

# A multi scale problem for thermal sciences

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## Abstract

When designing an aircraft, it is necessary to know accurately the temperatures in the different parts so as to validate the integrity of the structure and of the systems for different scenarios. The growing role of composite materials, which are 20 to 100 times less good conductors than metal structures make the need to have control over the temperatures in the parts of the plane all the more dire. Heat sources could be the sun or an electric device. One

We will study convection in air at temperatures between about  $10^\circ$  and  $100^\circ$ . The simplified problem which is posed in this document represents the insertion of an electronic device (called the Equipment (**E**)) in a larger bay (called the Cabin (**C**)). We are interested in the development of an efficient numerical resolution that enables to test quite rapidly and accurately several positioning of the equipment inside the cabin for control purposes (either by active regulation including a control loop or by passive regulation by choosing adequate materials or input flows). We are interested in the extremal temperatures inside the Equipment and the mean temperature inside the cabin.

## 1 Purpose

The cavity is embedded in a uniform external temperature  $T_{ext}$ , and surrounded by a heat insulation of constant thickness and thermal conductivity  $\kappa_{isol}$ . Inside the cavity, we consider an equipment, and conditioned air injected through a constant diameter injection nozzles with a velocity  $U$  and a temperature  $T$ .

## 2

### 2.1 Notations

### 2.2 System of equations

#### 2.2.1 Navier Stokes for incompressible fluids

Let us consider  $\Omega$  a bounded volume of  $\mathbb{R}^2$  ( $\mathbb{R}^2 = \Omega \cup \Omega^c$ ). The boundary of  $\Omega$  is denoted  $\partial\Omega$ . This domain is composed of two subdomains  $\Omega_C$  (cabin) and  $\omega_E$  (equipment) ( $\Omega = \Omega_C \cup \omega_E$ ).

The system of equations is the Navier Stokes system for incompressible fluids:

$$\left\{ \begin{array}{l} p = r\rho T \\ \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \\ \rho \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} \right) = \rho \vec{g} - \vec{\nabla} p + \vec{\nabla} \cdot [\mu (\vec{\nabla} \vec{v} + (\vec{\nabla} \vec{v})^T) + (\kappa - \frac{2}{3}\mu) \vec{U} \vec{\nabla} \cdot \vec{v}] \\ \rho c_p \left( \frac{\partial T}{\partial t} + \vec{v} \cdot \vec{\nabla} T \right) = -\vec{\nabla} \cdot (q^{cd} + \vec{q}^r) + P + T\beta \frac{dp}{dt} + \mu \phi_D \end{array} \right. \quad \forall x \in \Omega, \forall t \in \mathbb{R}_+,$$

### 2.2.2 Approximation of the model: Navier Stokes Boussinesq model

- Approximation for the fluid
  - Boussinesq approximation:  $\rho = \rho(T)$
  - Simplified Boussinesq approximation:
    - \*  $\rho(T) = \rho_0(1 - \frac{T-T_0}{T_0})$  in  $\rho \vec{g}$
    - \*  $\rho(T) = \rho_0$  elsewhere
- Other approximations:
  - 2D,
  - parameters are invariable with  $T$  and  $p$ ,
  - $\phi_D = 0, \frac{dp}{dt} = 0$ ,
  - $P = 0$  (volumic power dissipation rigourously true in our cases),
  - $\vec{q}^r = \vec{0}$  (true in the gas and temporarily between walls),
  - $\vec{\nabla} \vec{v} = (\vec{\nabla} \vec{v})^T$  (no rotation),
  - $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \Leftrightarrow \vec{U} \vec{\nabla} \cdot \vec{v} = 0$  (isovolume).

The system of equation thus becomes:

$$\left\{ \begin{array}{l} \vec{\nabla} \cdot \vec{v} = 0, \\ \rho_0 \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} \right) = 2\rho_0 \vec{g} - \frac{\rho_0}{T_0} T \vec{g} - \vec{\nabla} p + \mu \vec{\nabla} \cdot \vec{\nabla} \vec{v}, \\ \rho_0 c_p \left( \frac{\partial T}{\partial t} + \vec{v} \cdot \vec{\nabla} T \right) = \underbrace{\lambda \vec{\nabla} \cdot \vec{\nabla} T}_{-\vec{\nabla} q^{cd}} \end{array} \right. \quad \forall x \in \Omega, \forall t \in \mathbb{R}_+,$$

## 2.3 Multi scale problem

A compartment C and an electronic equipment E. The compartment is supposed to be much larger than the electronic equipment ( $vol(\Omega_C) \gg vol(\omega_E)$ ). The internal energy of the equipment is supposed to be smaller than the one contained in the cavity without the equipment.

