Deformable vesicles in hydrodynamic flows

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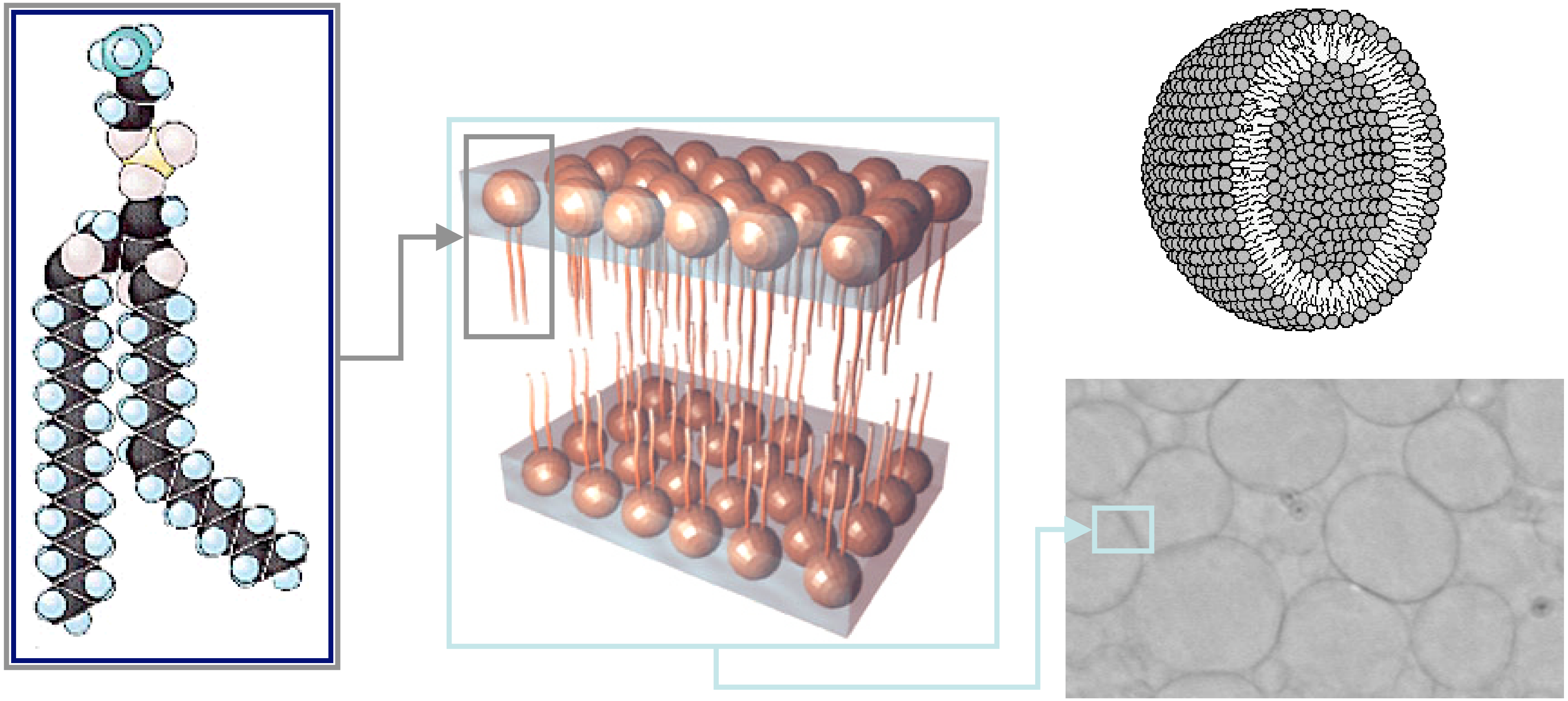
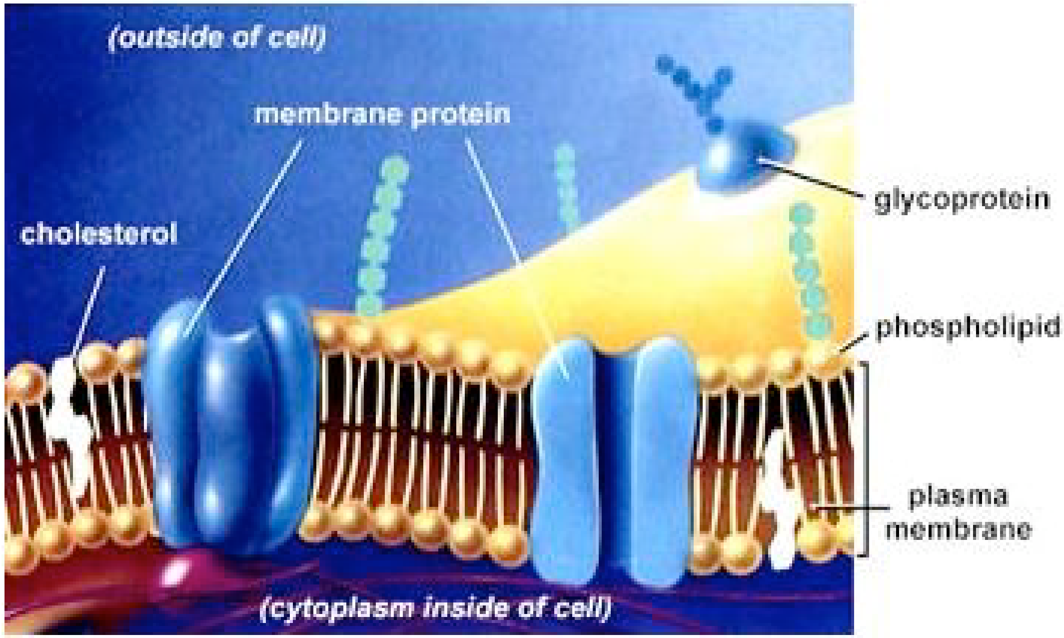
Outline

