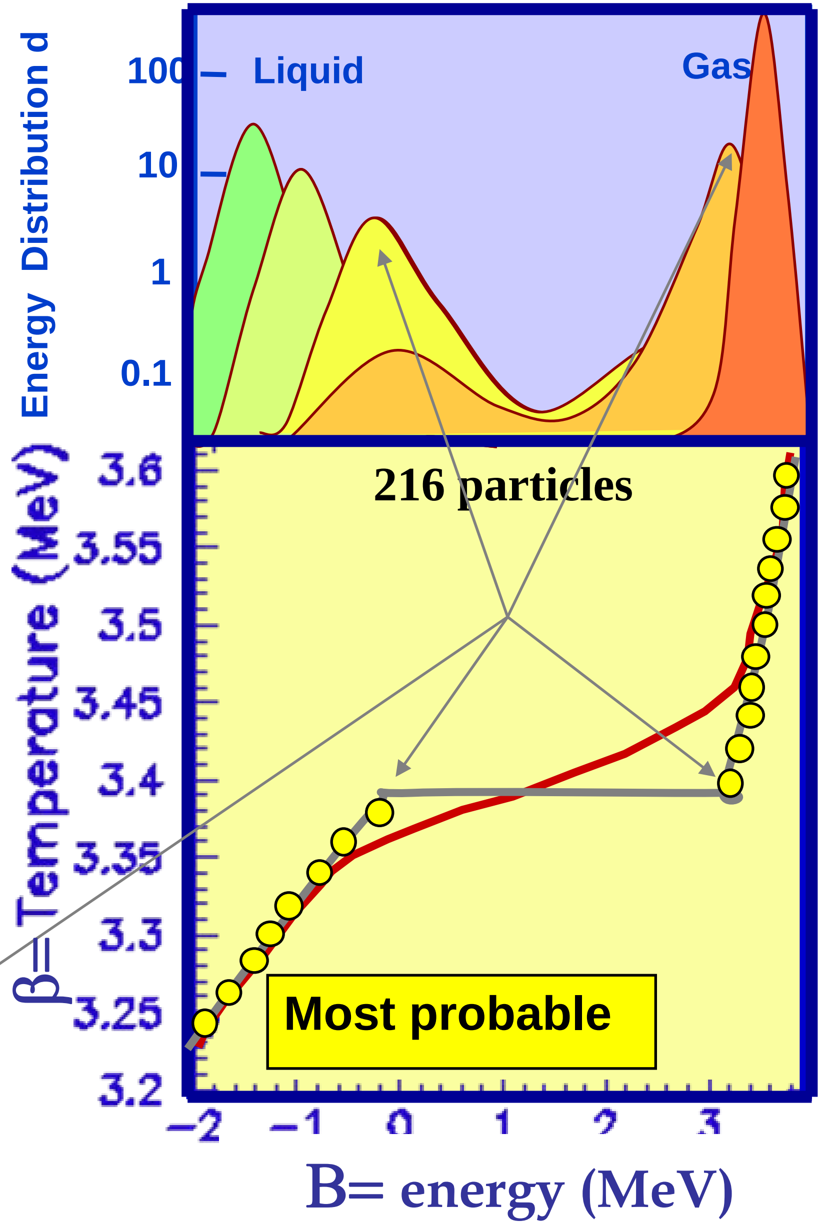
- **Energy distribution at a fixed temperature**

