Gyrokinetic simulations of magnetic fusion plasmas

Tutorial 2

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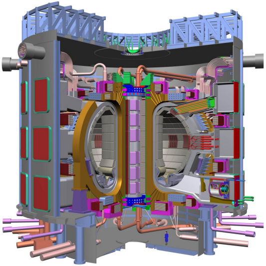
Summary of Tutorial 1

Understanding and predicting physics in ITER

Predicting density and temperature in magnetised plasma is a subject of utmost importance in view of understanding and optimizing experiments in the present fusion devices and also for designing future reactors.

- Certainty : Turbulence limit the maximal value reachable for n and T
  - Generate loss of heat and particles
  - $\searrow$ Confinement properties of the magnetic configuration



Turbulence study in tokamak plasmas

Plasma turbulence

How to model plasma for turbulence study ?

$\Downarrow$

Kinetic turbulence is the best candidate

$\Downarrow$

Vlasov-Maxwell system

$\Downarrow$

A reduced electrostatic model: Vlasov-Poisson system

Some useful Vlasov equation properties

Advective form of Vlasov equation

- Advective form:

$$\frac{\partial}{\partial t} f(\mathbf{Z}, t) + \mathbf{U}(\mathbf{Z}, t) \cdot \nabla_{\mathbf{z}} f(\mathbf{Z}, t) = 0 \qquad (1)$$

- Another equivalent writing of the equation (1) is

$$\frac{\partial f}{\partial t} + \frac{\mathrm{d}\mathbf{Z}}{\mathrm{d}t} \cdot \nabla_{\mathbf{z}} f = 0$$

because of the characteristic equation

$$\frac{\mathrm{d}\mathbf{Z}}{\mathrm{d}t} = \mathbf{U}(\mathbf{Z}(t), t)$$

f constant along characteristics

- Which gives that the total time derivative of f

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \partial_t f + \frac{\mathrm{d}\mathbf{Z}}{\mathrm{d}t} \cdot \nabla_{\mathbf{z}} f$$

is equal to 0, i.e:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = 0 \tag{2}$$

- Fundamental property of the Vlasov equation: the distribution function f is constant along its characteristics.
- As we will see later, this property is one of the foundation of the semi-Lagrangian numerical approach.

Conservative form of Vlasov equation

- For the Vlasov equation the phase space element is incompressible
- The Liouville theorem applies– $\nabla_{\mathbf{z}} \mathbf{U} = 0$

Then the previous advective form of the Vlasov equation (1) is equivalent to the following equation ➠ conservative form of the Vlasov equation:

$$\frac{\partial}{\partial t} f(\mathbf{Z}, t) + \nabla_{\mathbf{z}} \cdot (\mathbf{U}(\mathbf{Z}, t)\, f(\mathbf{Z}, t)) = 0 \tag{3}$$

because

$$\nabla_{\mathbf{z}} \cdot (\mathbf{U}\, f) = \mathbf{U} \cdot \nabla_{\mathbf{z}} f + f \cdot \nabla_{\mathbf{z}} \mathbf{U} = \mathbf{U} \cdot \nabla_{\mathbf{z}} f$$

- The Liouville theorem expresses therefore the fact that the advective form and the conservative form of the Vlasov equation are equivalent.

- We will see later that both forms are used depending on the numerical scheme which is chosen to solve the system.

Kinetic theory

$$\Downarrow$$

Gyrokinetic theory

From kinetics to gyro-kinetics

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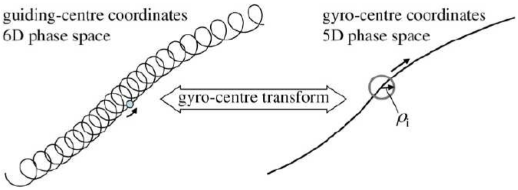
- Fusion plasma turbulence is low frequency:

$$\omega_{turb} \sim \omega_{*i} \sim (k_\theta \rho_i)\frac{v_{th}}{L_p} \sim 10^5 s^{-1} \ll \omega_{ci} = \frac{eB}{m_i} \sim 10^8 s^{-1}$$