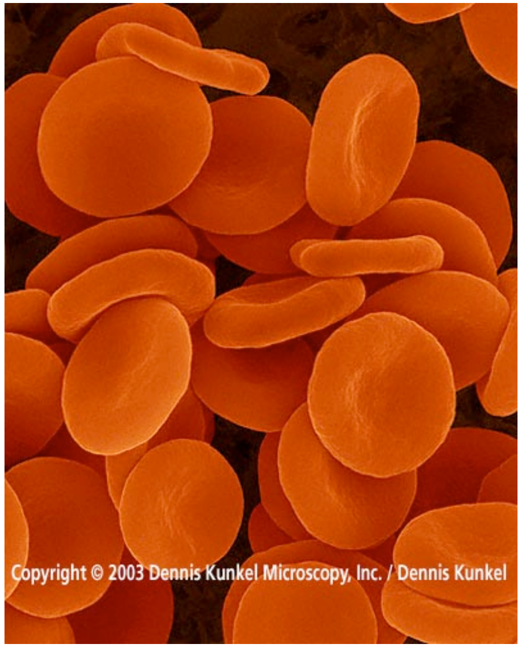
- vesicles: properties
- simple shear flows and rheology
- interactions with walls and lift
- microchannel flows

Giant Unilamellar Vesicles (GUV)



Vesicles as mechanical models for cells

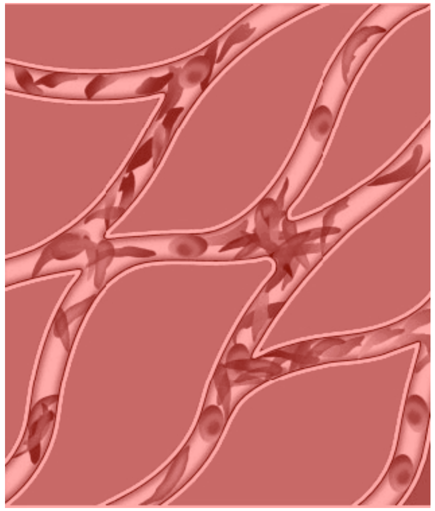




Blood flow and its disorders

- Rheology of blood
- Circulatory disorders
  - Anemia
  - Atherosclerosis
- Diagnostic tools