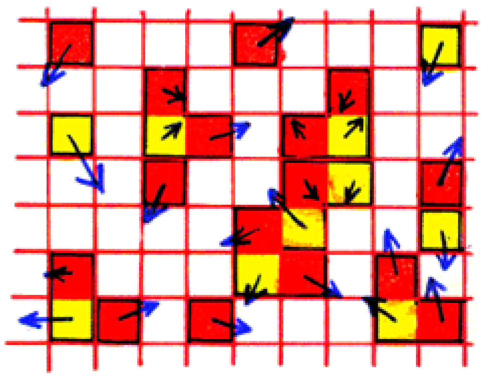




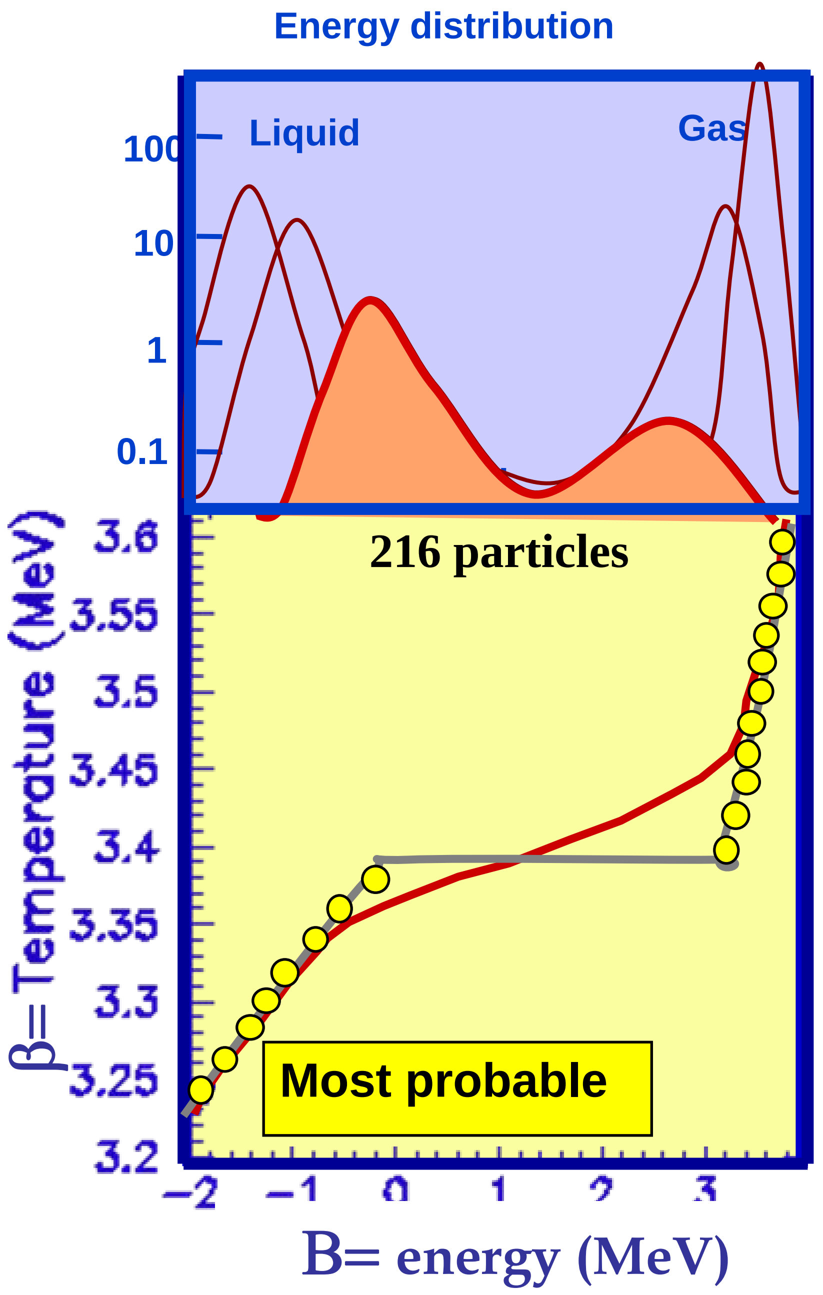
- Energy distribution at a fixed temperature

- Discontinuity of the most probable





- Energy distribution at a fixed temperature
  - Bimodal distribution
- Discontinuity of the most probable