Low coupling hypothesis:

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## 3 Toy model

### 3.1 Cavity

#### 3.1.1 Geometries

Two rectangular cavities can be defined (see figure 1 and figure 2).

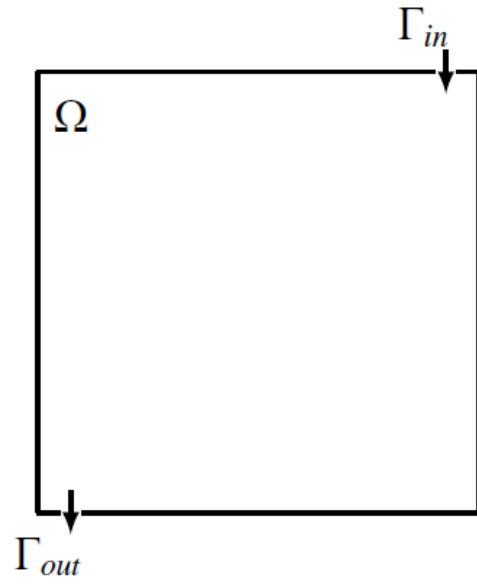


Figure 1:

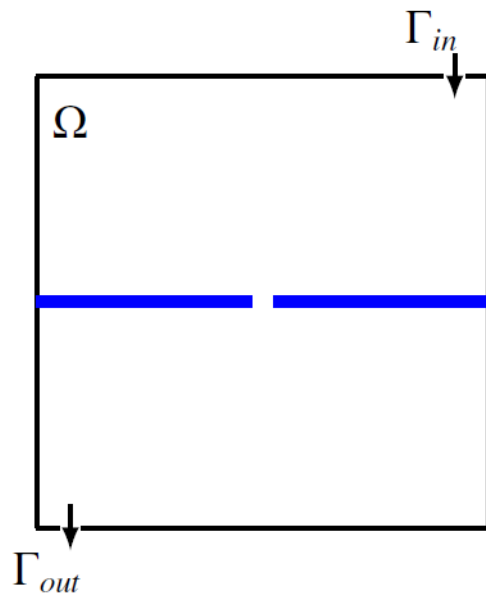


Figure 2:

### 3.1.2 Outputs of interest

- Punctual analysis:  $x \in \Omega$ ,  $T_x(\cdot) = T(x, \cdot)$
- Zonal analysis:  $\mathcal{D} \subset \Omega$ ,  $T_{\mathcal{D}}(\cdot) = \frac{1}{\mathcal{D}} \int_{\mathcal{D}} T(x, \cdot) dx$
- Global analysis:  $\Omega$ ,  $T_{\Omega}(\cdot) = \frac{1}{\Omega} \int_{\Omega} T(x, \cdot) dx$

## 3.2 Cooling of electronic components: a simplified 2D heat transfer model

We now present a simplified model of electronic equipment.

### 3.2.1 Problem description

We consider a 2D model representative of the neighboring of an electronic component submitted to a cooling air flow. It is described by four geometrical domains in  $\mathbb{R}^2$  named  $\Omega_i, i = 1, 2, 3, 4$ , see figure 3. We suppose the velocity  $\mathbf{v}$  is known in each domain — for instance in  $\Omega_4$  it is the solution of previous Navier-Stokes computations. — The temperature  $T$  of the domain  $\Omega = \cup_{i=1}^4 \Omega_i$  is then solution of heat transfer equation :

$$\rho C_i \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) - \nabla \cdot (k_i \nabla T) = Q_i, \quad i = 1, 2, 3, 4 \quad (1)$$

where  $t$  is the time and in each sub-domain  $\Omega_i$ ,  $\rho C_i$  is the volumic thermal capacity ,  $k_i$  is thermal conductivity and  $Q_i$  is a volumic heat dissipated.