elliptocytosis

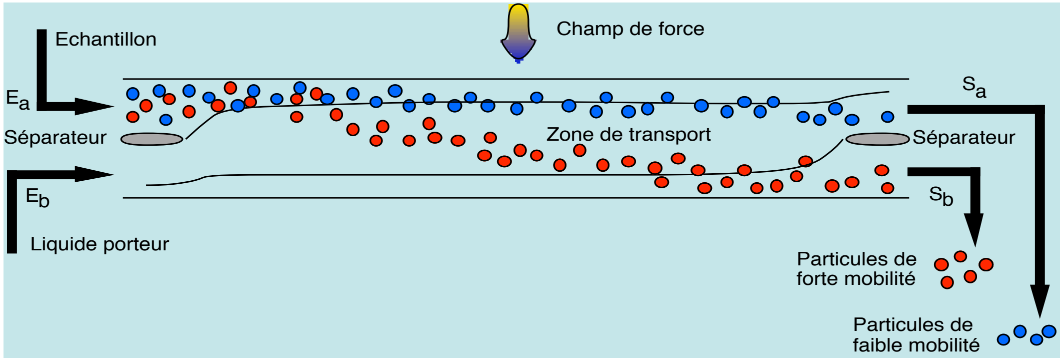


sickle cell anemia

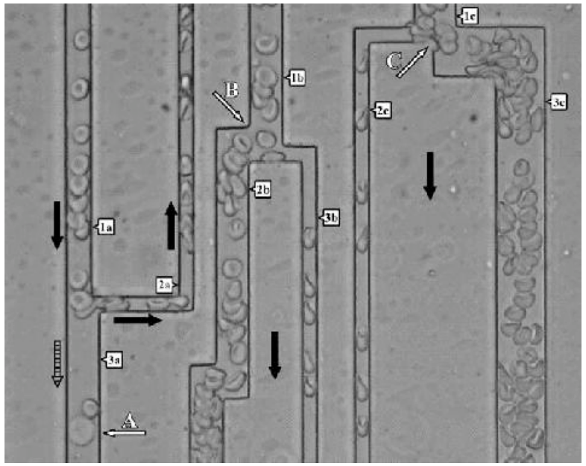
Collaboration B. Polack, Hematology / TIMC, CNRS/INSERM, CHU-Grenoble

Soft objects in microfluidic devices

Cell sorting



(Coll. M. Hoyos, P. Kurowski, PMMH, ESPCI Paris)



Shevkoplyas et al., Microvascular Research 65 (2003)

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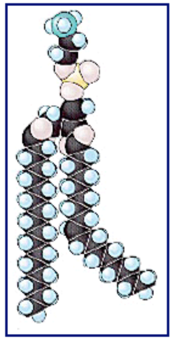
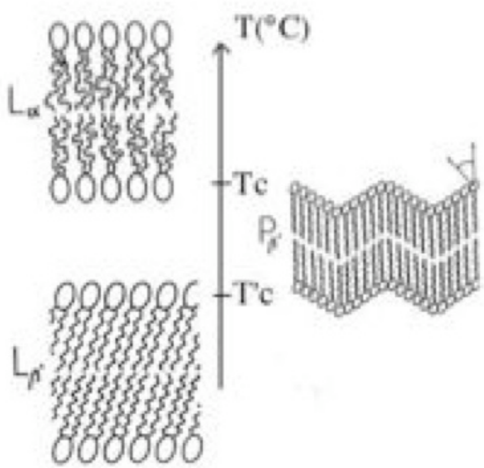
Lipid bilayers

- Surfactants with very low solubility : CMC~10^{-10} mol/l
- Forms bilayers
- High resistance to surface change
- Gel or liquid phase depending on temperature





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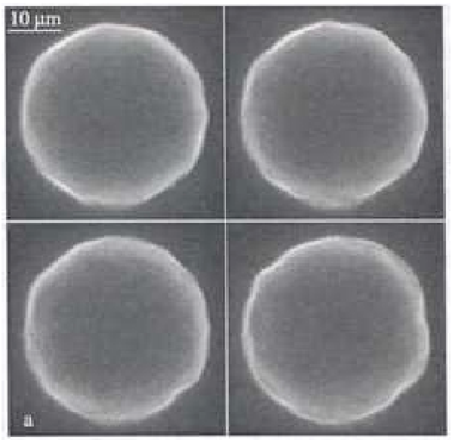
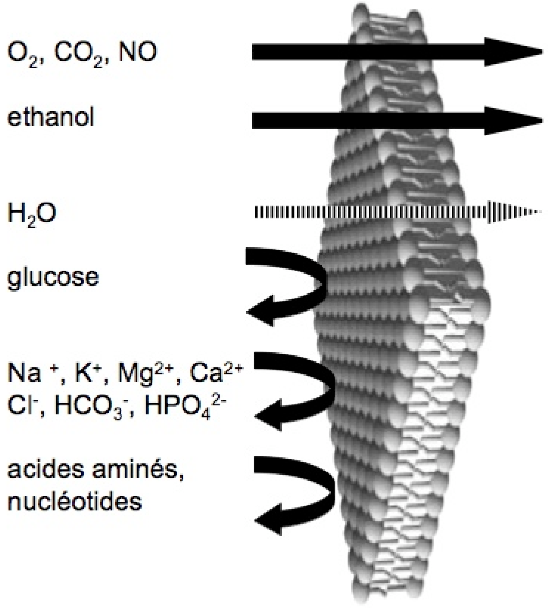
Mechanical and physical properties