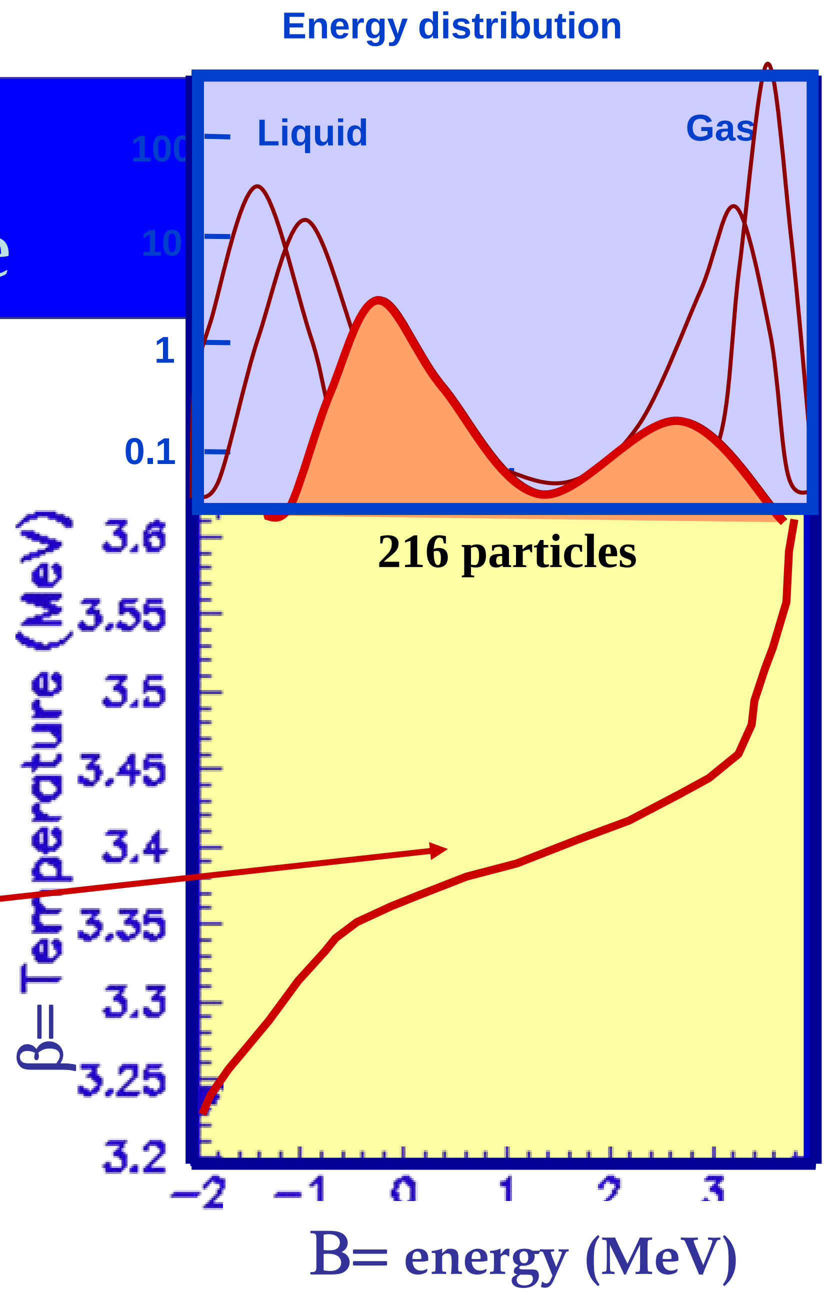
Bimodality and ensemble inequivalence

- β controlled

$$p_\beta^{(n)} = Z_\beta^{-1} e^{-\beta B^{(n)}}$$

$$p_\beta(B) = Z_\beta^{-1} \sum_n \delta\left(B^{(n)} - B\right) e^{-\beta B}$$

$$< B >= -\partial_\beta \log Z_\beta$$



- **β controlled**

$$p_\beta^{(n)} = Z_\beta^{-1} e^{-\beta B^{(n)}}$$

$$p_\beta(B) = Z_\beta^{-1} \sum_n \delta\left(B^{(n)} - B\right) e^{-\beta B}$$

$$< B >= -\partial_\beta \log Z_\beta$$

