APFEL project.

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Plan

Isothermal two-fluid Euler-Lorentz system.

$$\begin{cases} \partial_t n_i + \nabla_{\mathbf{x}} \cdot \mathbf{q}_i = 0\,, \\ \partial_t \mathbf{q}_i + \nabla_{\mathbf{x}} \cdot \left(\dfrac{\mathbf{q}_i \otimes \mathbf{q}_i}{n_i} \right) + \dfrac{k_B\, T_i}{m_i} \nabla_{\mathbf{x}} n_i = \dfrac{e}{m_i} \left(-n_i\, \nabla_{\mathbf{x}} \phi + \mathbf{q}_i \times \mathbf{B} \right)\,, \\ \partial_t n_e + \nabla_{\mathbf{x}} \cdot \mathbf{q}_e = 0\,, \\ \partial_t \mathbf{q}_e + \nabla_{\mathbf{x}} \cdot \left(\dfrac{\mathbf{q}_e \otimes \mathbf{q}_e}{n_e} \right) + \dfrac{k_B\, T_e}{m_e} \nabla_{\mathbf{x}} n_e = -\dfrac{e}{m_e} \left(-n_e\, \nabla_{\mathbf{x}} \phi + \mathbf{q}_e \times \mathbf{B} \right)\,. \end{cases}$$

- $(n_i, \mathbf{q}_i)$ (ions), $(n_e, \mathbf{q}_e)$ (electrons), ϕ (potential) : unknowns,
- T_i, m_i, T_e, m_e, k_B : given,
- $\mathbf{B}$ (magnetic field) : given.

Quasi-neutrality equation $n_i = n_e = n$.

Scaling.

Low Mach number regime & strong magnetic field :

$$\epsilon = m_e/m_i, \quad \overline{M}^2 = \tau\,, \quad \bar{t}\,\overline{\omega} = \frac{1}{\tau}.$$

Euler-Lorentz model in the low Mach regime :

$$\left\{\begin{array}{l}
\partial_t n^\tau + \nabla_{\mathbf{x}} \cdot \mathbf{q}_i^\tau = 0\,, \\
\tau \left[\partial_t \mathbf{q}_i^\tau + \nabla_{\mathbf{x}} \cdot \left(\dfrac{\mathbf{q}_i^\tau \otimes \mathbf{q}_i^\tau}{n^\tau}\right)\right] + \nabla_{\mathbf{x}} n^\tau = -n^\tau\, \nabla_{\mathbf{x}} \phi^\tau + \mathbf{q}_i^\tau \times \mathbf{B}\,, \\
\partial_t n^\tau + \nabla_{\mathbf{x}} \cdot \mathbf{q}_e^\tau = 0\,, \\
\epsilon\, \tau \left[\partial_t \mathbf{q}_e^\tau + \nabla_{\mathbf{x}} \cdot \left(\dfrac{\mathbf{q}_e^\tau \otimes \mathbf{q}_e^\tau}{n^\tau}\right)\right] + T_e\, \nabla_{\mathbf{x}} n^\tau = n^\tau\, \nabla_{\mathbf{x}} \phi^\tau - \mathbf{q}_e^\tau \times \mathbf{B}\,.
\end{array}\right.$$

Drift limit.

If $\tau \to 0$,

$$\left\{ \begin{array}{l} \partial_t n^0 + \nabla_{\mathbf{x}} \cdot \mathbf{q}_i^0 = 0\,, \\ \nabla_{\mathbf{x}} n^0 = -n^0\, \nabla_{\mathbf{x}} \phi^0 + \mathbf{q}_i^0 \times \mathbf{B}\,, \\ \partial_t n^0 + \nabla_{\mathbf{x}} \cdot \mathbf{q}_e^0 = 0\,, \\ T_e\, \nabla_{\mathbf{x}} n^0 = n^0\, \nabla_{\mathbf{x}} \phi^0 - \mathbf{q}_e^0 \times \mathbf{B}\,, \end{array} \right. \tag{1}$$

Degond, Deluzet, Sangam, Vignal - *J. Comput. Phys.* **228** (2009).

Brull, Degond, Deluzet - in preparation.

Reformulation of the drift limit model.

$$\left\{\begin{array}{l}\partial_t n^0 + \nabla_{\mathbf{x}} \cdot \mathbf{q}_i^0 = 0\,, \\ \mathbf{b} \cdot \nabla_{\mathbf{x}} n^0 = -n^0\, \mathbf{b} \cdot \nabla_{\mathbf{x}} \phi^0\,, \\ (\mathbf{q}_i^0)_\perp = \dfrac{1}{\|\mathbf{B}\|} \mathbf{b} \times \left(\nabla_{\mathbf{x}} n^0 + n^0\, \nabla_{\mathbf{x}} \phi^0\right), \\ \partial_t n^0 + \nabla_{\mathbf{x}} \cdot \mathbf{q}_e^0 = 0\,, \\ T_e\, \mathbf{b} \cdot \nabla_{\mathbf{x}} n^0 = n^0\, \mathbf{b} \cdot \nabla_{\mathbf{x}} \phi^0\,, \\ (\mathbf{q}_e^0)_\perp = \dfrac{1}{\|\mathbf{B}\|} \mathbf{b} \times \left(-T_e\, \nabla_{\mathbf{x}} n^0 + n^0\, \nabla_{\mathbf{x}} \phi^0\right), \end{array}\right. \tag{2}$$