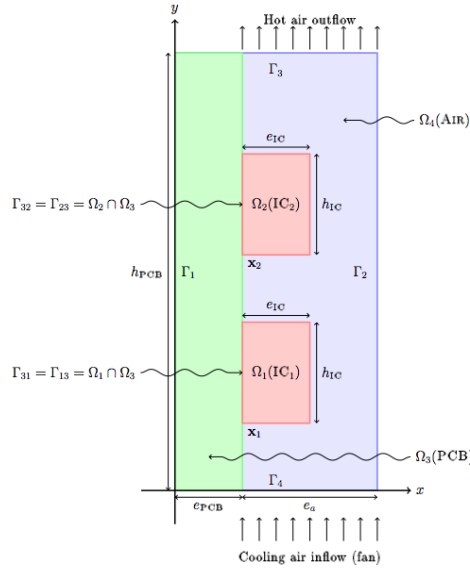


Figure 3: Geometry of  $\Omega = \cup_{i=1}^4 \Omega_i$  with  $\partial\Omega = \cup_{i=1}^4 \Gamma_i$

One should notice that the convection term in heat transfer equation may lead to spatial oscillations which can be overcome by Petrov-Galerkin type or continuous interior penalty stabilization techniques.

Integrated circuits (ICs) (domains  $\Omega_1$  and  $\Omega_2$ ) are respectively soldered on PCB at  $\mathbf{x}_1 = (e_{\text{PCB}}, h_1)$  and  $\mathbf{x}_2 = (e_{\text{PCB}}, h_2)$ . They are considered as rectangles with width  $e_{\text{IC}}$  and height  $h_{\text{IC}}$ . The printed circuit board (PCB) is a rectangle  $\Omega_3$  of width  $e_{\text{PCB}}$  and height  $h_{\text{PCB}}$ . The air (AIR) is flowing along the PCB in domain  $\Omega_4$ . Speed in the air channel  $\Omega_4$  is supposed to have a parabolic profile function of  $x$  coordinate. Its expression is simplified as follows (notice that  $\mathbf{v}$  is normally solution of Navier-Stokes equations; the resulting temperature and velocity fields should be quite different from that simplified model). We have for all  $0 \leq y \leq h_{\text{PCB}}$

$$\begin{aligned} e_{\text{PCB}} + e_{\text{IC}} \leq x \leq e_{\text{PCB}} + e_a, \quad \mathbf{v} &= \frac{3}{2(e_a - e_{\text{IC}})} D \left( 1 - \left( \frac{x - ((e_a + e_{\text{IC}})/2 + e_{\text{PCB}})}{(e_a - e_{\text{IC}})/2} \right)^2 \right) f(t) \mathbf{y} \\ e_{\text{PCB}} \leq x \leq e_{\text{PCB}} + e_{\text{IC}}, \quad \mathbf{v} &= 0 \end{aligned} \quad (2)$$

where  $f$  is a function of time modelling the starting of the PCB ventilation, *i.e.*

$$f(t) = 1 - \exp\left(-\frac{t}{3}\right), \quad (3)$$

$D$  is the air flow rate, see table 1 and  $\mathbf{y} = (0, 1)^T$  is the unit vector along the  $y$  axis. A quick verification shows that

$$\int_{\Gamma_4 \cap \Omega_4} \mathbf{v} \cdot \mathbf{n} = \int_{\Gamma_4 \cap \Omega_4} v_y = D \quad (4)$$

The medium velocity  $\mathbf{v}_i = \mathbf{0}$ ,  $i = 1, 2, 3$  in the solid domains  $\Omega_i$ ,  $i = 1, 2, 3$ . ICs dissipate heat, we have respectively

$$\begin{aligned} Q_1 &= Q(1 - \exp(-t)) && \text{in } \Omega_1 \\ Q_2 &= Q(1 - \exp(-t)) && \text{in } \Omega_2 \end{aligned} \quad (5)$$

where  $Q$  is defined in table 1.

We shall denote  $\mathbf{n}_{|\Omega_i} = \mathbf{n}_i$  the unit outward normal to  $\Omega_i$  and  $\mathbf{n}_{|\Omega_j} = \mathbf{n}_j$  the unit outward normal to  $\Omega_j$ .

