ISOPIC: Axisymmetric PIC code based on isogeometric analysis

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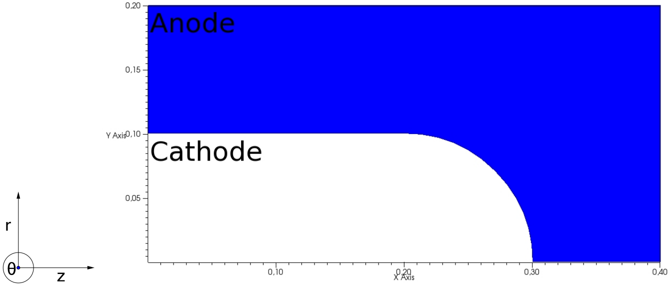
Outline

Goal

Our aim: to simulate the emission of electrons in a diode, using the Nurbs and the 2D axisymmetric geometry.



Diode

Vlasov-Maxwell

We solve the Vlasov-Maxwell system:

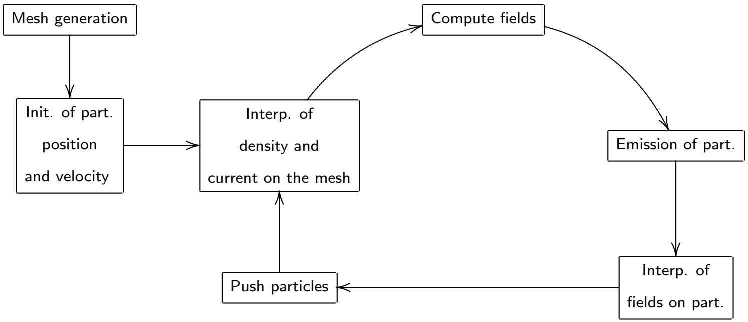
$$\left\{ \begin{array}{c} \frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} - (\mathbf{E} + \mathbf{v} \wedge \mathbf{B})\frac{\partial f}{\partial \mathbf{v}} = 0, \\ -\frac{\partial \mathbf{E}}{\partial t} + \mathbf{rotB} = \mathbf{J}, \\ \frac{\partial \mathbf{B}}{\partial t} + \mathbf{rotE} = 0, \\ div\mathbf{E} = \rho. \end{array} \right.$$

To solve Maxwell we have adapted a code using Splines.

2D axisymmetric geometry

- The diode is cylindrical but symmetric in the θ direction.
- We are on a 2D axisymmetric geometry.
- We solve Vlasov with a Particle-In-Cell (PIC) method:
  we consider N particles, their position $\mathbf{x_k}$, velocity $\mathbf{v_k}$ and weight ω_k,
  we approach f by $f_N(\mathbf{x}, \mathbf{v}, t) = \sum_k \omega_k \delta(\mathbf{x} - \mathbf{x_k}(t))\delta(\mathbf{v} - \mathbf{v_k}(t))$.
- We move particles (electrons) with the equations of motion in this geometry.

PIC method


Mesh generation
Init. of part. position and velocity
Interp. of density and current on the mesh
Compute fields
Emission of part.
Interp. of fields on part.
Push particles

Splines are smooth piecewise polynomial functions.

Let $T = (t_i)_{1 \leqslant i \leqslant N+k}$ be a non-decreasing sequence of knots.

Definition (B-Spline)

The i-th B-Spline of order k is defined by the recurrence relation:

$$N_j^k = w_j^k N_j^{k-1} + (1 - w_{j+1}^k) N_{j+1}^{k-1}$$

$$\textit{where} \qquad w_j^k(x) = \frac{x - t_j}{t_{j+k-1} - t_j}, \quad N_j^1(x) = \chi_{[t_j, t_{j+1}[}(x)$$