Consequence : $\mathbf{b} \cdot \nabla_{\mathbf{x}} n^0 = \mathbf{b} \cdot \nabla_{\mathbf{x}} \phi^0 = 0.$

Time semi-discretization.

$$
\left\{
\begin{aligned}
&\frac{n^{\tau,m+1}-n^{\tau,m}}{\Delta t}+\nabla_{\mathbf{x}}\cdot\left(\left(\mathbf{b}^{m+1}\otimes\mathbf{b}^{m+1}\right)\mathbf{q}_i^{\tau,m+1}\right)\\
&\qquad\qquad\qquad\qquad+\nabla_{\mathbf{x}}\cdot\left(\left(\mathbb{I}-\mathbf{b}^{m+1}\otimes\mathbf{b}^{m+1}\right)\mathbf{q}_i^{\tau,m}\right)=0\,,\\
&\frac{\mathbf{q}_i^{\tau,m+1}-\mathbf{q}_i^{\tau,m}}{\Delta t}+\nabla_{\mathbf{x}}\cdot\left(\frac{\mathbf{q}_i^{\tau,m}\otimes\mathbf{q}_i^{\tau,m}}{n^{\tau,m}}\right)+\frac{1}{\tau}\nabla_{\mathbf{x}}n^{\tau,m+1}\\
&\qquad\qquad=\frac{1}{\tau}\left[-n^{\tau,m+1}\,\nabla_{\mathbf{x}}\phi^{\tau,m+1}+\mathbf{q}_i^{\tau,m+1}\times\mathbf{B}^{m+1}\right],\\
&\frac{n^{\tau,m+1}-n^{\tau,m}}{\Delta t}+\nabla_{\mathbf{x}}\cdot\left(\left(\mathbf{b}^{m+1}\otimes\mathbf{b}^{m+1}\right)\mathbf{q}_e^{\tau,m+1}\right)\\
&\qquad\qquad\qquad\qquad+\nabla_{\mathbf{x}}\cdot\left(\left(\mathbb{I}-\mathbf{b}^{m+1}\otimes\mathbf{b}^{m+1}\right)\mathbf{q}_e^{\tau,m}\right)=0\,,\\
&\frac{\mathbf{q}_e^{\tau,m+1}-\mathbf{q}_e^{\tau,m}}{\Delta t}+\nabla_{\mathbf{x}}\cdot\left(\frac{\mathbf{q}_e^{\tau,m}\otimes\mathbf{q}_e^{\tau,m}}{n^{\tau,m}}\right)+\frac{T_e}{\epsilon\,\tau}\nabla_{\mathbf{x}}n^{\tau,m+1}\\
&\qquad\qquad=\frac{1}{\epsilon\,\tau}\left[n^{\tau,m+1}\,\nabla_{\mathbf{x}}\phi^{\tau,m+1}-\mathbf{q}_e^{\tau,m+1}\times\mathbf{B}^{m+1}\right].
\end{aligned}
\right.
$$

Perpendicular components of $\mathbf{q}_i$ and $\mathbf{q}_e$.

$$\begin{aligned}
(\mathbf{q}_i^{\tau,m+1})_\perp^{m+1} - \frac{\tau}{\Delta t\,\|\mathbf{B}^{m+1}\|}\mathbf{b}^{m+1} \times (\mathbf{q}_i^{\tau,m+1})_\perp^{m+1}
&= \frac{1}{\|\mathbf{B}^{m+1}\|}\mathbf{b}^{m+1} \times \left[\nabla_{\mathbf{x}} n^{\tau,m+1} + n^{\tau,m+1}\,\nabla_{\mathbf{x}}\phi^{\tau,m+1}\right] \\
&+ \frac{\tau}{\|\mathbf{B}^{m+1}\|}\mathbf{b}^{m+1} \times \left[- \frac{\mathbf{q}_i^{\tau,m}}{\Delta t} + \nabla_{\mathbf{x}} \cdot \left(\frac{\mathbf{q}_i^{\tau,m} \otimes \mathbf{q}_i^{\tau,m}}{n^{\tau,m}} \right)\right],
\end{aligned}$$

$$\begin{aligned}
(\mathbf{q}_e^{\tau,m+1})_\perp^{m+1} + \frac{\epsilon\,\tau}{\Delta t\,\|\mathbf{B}^{m+1}\|}\mathbf{b}^{m+1} \times (\mathbf{q}_e^{\tau,m+1})_\perp^{m+1}
&= \frac{1}{\|\mathbf{B}^{m+1}\|}\mathbf{b}^{m+1} \times \left[- T_e\,\nabla_{\mathbf{x}} n^{\tau,m+1} + n^{\tau,m+1}\,\nabla_{\mathbf{x}}\phi^{\tau,m+1}\right] \\
&- \frac{\epsilon\,\tau}{\|\mathbf{B}^{m+1}\|}\mathbf{b}^{m+1} \times \left[- \frac{\mathbf{q}_e^{\tau,m}}{\Delta t} + \nabla_{\mathbf{x}} \cdot \left(\frac{\mathbf{q}_e^{\tau,m} \otimes \mathbf{q}_e^{\tau,m}}{n^{\tau,m}} \right)\right],
\end{aligned}$$