**Boundary conditions** We set

1. on  $\Gamma_3$ , a zero flux (Neumann) condition

$$-k_3 \nabla T \cdot \mathbf{n}_3 = -k_4 \nabla T \cdot \mathbf{n}_4 = 0; \quad (6)$$

2. on  $\Gamma_4$ , ( $0 \leq x \leq e_{\text{PCB}} + e_a, y = 0$ ) the temperature is set (Dirichlet condition)

$$T = T_0; \quad (7)$$

3. between  $\Gamma_1$  and  $\Gamma_2$ , periodic conditions

$$\begin{aligned} T \Big|_{\mathbf{x}=0} &= T \Big|_{\mathbf{x}=e_{\text{PCB}}+e_a} \\ -k_3 \nabla T \cdot \mathbf{n}_3 \Big|_{\mathbf{x}=0} &= k_4 \nabla T \cdot \mathbf{n}_4 \Big|_{\mathbf{x}=e_{\text{PCB}}+e_a}; \end{aligned} \quad (8)$$

4. at interfaces between the ICs and PCB, there is a thermal contact conductance:

$$\begin{aligned} -k_1 \nabla T \cdot \mathbf{n}_1 &= k_3 \nabla T \cdot \mathbf{n}_3 = r_{13} (T_{\partial\Omega_1} - T_{\partial\Omega_3}) \\ -k_2 \nabla T \cdot \mathbf{n}_2 &= k_3 \nabla T \cdot \mathbf{n}_3 = r_{23} (T_{\partial\Omega_2} - T_{\partial\Omega_3}); \end{aligned} \quad (9)$$

5. on other internal boundaries, the continuity of the heat flux and temperature, on  $\Gamma_{ij} = \Omega_i \cap \Omega_j \neq \emptyset$

$$\begin{aligned} T_i &= T_j \\ k_i \nabla T \cdot \mathbf{n}_i &= -k_j \nabla T \cdot \mathbf{n}_j. \end{aligned} \quad (10)$$

**Initial condition** At  $t = 0s$ , we set  $T = T_0$ .

### 3.2.2 Inputs

The table 1 displays the various fixed and variables parameters of this test-case.

Name	Description	Nominal Value	Range	Units
Parameters				
$t$	time		[0, 1500]	$s$
$Q$	heat source	$10^6$	[0, $10^6$ ]	$W \cdot m^{-3}$
IC Parameters				
$k_1 = k_2 = k_{IC}$	thermal conductivity	2	[0.2, 150]	$W \cdot m^{-1} \cdot K^{-1}$
$r_{13} = r_{23} = r$	thermal conductance	100	[ $10^{-1}$ , $10^2$ ]	$W \cdot m^{-2} \cdot K^{-1}$
$\rho C_{IC}$	heat capacity	$1.4 \cdot 10^6$		$J \cdot m^{-3} \cdot K^{-1}$
$e_{IC}$	thickness	$2 \cdot 10^{-3}$		$m$
$h_{IC} = L_{IC}$	height	$2 \cdot 10^{-2}$		$m$
$h_1$	height	$2 \cdot 10^{-2}$		$m$
$h_2$	height	$7 \cdot 10^{-2}$		$m$
PCB Parameters				
$k_3 = k_{PCB}$	thermal conductivity	0.2		$W \cdot m^{-1} \cdot K^{-1}$
$\rho C_3$	heat capacity	$2 \cdot 10^6$		$J \cdot m^{-3} \cdot K^{-1}$
$e_{PCB}$	thickness	$2 \cdot 10^{-3}$		$m$
$h_{PCB}$	height	$13 \cdot 10^{-2}$		$m$
Air Parameters				
$T_0$	Inflow temperature	300		$K$
$D$	Inflow rate	$7 \cdot 10^{-3}$	[ $5 \cdot 10^{-4}$ , $10^{-2}$ ]	$m^2 \cdot s^{-1}$
$k_4$	thermal conductivity	$3 \cdot 10^{-2}$		$W \cdot m^{-1} \cdot K^{-1}$
$\rho C_4$	heat capacity	1100		$J \cdot m^{-3} \cdot K^{-1}$
$e_a$	thickness	$4 \cdot 10^{-3}$	[ $2.5 \cdot 10^{-3}$ , $5 \cdot 10^{-2}$ ]	$m$