- Phase space reduction: fast gyro-motion is averaged out

$$f(\mathbf{x}, v_\parallel, v_\perp, \varphi_c, t) \rightarrow \bar{f}(\mathbf{x}_G, v_{G\parallel}, \mu, t)$$



Gyrokinetic ordering in a small parameter ϵ_g (1/3)

- Besides, experimental observations in core plasmas of magnetic confinement fusion devices suggest that small scale turbulence, responsible for anomalous transport, obeys the following ordering in a small parameter ϵ_g
- Slow time variation as compared to the gyro-motion time scale

$$\omega/\omega_{ci} \sim \epsilon_g \ll 1 \qquad (\omega_{ci} = eB/m_i)$$

- Spatial equilibrium scale much larger than the Larmor radius

$$\rho/L_n \sim \rho/L_T \equiv \epsilon_g \ll 1$$

where $L_n = |\nabla \ln n_0|^{-1}$ and $L_T = |\nabla \ln T|^{-1}$ the characteristic lengths of n_0 and T.

Gyrokinetic ordering in a small parameter ϵ_g (2/3)

- Small perturbation of the magnetic field

$$B/\delta B \sim \epsilon_g \ll 1$$

where B and δB are respectively the equilibrium and the perturbed magnetic field

- Strong anisotropy, i.e only perpendicular gradients of the fluctuating quantities can be large ($k_\perp \rho \sim 1$, $k_\| \rho \sim \epsilon_g$)

$$k_\| / k_\perp \sim \epsilon_g \ll 1$$

where $k_\| = \mathbf{k} \cdot \mathbf{b}$ and $k_\perp = |\mathbf{k} \times \mathbf{b}|$ are parallel and perpendicular components of the wave vector $\mathbf{k}$ with $\mathbf{b} = \mathbf{B}/B$

- Small amplitude perturbations, i.e energy of perturbation much smaller than the thermal energy

$$e\phi / T_e \sim \epsilon_g \ll 1$$

Gyrokinetic model: Reduction from 6D to 5D

- The gyrokinetic model is a Vlasov-Maxwell on which the previous ordering is imposed
- Performed by eliminating high-frequency processes characterized by $\omega > \Omega_s$.
- The phase space is reduced from 6 to 5 dimensions, while retaining crucial kinetic effects such as finite Larmor radius effects.

Numerical gain

- Numerically speaking, the computational cost is dramatically reduced because the limitations on the time step and the grid discretization are relaxed from

$$\omega_{ps}\,\Delta t < 1 \quad \text{and} \quad \Delta x < \lambda_{Ds}$$

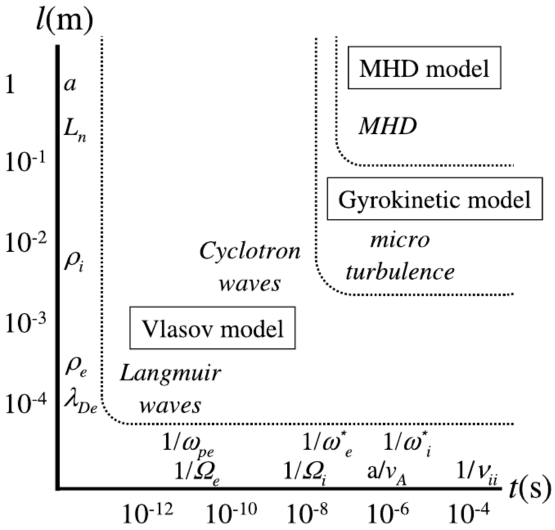
to

$$\omega_s^*\,\Delta t < 1 \quad \text{and} \quad \Delta x < \rho_s$$

with ω_{ps} the plasma oscillation frequency and λ_{Ds} the Debye length

- A gain of more than 2 order of magnitude in spatial and temporal discretization