Parallel components of $\mathbf{q}_i$ and $\mathbf{q}_e$.

$$
\begin{aligned}
&(\mathbf{b}^{m+1} \otimes \mathbf{b}^{m+1})\, \mathbf{q}_i^{\tau,m+1} - (\mathbf{b}^{m+1} \otimes \mathbf{b}^{m+1})\, \mathbf{q}_i^{\tau,m} \\
&\quad + \Delta t\, (\mathbf{b}^{m+1} \otimes \mathbf{b}^{m+1})\, \nabla_{\mathbf{x}} \cdot \left(\frac{\mathbf{q}_i^{\tau,m} \otimes \mathbf{q}_i^{\tau,m}}{n^{\tau,m}} \right) \\
&\quad + \frac{\Delta t}{\tau} (\mathbf{b}^{m+1} \otimes \mathbf{b}^{m+1}) \left[\nabla_{\mathbf{x}} n^{\tau,m+1} + n^{\tau,m+1}\, \nabla_{\mathbf{x}} \phi^{\tau,m+1} \right] = 0\,,
\end{aligned}
$$

$$
\begin{aligned}
&(\mathbf{b}^{m+1} \otimes \mathbf{b}^{m+1})\, \mathbf{q}_e^{\tau,m+1} - (\mathbf{b}^{m+1} \otimes \mathbf{b}^{m+1})\, \mathbf{q}_e^{\tau,m} \\
&\quad + \Delta t\, (\mathbf{b}^{m+1} \otimes \mathbf{b}^{m+1})\, \nabla_{\mathbf{x}} \cdot \left(\frac{\mathbf{q}_e^{\tau,m} \otimes \mathbf{q}_e^{\tau,m}}{n^{\tau,m}} \right) \\
&\quad + \frac{\Delta t}{\epsilon\, \tau} (\mathbf{b}^{m+1} \otimes \mathbf{b}^{m+1}) \left[T_e\, \nabla_{\mathbf{x}} n^{\tau,m+1} - n^{\tau,m+1}\, \nabla_{\mathbf{x}} \phi^{\tau,m+1} \right] = 0\,.
\end{aligned}
$$

Diffusion problems for $n^{\tau,m+1}$ and $\phi^{\tau,m+1}$.

$$- \nabla_{\mathbf{x}} \cdot \left((\mathbf{b}^{m+1} \otimes \mathbf{b}^{m+1}) \nabla_{\mathbf{x}} n^{\tau,m+1} \right) + \lambda \, \tau \, n^{\tau,m+1} = \tau \, R^{\tau,m+1} , \quad (3)$$

$$- \nabla_{\mathbf{x}} \cdot \left(n^{\tau,m+1} (\mathbf{b}^{m+1} \otimes \mathbf{b}^{m+1}) \nabla_{\mathbf{x}} \phi^{\tau,m+1} \right) = \tau \, S^{\tau,m+1} , \quad (4)$$

To be consistent with the limit $\Longrightarrow$ Neumann type boundary conditions :

$$\begin{cases} \mathbf{b}^{m+1} \cdot \nabla_{\mathbf{x}} n^{\tau,m+1} = 0 \, , & \text{on } \partial\Omega, \\ \mathbf{b}^{m+1} \cdot \nabla_{\mathbf{x}} \phi^{\tau,m+1} = 0 \, , & \text{on } \partial\Omega. \end{cases} \quad (5)$$

(3)-(5.a) is well posed for any $\tau > 0$ but ill-posed when $\tau = 0$.
(4)-(5.b) is ill posed for any $\tau \geq 0$.

Perturbed Euler-Lorentz system for a two fluid model.

Introduce small perturbations in mass conservation equations :

$$\begin{cases} \partial_t n^\tau + C_1\, \partial_t \phi^\tau + \nabla_{\mathbf{x}} \cdot \mathbf{q}_i^\tau = 0\,, \\ \tau \left[\partial_t \mathbf{q}_i^\tau + \nabla_{\mathbf{x}} \cdot \left(\dfrac{\mathbf{q}_i^\tau \otimes \mathbf{q}_i^\tau}{n^\tau} \right) \right] + \nabla_{\mathbf{x}} n^\tau = -n^\tau\, \nabla_{\mathbf{x}} \phi^\tau + \mathbf{q}_i^\tau \times \mathbf{B}\,, \\ \partial_t n^\tau + C_2\, \partial_t \phi^\tau + \nabla_{\mathbf{x}} \cdot \mathbf{q}_e^\tau = 0\,, \\ \epsilon\, \tau \left[\partial_t \mathbf{q}_e^\tau + \nabla_{\mathbf{x}} \cdot \left(\dfrac{\mathbf{q}_e^\tau \otimes \mathbf{q}_e^\tau}{n^\tau} \right) \right] + T_e\, \nabla_{\mathbf{x}} n^\tau = n^\tau\, \nabla_{\mathbf{x}} \phi^\tau - \mathbf{q}_e^\tau \times \mathbf{B}\,. \end{cases}$$

$C_1, C_2 > 0$ free parameters chosen later.