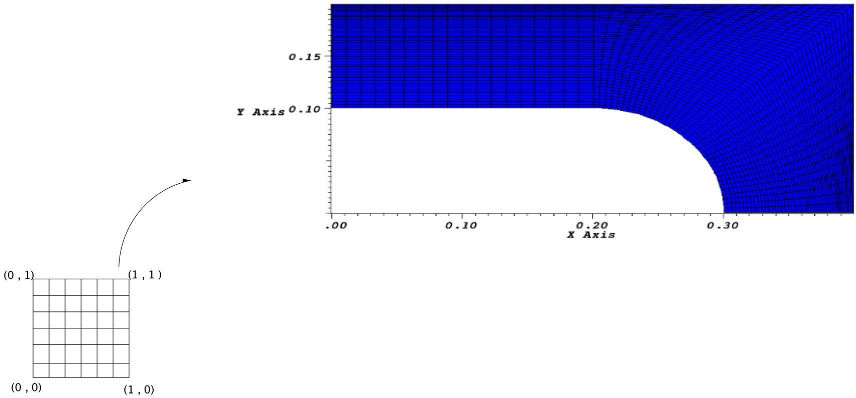
Definition (NURBS)

The i-th NURBS of order k associated to the knot vector T and the weights ω, is defined by

$$R_i^k = \frac{\omega_i N_i^k}{\sum_{j=1}^N \omega_j N_j^k}.$$

Splines and NURBS for physical domain

We use the NURBS to build the mapping between a patch and our physical domain, then we enrich the patch by Splines.



Equations of motion

Whatever coordinates system $\left(\mathbf{X}, \dot{\mathbf{X}}\right)$ we choose, we can find the equations of motion with the help of the Euler-Lagrange equations:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{X}_i} = \frac{\partial \mathcal{L}}{\partial X_i},$$

where $\mathcal{L}$ is a function of $\left(\mathbf{X}, \dot{\mathbf{X}}, t\right)$ called Lagrangian.

Relativistic Lagrangian in cartesian coordinates

The Lagrangian of a special relativistic test particle in an electromagnetic field is

$$L(\mathbf{X}, \dot{\mathbf{X}}, t) = \frac{-m\, c^2}{\gamma} + e(\mathbf{A} \cdot \frac{\vec{P}}{m\, \gamma} - \phi),$$

where e, m are the charge and the mass of the particle,

$\mathbf{A} = (A_x, A_y, A_z)$ corresponds to the potential vector: $\mathbf{B} = \mathbf{curl}\, \mathbf{A}$,

ϕ is the Poisson function such that $\mathbf{E} = -\nabla\phi - \frac{\partial}{\partial t}\mathbf{A}$.

Equations of motion in cartesian coordinates

With the help of the Euler-Lagrange equations we obtain the equations of motion in this coordinates system:

$$\dot{\mathbf{P}} = \mathbf{E} + \frac{\vec{P}}{m\gamma} \times \mathbf{B}$$

where the generalized momenta are $\mathbf{P} = m\gamma\dot{\mathbf{X}}$ with $\gamma = \frac{1}{\sqrt{1-\frac{\dot{\mathbf{x}}^2}{c^2}}}$.

Relativistic equations of motion in axisymmetric coordinates

In axisymmetric geometry: $(r\cos\theta, r\sin\theta, z) = F(r,\theta,z)$. The relativistic Lagrangian is:

$$L(\mathbf{X}_{axi}, \dot{\mathbf{X}}_{axi}, t) = \frac{-m\,c^2}{\gamma_{axi}} + e(\mathbf{A}\cdot\begin{pmatrix}\dot{z}\\ \dot{r}\\ \dot{\theta}\end{pmatrix} - \phi).$$

The equations of motion become

$$\begin{pmatrix} r\,\dot{P}_z \\ r\,\dot{P}_r - P_\theta\,\dot{\theta} \\ \frac{\dot{P}_\theta}{r}\end{pmatrix} = e\left[\begin{pmatrix} r\,E_z \\ r\,E_r \\ \frac{E_\theta}{r}\end{pmatrix} + \begin{pmatrix}\dot{z}\\ \dot{r}\\ \dot{\theta}\end{pmatrix} \times \begin{pmatrix} B_z \\ B_r \\ B_\theta\end{pmatrix}\right]$$

where the generalized momenta become
$\mathbf{P}_{axi} = m\,\gamma_{axi}\left(\dot{r},\ r^2\dot{\theta},\ \dot{z}\right)$ where $\gamma_{axi} = \frac{1}{\sqrt{1-\frac{(\mathbf{v}_{axi})^2}{c^2}}}$.