Typical space and time range scales



μ an adiabatic invariant

- It is also important to note that the magnetic moment,

$$\mu_s = m_s v_\perp^2/(2B)$$

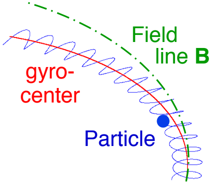
  becomes an adiabatic invariant.
- In terms of simulation cost, this last point is convenient because μ_s plays the role of a parameter.
- This means that the problem to treat is not a true 5D problem but rather a 4D problem parametrized by μ_s.
- Note that μ_s looses its invariance property in the presence of collisions.
- Such a numerical drawback can be overcome by considering reduced collisions operators acting in the $v_\|$ space only, while still recovering the results of the neoclassical theory *[Garbet, PoP 2009]*.

Road map of gyro-kinetic theory

- Two main challenges for the theory:

  1. To transform Vlasov eq. df/dt=0
     into the gyro-kinetic eq. governing $\bar{f}$ dynamics
     $\Rightarrow$ gyro-center eqs. of motion

  2. To write Maxwell's eqs. in terms of $\bar{f}$



- Modern formulation:
  Lagrangian formalism & Lie perturbation theory
  [Brizard-Hahm, *Rev. Mod. Phys.* (2007)]

Gyrokinetic equation

The resulting gyrokinetic equation is today the most advanced framework to describe plasma micro-turbulence.

$$B_{\parallel}^{*}\frac{\partial \bar{f}_s}{\partial t} + \nabla \cdot \left(B_{\parallel}^{*}\frac{\mathrm{d}\mathbf{x}_G}{\mathrm{d}t}\ \bar{f}_s \right) + \frac{\partial}{\partial v_{G\parallel}} \left(B_{\parallel}^{*}\frac{\mathrm{d}v_{G\parallel}}{\mathrm{d}t}\ \bar{f}_s \right) = 0 \qquad (4)$$

In the electrostatic limit, the equations of motion of the guiding centers are given below:

$$B_{\parallel}^{*}\frac{\mathrm{d}\mathbf{x}_G}{\mathrm{d}t} = v_{G\parallel}\mathbf{B}_{\parallel}^{*} + \frac{\mathbf{b}}{e_s} \times \nabla \Xi \qquad (5)$$

$$B_{\parallel}^{*}\frac{\mathrm{d}v_{G\parallel}}{\mathrm{d}t} = -\frac{\mathbf{B}_{\parallel}^{*}}{m_s} \cdot \nabla \Xi \qquad (6)$$

with

$$\nabla \Xi = \mu_s \nabla B + e_s \nabla \bar{\phi} \quad \text{and} \quad \mathbf{B}_{\parallel}^{*} = \mathbf{B} + (m_s/e_s)\, v_{G\parallel} \nabla \times \mathbf{b}$$

References for modern gyrokinetic derivation

- For an overview and a modern formulation of the gyrokinetic derivation, see the review paper by A.J. Brizard and T.S. Hahm, *Foundations of nonlinear gyrokinetic theory*, Rev. Mod. Phys (2007).
- This new approach is based on Lagrangian formalism and Lie perturbation theory (see *e.g.* J.R Cary *[Physics Reports (1981)]*, J.R Cary and Littlejohn *[Annals of Physics (1983)]*
- The advantage of this approach is to preserve the first principles by construction, such as the symmetry and conservation properties of the Vlasov equation – particle number, momentum, energy and entropy.