Semi-discretization.

$$
\left\{
\begin{array}{l}
\dfrac{n^{\tau,m+1} - n^{\tau,m}}{\Delta t} + C_1 \dfrac{\phi^{\tau,m+1} - \phi^{\tau,m}}{\Delta t} + \nabla_{\mathbf{x}} \cdot \left((\mathbf{q}_i^{\tau,m+1})_{||}^{m+1} \right) \\
\qquad\qquad\qquad\qquad\qquad + \nabla_{\mathbf{x}} \cdot \left((\mathbf{q}_i^{\tau,m})_{\perp}^{m+1} \right) = 0 \,, \\
\dfrac{\mathbf{q}_i^{\tau,m+1} - \mathbf{q}_i^{\tau,m}}{\Delta t} + \nabla_{\mathbf{x}} \cdot \left(\dfrac{\mathbf{q}_i^{\tau,m} \otimes \mathbf{q}_i^{\tau,m}}{n^{\tau,m}} \right) + \dfrac{1}{\tau} \nabla_{\mathbf{x}} n^{\tau,m+1} \\
\qquad\qquad = \dfrac{1}{\tau} \left[- n^{\tau,m+1} \nabla_{\mathbf{x}} \phi^{\tau,m+1} + \mathbf{q}_i^{\tau,m+1} \times \mathbf{B}^{m+1} \right] , \\
\dfrac{n^{\tau,m+1} - n^{\tau,m}}{\Delta t} + C_2 \dfrac{\phi^{\tau,m+1} - \phi^{\tau,m}}{\Delta t} + \nabla_{\mathbf{x}} \cdot \left((\mathbf{q}_e^{\tau,m+1})_{||}^{m+1} \right) \\
\qquad\qquad\qquad\qquad\qquad + \nabla_{\mathbf{x}} \cdot \left((\mathbf{q}_e^{\tau,m})_{\perp}^{m+1} \right) = 0 \,, \\
\dfrac{\mathbf{q}_e^{\tau,m+1} - \mathbf{q}_e^{\tau,m}}{\Delta t} + \nabla_{\mathbf{x}} \cdot \left(\dfrac{\mathbf{q}_e^{\tau,m} \otimes \mathbf{q}_e^{\tau,m}}{n^{\tau,m}} \right) + \dfrac{T_e}{\epsilon\, \tau} \nabla_{\mathbf{x}} n^{\tau,m+1} \\
\qquad\qquad = \dfrac{1}{\epsilon\, \tau} \left[n^{\tau,m+1} \nabla_{\mathbf{x}} \phi^{\tau,m+1} - \mathbf{q}_e^{\tau,m+1} \times \mathbf{B}^{m+1} \right] .
\end{array}
\right.
$$

Problem on $n^{\tau,m+1}$ and $\phi^{\tau,m+1}$.

$$\left(1-\frac{\epsilon}{T_e}\right)\frac{n^{\tau,m+1}}{\Delta t}+\left(C_1-\frac{C_2\,\epsilon}{T_e}\right)\frac{\phi^{\tau,m+1}}{\Delta t}$$
$$-\left(1-\frac{\epsilon}{T_e}\right)\frac{\Delta t}{\tau}\nabla_{\mathbf{x}}\cdot\left(n^{\tau,m+1}\left(\mathbf{b}^{m+1}\otimes\mathbf{b}^{m+1}\right)\nabla_{\mathbf{x}}\phi^{\tau,m+1}\right)=RHS,$$

$$(1+\epsilon)\frac{n^{\tau,m+1}}{\Delta t}+\left(C_1+C_2\,\epsilon\right)\frac{\phi^{\tau,m+1}}{\Delta t}$$
$$-(1+\epsilon)\frac{\Delta t}{\tau}\nabla_{\mathbf{x}}\cdot\left(\left(\mathbf{b}^{m+1}\otimes\mathbf{b}^{m+1}\right)\nabla_{\mathbf{x}}n^{\tau,m+1}\right)=RHS.$$

Choice of C_1, C_2

$$C_1+\epsilon\,C_2=0,\qquad C_1-\frac{C_2\,\epsilon}{T_e}=C\,,$$

with $C>0$ small and fixed.

Problem on $n^{\tau,m+1}$ and $\phi^{\tau,m+1}$.