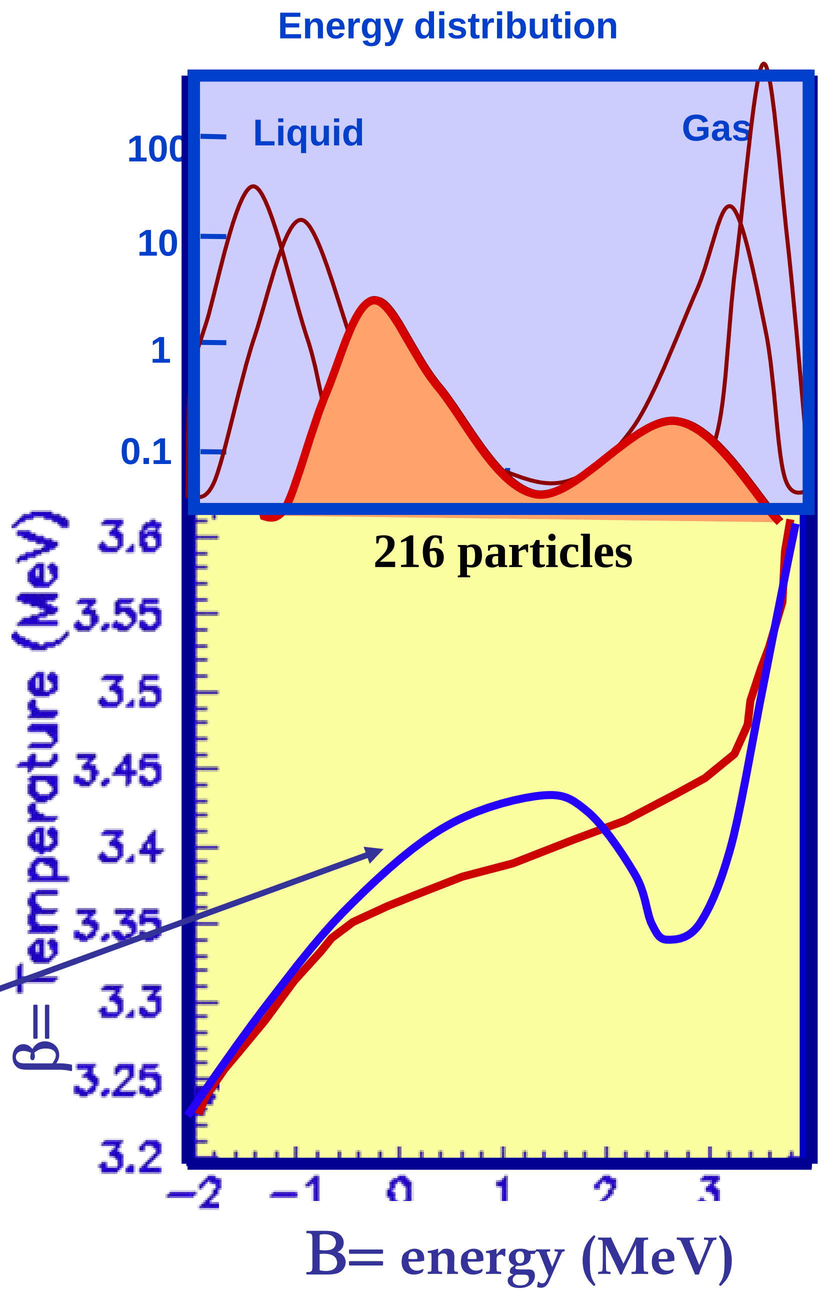
- **B controlled**

$$p_B^{(n)} = S_B^{-1} \delta\left(B^{(n)} - B\right)$$

$$S_B = -\sum_n p_B^{(n)} \log p_B^{(n)} = \sum_n \delta\left(B - B^{(n)}\right)$$

$$= \log p_{\tilde{\beta}}(B) + \log Z_{\tilde{\beta}} + \tilde{\beta} B$$

$$\beta = \partial_B S_B = \partial_B \log p_{\tilde{\beta}}(B) + \tilde{\beta}$$



Bimodality and ensemble inequivalence

- A first order phase transition occuring at the thermo limit:

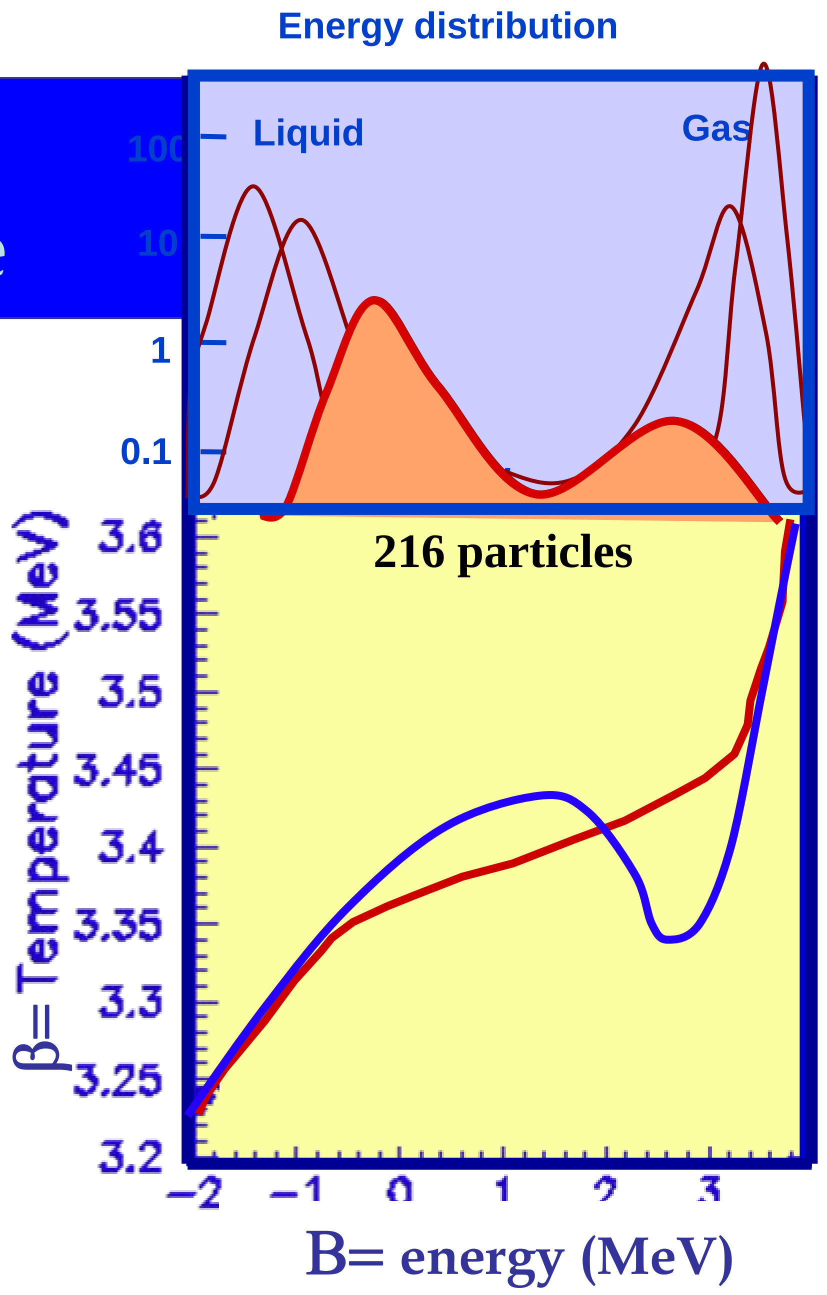
⇒ the distribution of the order parameter is bimodal in the finite system

⇒ The underlying entropy has a convex intruder

=> The EoS β<–>B is not the same if the controlled variable is extensive or intensive

⇒ The thermodynamics is not unique

⇒ In particular thermodynamics anomalies occur in the extensive ensemble: Negative heat capacity!