The gyro-kinetic equation

- The gyro-kinetic eq. exhibits a conservative form:

$$B_{\parallel}^{*}\frac{\partial \bar{f}_s}{\partial t} + \boldsymbol{\nabla}\cdot\left(B_{\parallel}^{*}\dot{\mathbf{x}}_G\ \bar{f}_s\right) + \frac{\partial}{\partial v_{G\parallel}}\left(B_{\parallel}^{*}\dot{v}_{G\parallel}\ \bar{f}_s\right) = 0$$

with $B_{\parallel}^{*} \sim B$

- Notice:
  - Similar structure as Vlasov eq. $\rightarrow$ conservation properties
  - Magnetic moment $\mu = \dfrac{mv_{\perp}^2}{2B}$ has become an (adiabatic) invariant $\rightarrow$ parameter (if collisionless)
  - Averaging process $\Rightarrow$ velocity drifts $\dot{\mathbf{x}}_G$ of the gyro-center

How to get drifts out of cyclotron motion?

Challenge: cutting the wings while preserving the motion

How to get drifts out of cyclotron motion?

- Adiabatic limit framework:

  Magnetic field evolves slowly w.r.t. ω_{ci}

$$\partial_t \log B \sim \mathbf{v} \cdot \boldsymbol{\nabla} \log B \ll \omega_c \quad \Rightarrow \quad \frac{\rho_s}{R} \sim \frac{m v_\parallel}{eBR} \sim \frac{m v_\perp}{eBR} \ll 1$$

- Scale separation:

  average over fast time scale

$$\begin{cases} \mathbf{v} = \mathbf{v}_G + \tilde{\mathbf{v}} \\ \mathbf{B} = \mathbf{B}_G + \tilde{\mathbf{B}} \\ \mathbf{E} = \mathbf{E}_G + \tilde{\mathbf{E}} \end{cases} \quad \text{with} \quad \langle \tilde{\mathbf{y}} \rangle \doteq \oint \frac{d\varphi_c}{2\pi} \tilde{\mathbf{y}} = 0$$

- Perturbation theory – Solving at leading orders the small parameter $\epsilon = \rho_s / R \ll 1$

Average over the cyclotron motion

- The gyro-radius $\boldsymbol{\rho}_s$ is transverse to $\mathbf{b} = \mathbf{B}/B$ and depends on the gyrophase angle φ_c:

$$\boldsymbol{\rho}_s = \frac{\mathbf{b} \times \mathbf{v}}{\Omega_s} = \rho_s \left[\cos\varphi_c \, \mathbf{e}_{\perp 1} + \sin\varphi_c \, \mathbf{e}_{\perp 2}\right] \tag{7}$$

  where $\mathbf{e}_{\perp 1}$ and $\mathbf{e}_{\perp 2}$ are the unit vectors of a cartesian basis in the plane perpendicular to the magnetic field direction $\mathbf{b}$.
- Let $\mathbf{x}_G$ be the guiding-center radial coordinate and $\mathbf{x}$ the position of the particle in the real space.
- These two quantities differ by a Larmor radius $\boldsymbol{\rho}_s$:

$$\mathbf{x} = \mathbf{x}_G + \boldsymbol{\rho}_s$$

Gyroaverage operator

- The gyro-average $\bar{g}$ of any function g depending on the spatial coordinates corresponds to the following operation:

$$\bar{g}(\mathbf{x}_G, v_\perp) = \oint_0^{2\pi} \frac{\mathrm{d}\varphi_c}{2\pi} g(\mathbf{x}) = \left\{ \oint_0^{2\pi} \frac{\mathrm{d}\varphi_c}{2\pi} \exp(\boldsymbol{\rho}_s \cdot \boldsymbol{\nabla}) \right\} g(\mathbf{x}_G)$$

- The operator $\mathrm{e}^{\boldsymbol{\rho}_s \cdot \boldsymbol{\nabla}}$ corresponds to the change of coordinates $(\mathbf{x}, \mathbf{p}) \rightarrow (\mathbf{x}_G, \mathbf{p}_G)$.
- The inverse operator governing the transformation $(\mathbf{x}_G, \mathbf{p}_G) \rightarrow (\mathbf{x}, \mathbf{p})$ simply reads $\mathrm{e}^{-\boldsymbol{\rho}_s \cdot \boldsymbol{\nabla}}$.
- This gyro-average process consists in computing an average on the Larmor circle. It tends to damp any fluctuation which develops at sub-Larmor scales.