Table 1: Table of fixed and variable parameters

### 3.2.3 Outputs, variables of interest

The outputs are (i) the mean temperature  $s_1(\mu)$  of the hottest IC

$$s_1(\mu) = \frac{1}{e_{\text{IC}} h_{\text{IC}}} \int_{\Omega_2} T \quad (11)$$

and (ii) mean temperature  $s_2(\mu)$  of the air at the outlet

$$s_2(\mu) = \frac{1}{e_a} \int_{\Omega_4 \cap \Gamma_3} T \quad (12)$$

both depends on the solution of (1) and are dependent on the parameter set  $\mu$ .

### 3.2.4 Methodology

The model can be described with increasing difficulty:

**Step 1:** Consider the problem as stationary, with only 2 varying parameters  $k_{\text{IC}}$  and  $r$  in their variation interval.

**Step 2:** Consider the problem step 1 plus variation of geometrical parameters  $e_a$ .

**Step 3:** Consider the problem as stationary with the six varying parameters listed in section 3.2.2.

**Step 4:** Consider the full transient problem with the six varying parameters as described above.

**Step 5:**  $D$  may be considered as a  $C^1$  function of time bounded in the defined interval.

**Next steps:** Depending on method capabilities, we can go on thermally dependant EDP coefficients, Navier-Stokes coupling or adding surface-to-surface radiation in boundary conditions. This should be discussed.

## 3.3 Cavity+equipment

Two rectangular cavities are defined see figure 4 and figure 5.

## 4 Problem

We propose to address the problem of thermal control in three steps with increasing difficulties:

1. Computation of the cabin without equipment
2. Computation of the equipment out of the cabin
3. Computation of both cabin and equipement.

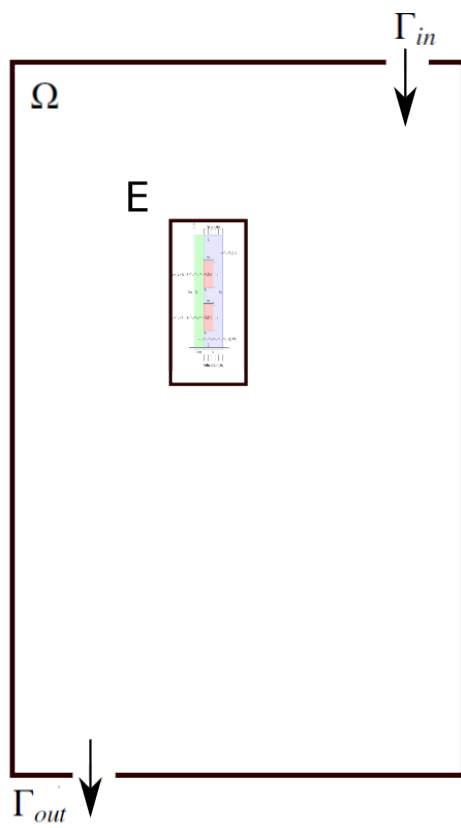


Figure 4:





Figure 5:

The prediction of temperature previously described can be involved in various contexts:

- In preliminary design, the main goal is to choose the values of the design parameters  $\kappa_{isol}$ ,  $C$ ,  $T_{ext}$ ,  $P$ ,  $T_b$  and  $T_{obs}$  which insure that the temperature field is between two critical values. In a first step, this is done by a direct propagation of uncertainties, but eventually, the objective will be the robust optimization of the parameters under uncertainties constraints. Moreover, a sensitivity analysis will determine the most important parameters for a robust control of the temperature.
- In a verification phase, the design parameters are determined and fixed and the requirement is to prove that the temperature has a target value or is not lower than a threshold.
- If the design involves different cooperants, the architect specifies the bounds of variation (or distributions of fluctuations) for the parameters  $\kappa_{isol}$ ,  $C$ ,  $T_{ext}$ ,  $P$ ,  $T_b$  and  $T_{obs}$ , which are consistent with a temperature between two given bounds.

If needed, the methodology can be developed on a simplified problem such as a single equation involving different domains.

## 5 Material

Numerical models of the equipment and the cavity will be distributed in FreeFEM++.