Multiscale Finite Element Method using IsoGeometric Analysis

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Abstract

The aim of this project, is to use B-splines/NURBS in a Multiscale Finite Element Method, as developed by Hou [3].

The underlying idea behind the IsoGeometric Analysis (IGA), developed recently by Hughes [1], is to use the functions (*B-splines/NURBS*) that describe the geometry in order to approach the numerical solution of pdes. In [2], the authors have applied it using the variational multiscale (VMS) method. In this project, we would like to develop a MsFEM based on IGA.

Using the classical MsFEM, we have to solve for each basis function φ_b^0, the following problem,

$$\begin{cases} \mathcal{L}^\epsilon \varphi_b = 0, & K \\ \varphi_b = \varphi_b^0, & \partial K \end{cases} \tag{1}$$

for each $K \subset \text{support}(\varphi_b^0)$. Where $\mathcal{L}^\epsilon$ denotes an elliptic differential operator.
One of the most interesting aspects of the IGA is the *k-refinement*: we can control the regularity of the Schoenberg space by increasing/decreasing the multiplicity of inserted knots. In this case, the support of the basis function is no more one cell, but can be $(p+1)^d$ cells in the case of maximum regularity. In IGA, the support is a cartesian grid in the parametric domain (patch). The number of cells will depend on the multiplicity of the correspondent knots sequence.
In the first part, we would like to study the impact of the *k-refinement* on the multiscale basis. We will compare these functions, in the case of classical formulation (using the classical boundary conditions), and a formulation on the whole support of the *B-splines*. In a second part, we will study the oversampling method. We may also study the impact of the normalization of splines on the approximation; the usual normalization is the partition unity property, but we can change the normalization in order to have integral one.
The study will be done firstly on a square as the support of a *B-spline/NURBS*. To take advantage of the tensor product property involved in the IGA, we may study the case of $1D$. In the case of a $2D$ problem, we will use the library *PyIGA* [4]. Finally, we will use a multipatchs description to treat the whole domain. A theoretical study can also be performed in order to prove the accuracy of the method, using high order *B-splines*.

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