In Fourier space ➠ Bessel operator

- Introducing $\hat{g}(\mathbf{k})$ the Fourier transform of g, with $\mathbf{k}$ the wave vector, then the operation of gyro-average reads:

$$
\begin{aligned}
\bar{g}(\mathbf{x}_G, v_\perp) &= \int_0^{2\pi} \frac{\mathrm{d}\varphi_c}{2\pi} \int_{-\infty}^{+\infty} \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\, \hat{g}(\mathbf{k}) \exp\{i\mathbf{k}\cdot(\mathbf{x}_G + \boldsymbol{\rho}_s)\} \\
&= \int_{-\infty}^{+\infty} \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \left[\int_0^{2\pi} \frac{\mathrm{d}\varphi_c}{2\pi} \exp(ik_\perp \rho_s \cos\varphi_c)\right] \hat{g}(\mathbf{k}) \exp(i\mathbf{k}\cdot\mathbf{x}_G) \\
&= \int_{-\infty}^{+\infty} \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} J_0(k_\perp \rho_s) \hat{g}(\mathbf{k}) \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}_G}
\end{aligned}
$$

- where, $k_\perp$ is the norm of the transverse component of the wave vector $\mathbf{k}_\perp = \mathbf{k} - (\mathbf{b}.\mathbf{k})\mathbf{b}$, and J_0 is the Bessel function of first order.

Scale separation: gyro-motion + drifts

- The dynamics of a non relativistic charged particle s in an electromagnetic field obeys the following equation:

$$m_s \frac{\mathrm{d}\mathbf{v}_s}{\mathrm{d}t} = e_s\{\mathbf{E}(\mathbf{x}, t) + \mathbf{v}_s \times \mathbf{B}(\mathbf{x}, t)\}$$

- Main idea: considering the fast time average of Newton's equations in the adiabatic limit
  - At leading order, $\langle \mathbf{B} \rangle$ can be approximated by its value at the position of the guiding-center $\mathbf{B}_G$
  - Conversely, there is no such a hierarchy for the velocities, $\tilde{\mathbf{v}}$ and $\mathbf{v}_G$ being of the same order of magnitude a priori.

Scale separation: gyro-motion + drifts

- Fast motion = cyclotron motion:

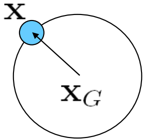
$$\frac{d\tilde{\mathbf{v}}}{dt} = \frac{e}{m}\tilde{\mathbf{v}} \times \mathbf{B}_G \longrightarrow \tilde{\mathbf{v}} = \frac{e}{m}\boldsymbol{\rho}_s \times \mathbf{B}_G$$

- Slow motion = drifts:

$$\frac{d\mathbf{v}_G}{dt} = \frac{e}{m}\left\{\mathbf{E}_G + \mathbf{v}_G \times \mathbf{B}_G + \langle \tilde{\mathbf{v}} \times \tilde{\mathbf{B}} \rangle\right\}$$

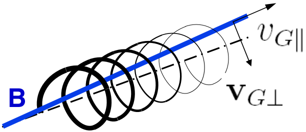
$$\mathbf{E}_G = \left\langle e^{\boldsymbol{\rho}_s \cdot \boldsymbol{\nabla}} \right\rangle \mathbf{E} \qquad \tilde{\mathbf{B}} \simeq (\boldsymbol{\rho}_s \cdot \boldsymbol{\nabla})\mathbf{B}_G$$

(adiabatic limit)



$$\Rightarrow \quad \langle \tilde{\mathbf{v}} \times \tilde{\mathbf{B}} \rangle = -\frac{\mu}{e}\boldsymbol{\nabla} B_G$$ at leading order in ϵ

Transverse drifts



- Transverse & parallel dynamics:

$$\mathbf{v}_G \equiv v_{G\|}\mathbf{b} + \mathbf{v}_{G\perp} \quad \text{(with } \mathbf{b} = \frac{\mathbf{B}}{B}\text{)}$$