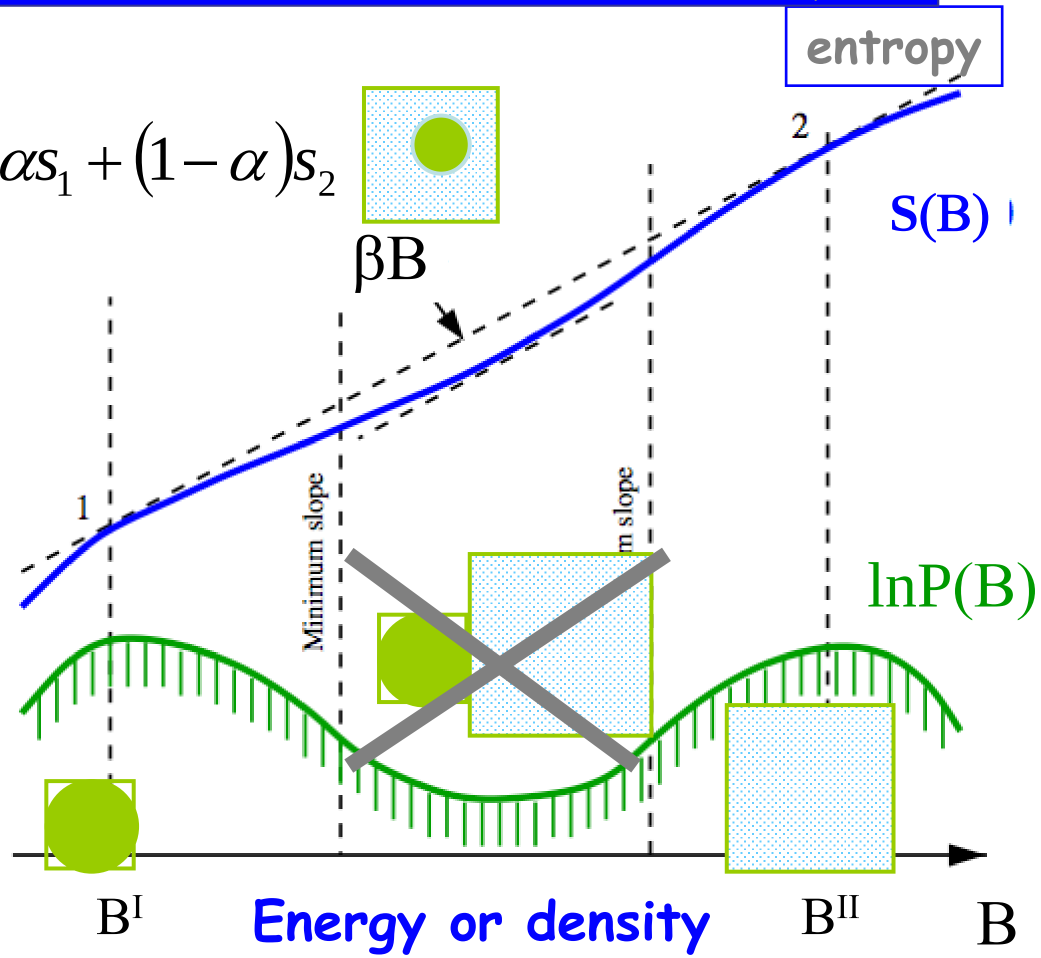
How can a convex entropy be stable ???

K.Binder D.P.Landau 1984 D.J.Wales R.S.Berry 1994

$$\frac{S(B)}{V} \neq s\left(\frac{B}{V}\right) \Rightarrow s < \alpha s_1 + (1-\alpha) s_2$$

- Interfaces cannot be neglected in a finite system
- Phase separation has an extra energy and entropy cost
- Convexities persist
- Local dishomogeneities arise