Lagrangian in patch

In our problem we resolve the Maxwell's equations in a patch so we also need resolve the equations of motion in this domain !!!!
We change coordinates
$G(\xi,\eta,\theta) = (z(\xi,\eta,),r(\xi,\eta,),\theta) = (z,r,\theta)$, we recompute the Lagrangian with the hypothesis that $\dot{\theta} = 0$:

$$L = \frac{-m\,c^2}{\gamma} + e\left(\mathbf{A}\cdot\begin{pmatrix}\dot{\xi}\\ \dot{\eta}\\ 0\end{pmatrix} - \phi\right),$$

where γ is equal to $\frac{1}{\sqrt{1-\frac{M_\xi\dot{\xi}^2+M_\eta\dot{\eta}^2+2M_{\xi\eta}\dot{\xi}\dot{\eta}}{c^2}}}$.

Equations of motion in patch

We deduce the equations of motion in undefined coordinates using the Euler-Lagrange equations, and we obtain:

$$\dot{P}_\xi = \frac{m\gamma}{2}\left[\left(\frac{\partial}{\partial\xi}\left(\frac{P_\xi}{m\gamma}\right)\right)\dot{\xi} + \left(\frac{\partial}{\partial\xi}\left(\frac{P_\eta}{m\gamma}\right)\right)\dot{\eta}\right] + e\left[E_\xi + \dot{\eta}B_\theta\, det(J)\right]$$

$$\dot{P}_\eta = \frac{m\gamma}{2}\left[\left(\frac{\partial}{\partial\eta}\left(\frac{P_\xi}{m\gamma}\right)\right)\dot{\xi} + \left(\frac{\partial}{\partial\eta}\left(\frac{P_\eta}{m\gamma}\right)\right)\dot{\eta}\right] + e\left[E_\eta - \dot{\xi}B_\theta\, det(J)\right]$$

where the generalized momenta are

$$\mathbf{P} = m\gamma\left(M_\xi\dot{\xi} + M_{\xi\eta}\dot{\eta},\ M_{\xi\eta}\dot{\xi} + M_\eta\dot{\eta}\right),$$

and

$$M_\xi = \left(\frac{\partial r}{\partial\xi}\right)^2 + \left(\frac{\partial z}{\partial\xi}\right)^2, \qquad M_\eta = \left(\frac{\partial r}{\partial\eta}\right)^2 + \left(\frac{\partial z}{\partial\eta}\right)^2$$

$$M_{\xi\eta} = \frac{\partial r}{\partial\xi}\frac{\partial r}{\partial\eta} + \frac{\partial z}{\partial\xi}\frac{\partial z}{\partial\eta}.$$

Numerical result for the equations of motion

Movement

How does a diode work?

A potential drop between the anode ($V_a > 0$) and the cathode ($V_c = 0$) is imposed and it extracts electrons.

Numerically:

we create a potential drop, and every two iterations we create a particle with a positive weight in a cell Ω if:

- Ω is on the surface of the cathode where we can inject particles,
- the charge $\int_\Omega \rho d\omega = -\sum_{k\in\Omega} \omega_k$ is greater than the circulation $\int_\Omega \vec{\nabla} \cdot \vec{E} d\omega = \int_{\partial\Omega} \vec{E} \cdot \vec{n} d\sigma$.

Our problem

We still try to program this potential drop...

Remark:

if we consider a stationary wave already inside the diode, the electrons are extracted as expected.

We can so suppose that our condition to extract the particles is well implemented.

Extraction on a rectangle

Extraction on a rectangle

Conclusions:

- we have created the mesh of the physical domain,
- we have resolved Maxwell's equations and computed the current,
- we have resolved the equations of motion in axisymmetric geometry and on the patch, in relativistic and in non relativistic,
- we have implemented the condition to extract electrons.

Perspectives:

- we have to implement the physical condition on the boundaries i.e. Silver-Muller's condition,
- and the potential drop.