- Projection on the transverse plane ($\mathbb{I}_\perp = \mathbb{I} - \mathbf{b}\otimes\mathbf{b}$):

$$\left.\frac{d\mathbf{v}_{G\perp}}{dt}\right|_\perp + \left.\frac{d\mathbf{v}_\|}{dt}\right|_\perp = \frac{e}{m}\left(\mathbf{E}_{G\perp} + \mathbf{v}_G \times \mathbf{B}\right) - \frac{\mu}{m}\nabla_\perp B$$

$$\sim \epsilon^2 \qquad {v_{G\|}}^2 \frac{\mathbf{N}}{R}$$

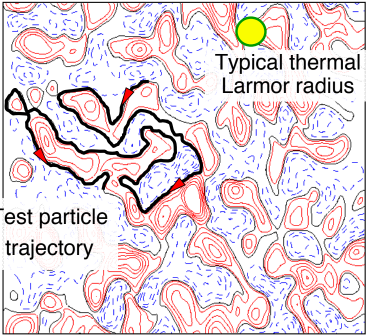
$$\mathbf{v}_{G\perp} \simeq \underbrace{\frac{\mathbf{B}\times\nabla\langle\phi\rangle}{B^2}}_{\text{electric drift } \mathbf{v}_E} + \underbrace{\frac{m{v_{G\|}}^2 + \mu B}{eB}\frac{\mathbf{B}\times\nabla B}{B^2}}_{\text{curvature + }\nabla\text{B drifts } \mathbf{v}_d} + o(\beta)$$

Physics of electric drift

Electric drift $\mathbf{v}_E$ $\Rightarrow$ Turbulent transport:

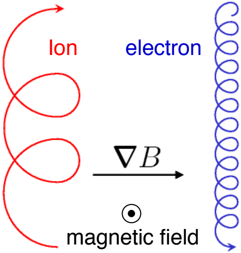
- ϕ ~ analogous to stream function in neutral fluid dynamics
- At leading order, particles move at ϕ=cst (motion invariant if **B**=cst and $\partial_t\phi=0$)
- Larger $\perp$ excursion than Larmor radius
- Heat transport requires non vanishing phase shift between δp and $\delta\phi$

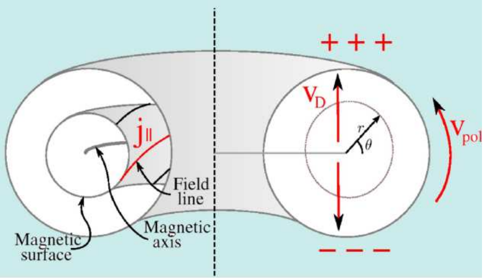
iso-contours of electric potential ϕ



Physics of curvature+∇B drifts

- Curvature+∇B drifts $\mathbf{v}_d$ ⇒ Vertical charge separation:





- Return current : parallel (electron) current (Pfirsch-Schlüter)
  polarization (ion) current

Parallel dynamics

- Parallel projection of Newton's eq.

$$\frac{dv_{G\|}}{dt} + \underbrace{\frac{d\mathbf{v}_{G\perp}}{dt} \cdot \mathbf{b}} = -\frac{e}{m}\nabla_{\|}\langle\phi\rangle - \frac{\mu}{m}\nabla_{\|}B$$

$$v_{G\|}\left(\nabla_{\|}\mathbf{v}_{G\perp}\right).\mathbf{b} + o\left(\epsilon^2\right)$$

non-vanishing contribution from ⊥ dynamics

- Parallel trapping & coupling $\mathbf{v}_d.\mathbf{E}_G$

$$\frac{dv_{G\|}}{dt} = -\frac{e}{m}\nabla_{\|}\langle\phi\rangle - \frac{\mu}{m}\nabla_{\|}B - \underbrace{v_{G\|}\frac{\mathbf{B}\times\nabla B}{B^3}\cdot\nabla\langle\phi\rangle} + o(\beta)$$