$$P_\beta(B) \propto e^{S(B)-\beta B}$$

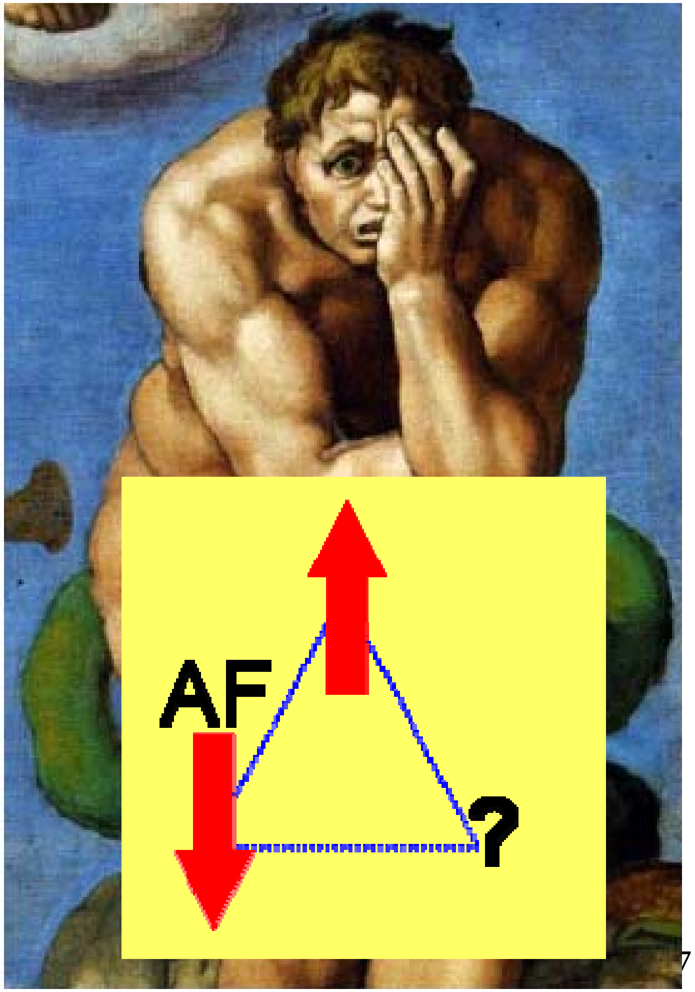


Frustration and dishomogeneous phases

- **Frustration** is a generic phenomenon in physics
- It occurs whenever matter is subject to opposite interactions on comparable length scales
- Global variations of B are replaced by local variations of B