Inversion of the system $\Rightarrow C_1 = \dfrac{T_e\, C}{1+T_e}$ and $C_2 = -\dfrac{T_e\, C}{\epsilon\,(1+T_e)}$.

$n^{\tau,m+1}$ and $\phi^{\tau,m+1}$ are respectively solution of

$$\begin{cases} -\nabla_{\mathbf{x}} \cdot \left((\mathbf{b} \otimes \mathbf{b})\, \nabla_{\mathbf{x}} n^{\tau} \right) + \tau\, \lambda_1\, n^{\tau} = \tau\, R^{\tau}\,, & \text{on } \Omega, \\ \mathbf{b} \cdot \nabla_{\mathbf{x}} n^{\tau} = 0\,, & \text{on } \partial\Omega, \end{cases}$$

$$\begin{cases} -\nabla_{\mathbf{x}} \cdot \left(n^{\tau}\, (\mathbf{b} \otimes \mathbf{b})\, \nabla_{\mathbf{x}} \phi^{\tau} \right) + \tau\, \lambda_2\, C\, \phi^{\tau} = \tau\, S^{\tau}\,, & \text{on } \Omega, \\ \mathbf{b} \cdot \nabla_{\mathbf{x}} \phi^{\tau} = 0\,, & \text{on } \partial\Omega. \end{cases}$$

Resolution : Brull-Degond-Deluzet's method.

Brull, Degond, Deluzet - in preparation.

Numerical results.

Stationary case : $\mathbf{B} = (B_x, B_y, 0)$ uniform, $\mathbf{q}_i^\tau = \mathbf{q}_e^\tau = \mathbf{B}$, $n^\tau = n_0$ and $\phi^\tau = \phi_0$ constants.

Perturbation : replace n^τ by

$$n^\tau = n_0 + \tau \ \max(0,\ 1 - \eta\,((x-x_0)^2 + (y-y_0)^2))\,,$$

with η, x_0, y_0 given, and take $\mathbf{B} = (\sin\alpha, -\cos\alpha, 0)$ with α given.

Case 1.

- $n_0 = 4$, $\phi_0 = 2$, $\alpha = \pi/2$,
- $\eta = 80$, $x_0 = y_0 = 1.5$,
- $\tau = 10^{-4}$, $T_e = 1$, $\epsilon = 0.1$, $C = 0.002$,
- $\Omega = [1, 2]^2$, $Nx = Ny = 100$, $\Delta t = 10^{-7}$.

Case 1.

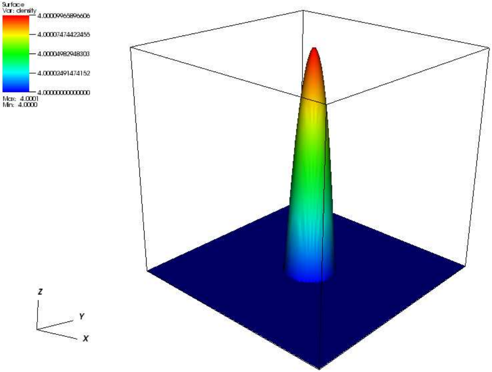


FIGURE: n^{τ} at time $t = 0$.

Case 1.

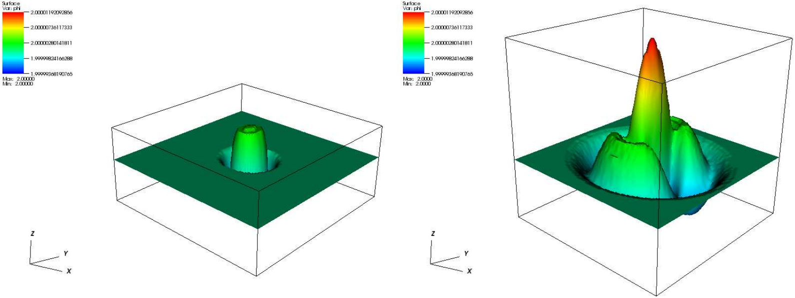


FIGURE: ϕ^τ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 1.

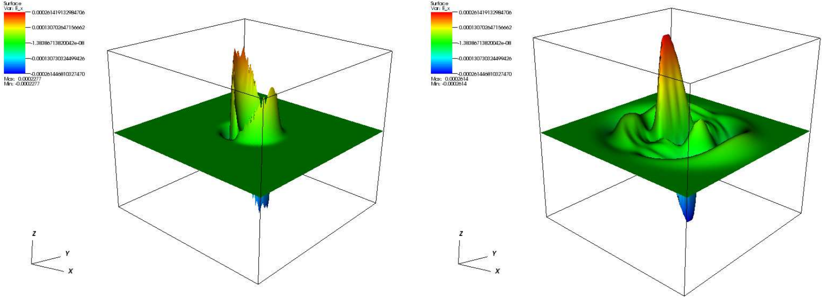


FIGURE: E_x^τ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 1.