Trapping in electric potential wells (turbulence)

Trapping in magnetic wells (magnetic equilibrium)

Coupling $\mathbf{v}_d.\mathbf{E}_G$

Poisson vs. quasi-neutrality

- Poisson equation:

$\sim$ (few ρ_i)$^{-2}$ — $\sim$ few % in the core

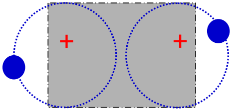
$\sim$ (few 4.10^{-3})$^{-2}$

$$\lambda_D^2 \; \nabla^2 \left(\frac{e\phi}{T_0} \right) = \frac{n_e - n_i}{n_0}$$

$$\lambda_D \approx 2.35\,10^{-5} (T_{[keV]}/n_{10^{20}m^{-3}})^{1/2} \; m$$

$\approx 10^{-4} \; m$ for Deuterium ions in ITER

- Safely replaced by quasi-neutrality (for ion turb.):



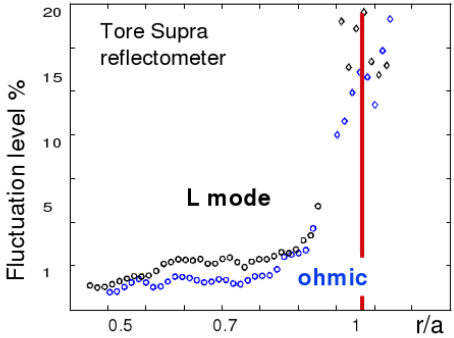
$n_e(\mathbf{x},t) = n_i(\mathbf{x},t)$

with $n_s(\mathbf{x},t) \doteq \int \mathrm{d}^3\mathbf{v} \, f_s(\mathbf{x},\mathbf{v},t)$

Pb: unknown function in GK theory ($n \neq n_G$)

Fluctuation level of few % in the core

Fluctuation level increases at the edge



P. Hennequin

Relation between $f(\mathbf{x}, v_\parallel, v_\perp, \varphi_c, t)$ and $\bar{f}(\mathbf{x}_G, v_{G\parallel}, \mu, t)$

- Infinitesimal canonical transformation theory:

$$\begin{cases} \mathbf{x} &= \mathbf{x}_G + \partial_{\mathbf{p}_G} S \\ \mathbf{p} &= \mathbf{p}_G - \partial_{\mathbf{x}_G} S \end{cases}$$

with S ~ generating function

- Transformation rule: $f(\mathbf{x}, \mathbf{p}) = f(\mathbf{x}_G, \mathbf{p}_G) + [f, S]_{\mathbf{x}_G, \mathbf{p}_G}$

- S obtained via the constraint imposed by gyro-kinetic framework:

$$\frac{\partial \bar{H}}{\partial \varphi_s} = 0 \quad \Rightarrow \quad S(\mathbf{x}, \mathbf{p}) = \int \frac{m\, \mathrm{d}\varphi_c}{B} \left\{ \phi(\mathbf{x}) - \langle \phi(\mathbf{x}_G, \mathbf{p}_G) \rangle \right\}$$

- It follows:

$$\begin{aligned} f(\mathbf{x}, \mathbf{v}, t) &= \bar{f}(\mathbf{x}_G, \mathbf{v}_G, t) \\ &+ \frac{e}{B} \left\{ \phi(\mathbf{x}, t) - \langle \phi(\mathbf{x}_G, \mathbf{v}_G) \rangle \right\} \partial_\mu \bar{f}_{eq}(\mathbf{x}_G, \mathbf{v}_G) \end{aligned}$$