=>Phase coexistence is quenched

=>dishomogeneous phases arise



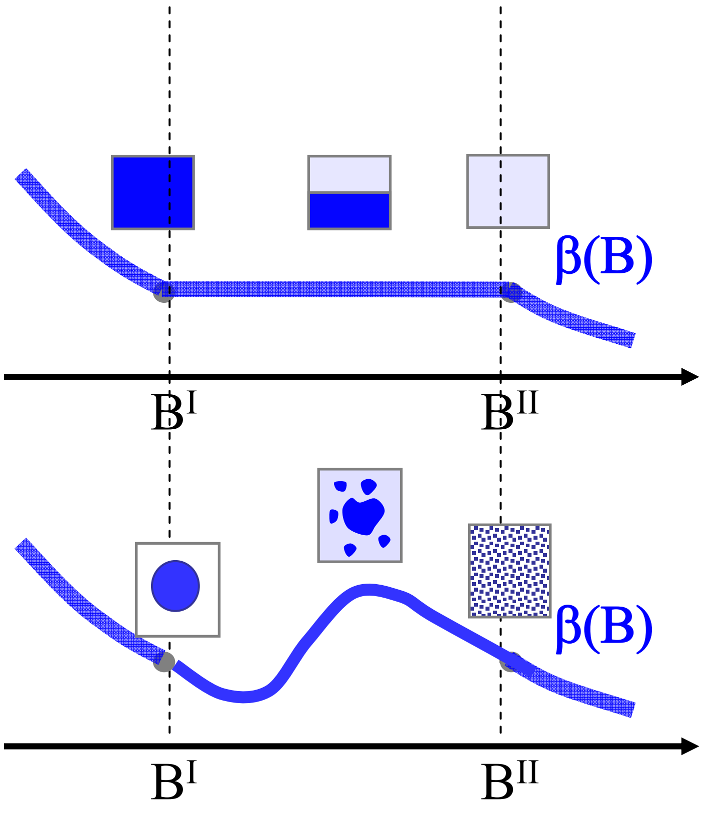
Frustration and dishomogeneous phases

- **Example I:** fluid transition in a finite system

$$E(N) = -e_{vol} N + e_{surf} N^{2/3}$$

=>Phase coexistence is quenched

=> dishomogeneous phases arise



Frustration and dishomogeneous phases

A.Raduta, F.G. 2011

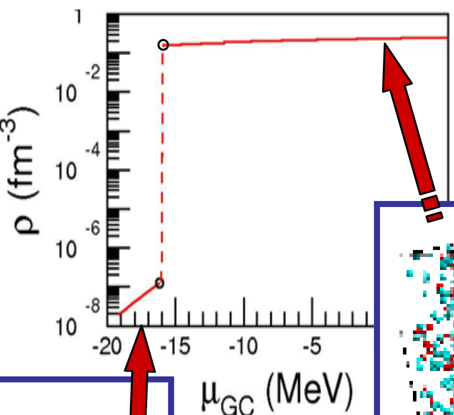
- **Example II:** crust structure of a neutron star (matter of neutrons, protons in an homogeneous electron background)

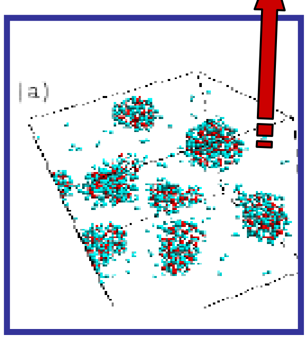
$$E = \alpha_{coul} \left\langle \left(\rho_p(r) - \rho_e \right)^2 N^{2/3}(r) \right\rangle - \alpha_{nucl} \rho$$

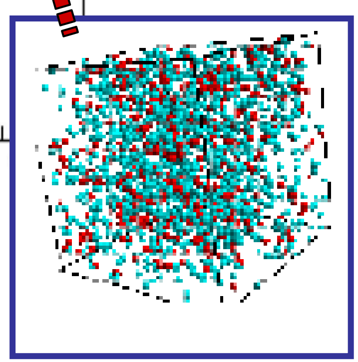
Frustration and dishomogeneous phases

A.Raduta, F.G. 2011

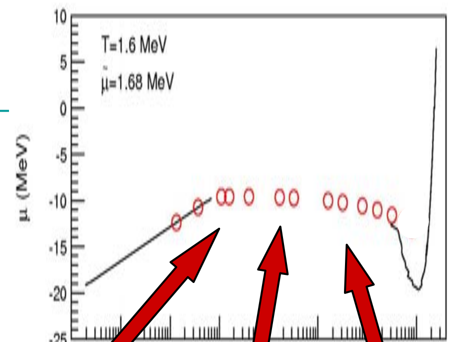
- **Example II:** crust structure of a neutron star (matter of neutrons, protons in an homogeneous electron background)

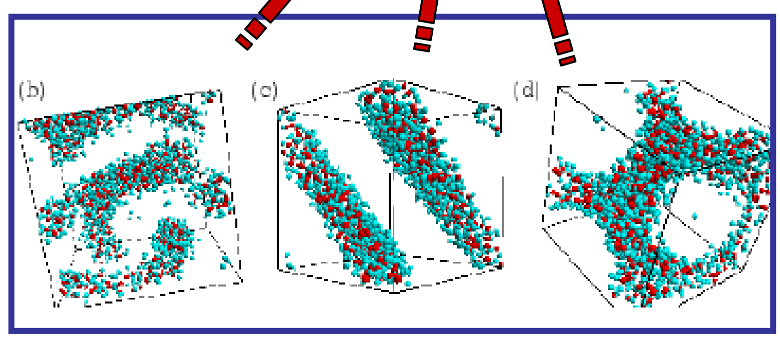






Extensive controlled





Intensive controlled

Summary

1. Phase transitions in physical systems
   - Concepts and definitions
   - Classical theory: Landau
2. First order transitions
   - Phase coexistence
   - Extension to many densities
3. Second order and critical phenomena
   - Divergence of the correlation length
   - Scale invariance and renormalization
4. Extension to finite systems
   - Precursor of phase transitions
   - Thermodynamic anomalies
   - Frustration and ensemble inequivalence