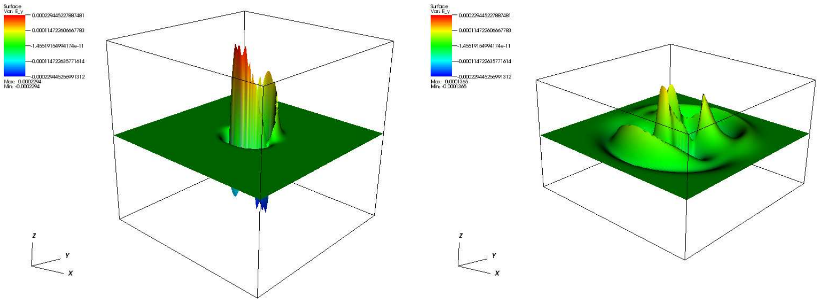


FIGURE: E_y^τ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 1.

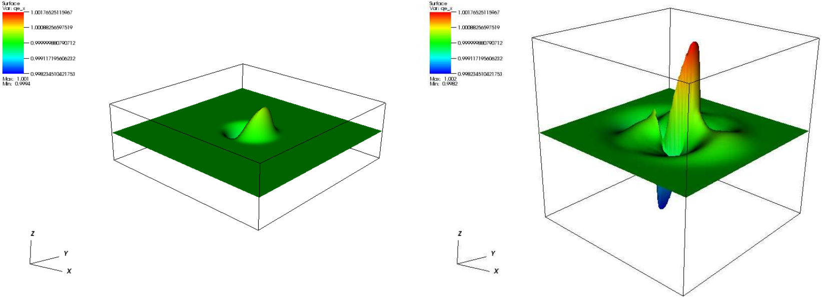


FIGURE: $q^{\tau}_{e,x}$ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 1.

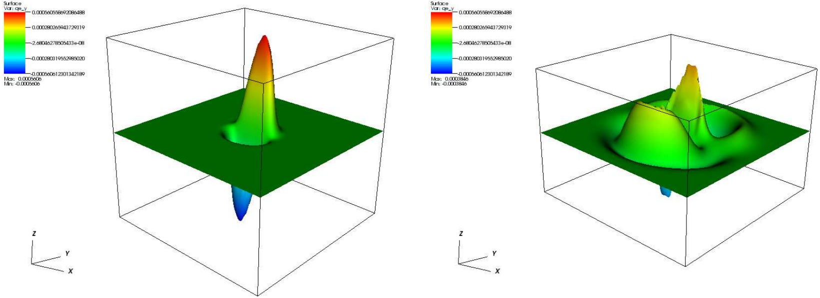
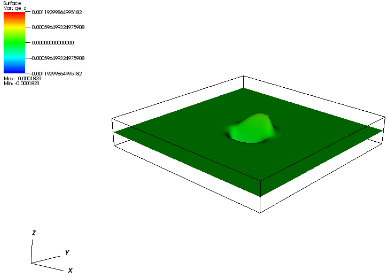


FIGURE: $q^{\tau}_{e,y}$ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 1.



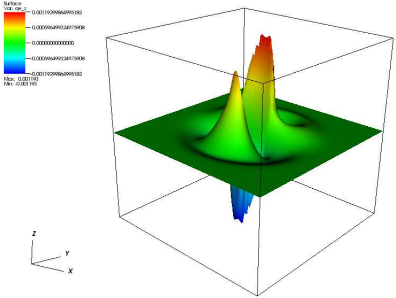


FIGURE: $q^{\tau}_{e,z}$ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 1.

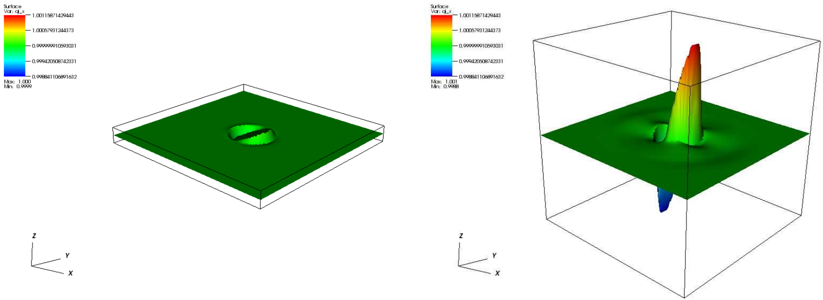


FIGURE: $q^{\tau}_{i,x}$ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 1.

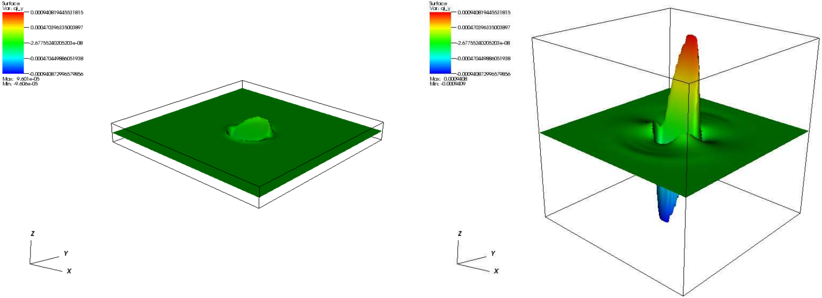
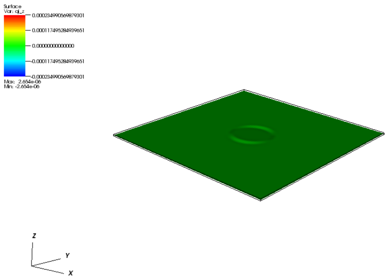


FIGURE: $q^{\tau}_{i,y}$ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 1.