Quasi-neutrality within GK framework

- Two contributions to $n_s(\mathbf{x},t)$ when replacing f by $\bar{f}$

$$n_s(\mathbf{x},t) = \underbrace{\int \mathrm{d}^3\mathbf{v}\,\bar{f}_s(\mathbf{x}_G,\mathbf{v}_G,t)}_{\int \mathcal{J}_v\,\mathrm{d}\mu\,\mathrm{d}v_{G\parallel}\,\langle \bar{f}_s(\mathbf{x},\mathbf{v},t)\rangle} + \underbrace{\int \mathrm{d}^3\mathbf{v}\frac{e_s}{B}\bar{f}_{eq,s}(\mathbf{x}_G,\mathbf{v}_G)\,\partial_\mu\bar{\phi}(\mathbf{x}_G,\mathbf{v}_G,t)}$$

Gyro-center density $n_{Gs}(\mathbf{x},t)$

Polarization density $n_{pol,s}(\mathbf{x},t)$

- In the $k_\perp\rho_s \ll 1$ limit only:

$$n_{pol,s}(\mathbf{x},t) = \nabla_\perp\cdot\left(\frac{m_s n_{eq,s}}{e_s B^2}\nabla_\perp\phi(\mathbf{x},t)\right)$$

- If electrons taken adiabatic:

$$\frac{e}{T_e}\left(\phi-\langle\phi\rangle_{FS}\right)-\frac{1}{n_{eq}}\nabla_\perp\cdot\left(\frac{m_s n_{eq}}{e_s B^2}\nabla_\perp\phi\right)=\frac{1}{n_{eq}}\int\mathcal{J}_v\mathrm{d}\mu\mathrm{d}v_{G\parallel}J.\bar{f}_i-1$$

Global Gyrokinetic system (1/2)

- The time evolution of the gyro-center distribution function $\bar{f}_i$ is given by the gyrokinetic Vlasov equation

$$\frac{\partial B_{\parallel}^{*}\bar{f}_s}{\partial t} + \boldsymbol{\nabla}\cdot\left(B_{\parallel}^{*}\frac{\mathrm{d}\mathbf{x}_G}{\mathrm{d}t}\,\bar{f}_s\right) + \frac{\partial}{\partial v_{G\parallel}}\left(B_{\parallel}^{*}\frac{\mathrm{d}v_{G\parallel}}{\mathrm{d}t}\,\bar{f}_s\right) = 0 \qquad (8)$$

- where the equations of motion of the guiding centers are given below

$$B_{\parallel}^{*}\frac{\mathrm{d}\mathbf{x}_G}{\mathrm{d}t} = v_{G\parallel}\mathbf{B}_{\parallel}^{*} + \frac{\mathbf{b}}{e_s}\times\boldsymbol{\nabla}\Xi \qquad (9)$$

$$B_{\parallel}^{*}\frac{\mathrm{d}v_{G\parallel}}{\mathrm{d}t} = -\frac{\mathbf{B}_{\parallel}^{*}}{m_s}\cdot\boldsymbol{\nabla}\Xi \qquad (10)$$

with

$$\boldsymbol{\nabla}\Xi = \mu_s\boldsymbol{\nabla}B + e_s\boldsymbol{\nabla}\bar{\phi} \quad \text{and} \quad \mathbf{B}_{\parallel}^{*} = \mathbf{B} + (m_s/e_s)\,v_{G\parallel}\boldsymbol{\nabla}\times\mathbf{b}$$

Global Gyrokinetic system (2/2)

- Self-consistently coupled to the quasi-neutrality equation

$$\frac{e}{T_e}\left(\phi - \langle\phi\rangle_{FS}\right) - \frac{1}{n_{eq}} \nabla_\perp \cdot \left(\frac{m_s n_{eq}}{e_s B^2} \nabla_\perp \phi \right) = \frac{1}{n_{eq}} \int \mathcal{J}_v \mathrm{d}\mu \mathrm{d}v_{G\parallel} J.\bar{f}_i - 1 \quad (11)$$

with $\langle\phi\rangle_{FS}$ the flux surface average of ϕ

- This system of equations (8)-(11) is the basis of the gyrokinetic codes.
- GK codes require state-of-the-art HPC techniques and must run efficiently on more than thousands processors.