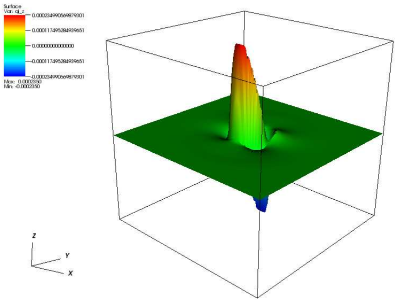


FIGURE: $q^{\tau}_{i,z}$ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 2.

- $n_0 = 4$, $\phi_0 = 2$, $\alpha = 2\pi/3$,
- $\eta = 80$, $x_0 = y_0 = 1.5$,
- $\tau = 10^{-4}$, $T_e = 1$, $\epsilon = 0.1$, $C = 0.002$,
- $\Omega = [1, 2]^2$, $Nx = Ny = 100$, $\Delta t = 10^{-7}$.

Case 2.

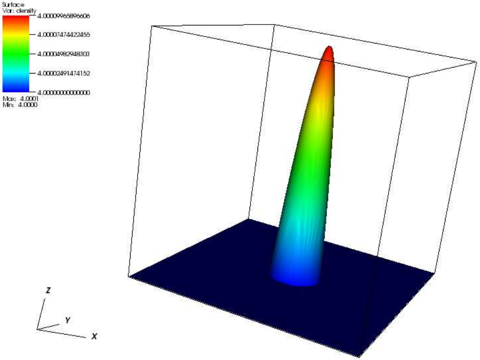
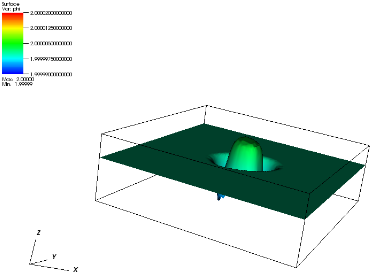


FIGURE: n^{τ} at time $t = 0$.

Case 2.



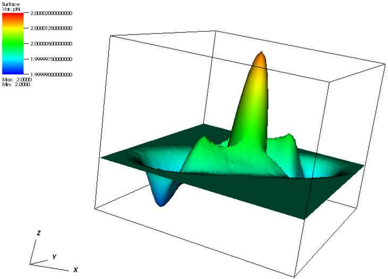


FIGURE: ϕ^τ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 2.

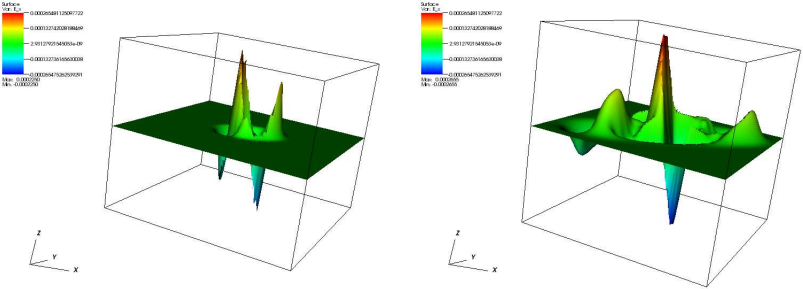


FIGURE: E_x^τ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 2.

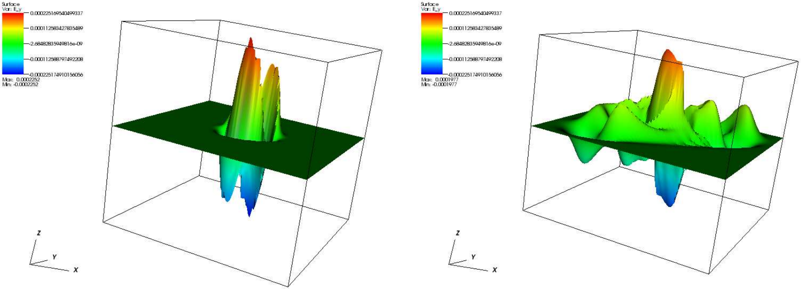
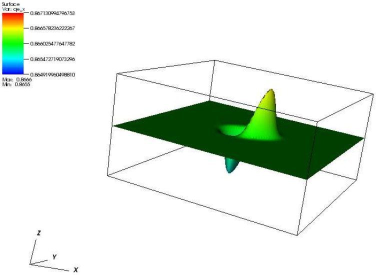


FIGURE: E_y^τ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 2.



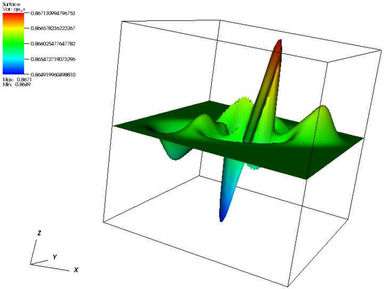
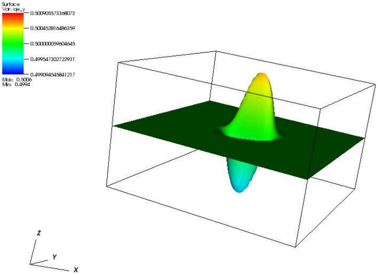


FIGURE: $q^{\tau}_{e,x}$ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 2.



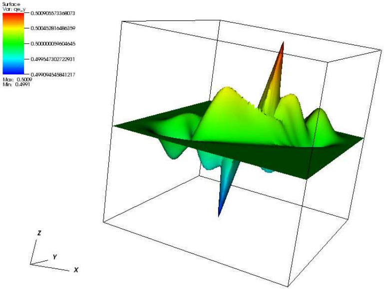


FIGURE: $q_{e,y}^{\tau}$ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 2.

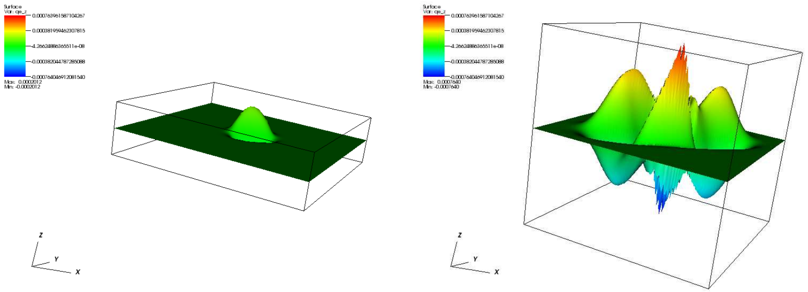


FIGURE: $q^{\tau}_{e,z}$ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 2.

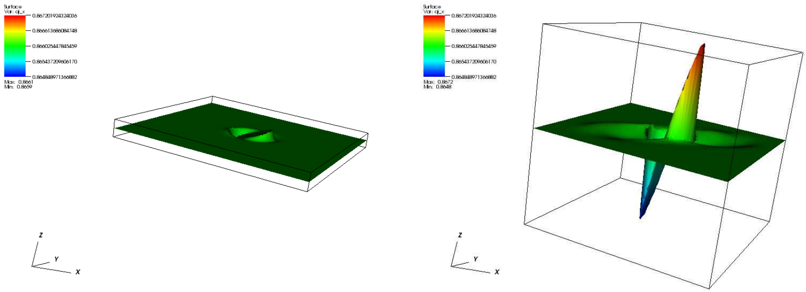
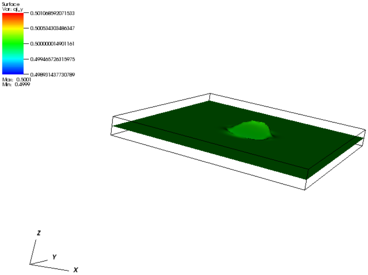
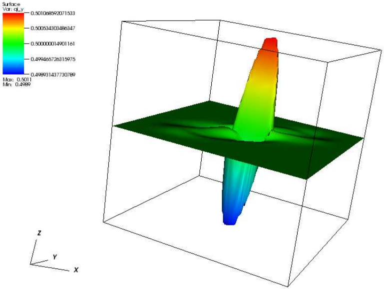


FIGURE: $q^{\tau}_{i,x}$ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Case 2.





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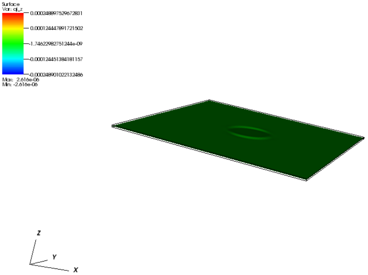

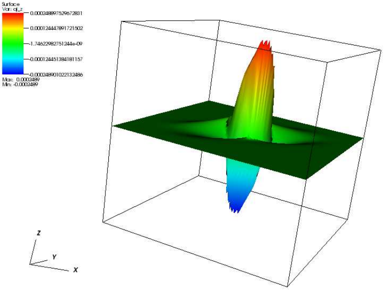

FIGURE: $q^{\tau}_{i,z}$ at time $t = 5 \times 10^{-7}$, $t = 4.5 \times 10^{-6}$.

Conclusions and perspectives

- The method works on the perturbed two-fluid Euler-Lorentz system,
- Successful tests when the perturbations do not reach $\partial\Omega$.

- Change the boundary conditions,
- Include energy conservation equations for each fluid,
- Coupling with Poisson or QN equation.

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THANKS FOR YOUR ATTENTION !