- Fluid membranes (DOPC) : no shear modulus, low surface shear viscosity
- Large stretch modulus (κ_s = 265 mN/m): <u>surface ~ constant</u>
- Low curvature energy ($\kappa_b = 10^{-19}$ J)
- Permeable to water, osmotic pressure imposes <u>volume ~ constant</u>





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Equilibrium shapes

$$E_{bending} = \frac{1}{2}\kappa \mathrm{H}^2 + \frac{1}{2}\kappa_g \mathrm{G}$$

Helfrich, Naturforsch. (1973)

Reduced volume v

$$v = V/(4\pi R_0^3/3)$$

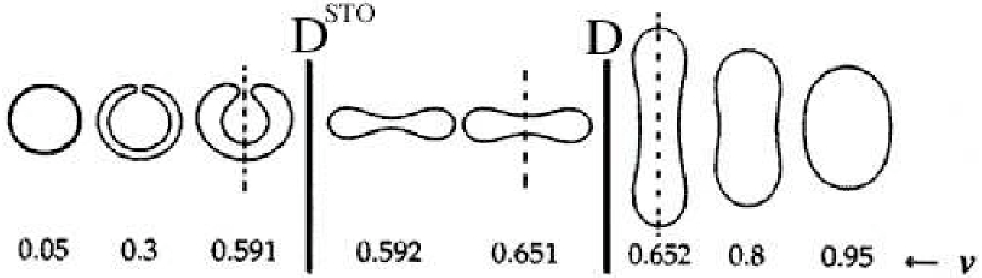
$$R_0 = (S/4\pi)^2$$

Excess area Δ

$$S = 4\pi + \Delta$$

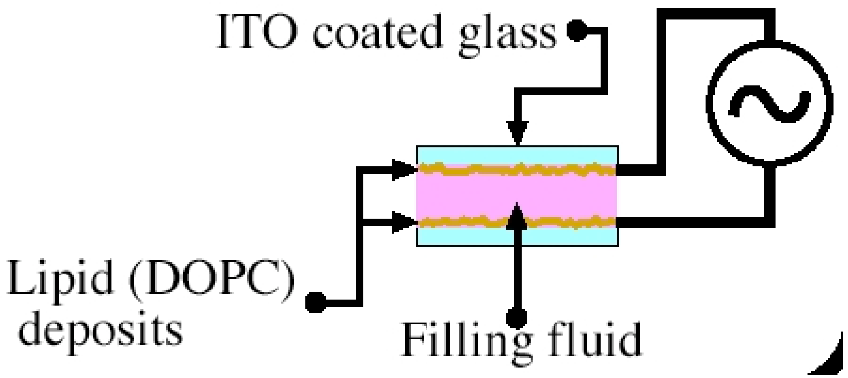
$$\Delta = 4\pi(\nu^{-2/3} - 1)$$

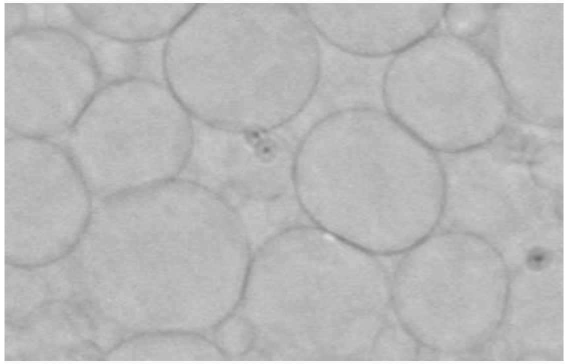
stomatocyte oblate prolate



Preparation: electroformation

- Composition:
  - lipids: DOPC, egg-PC (lecithin)...
  - solutions: glucose, sucrose, dextran
- Internal viscosities : 1-20 cP
- Densities: 1 - 1.05 g/cm^3
- Vesicle diameter: 1 - 100 μm





spectro
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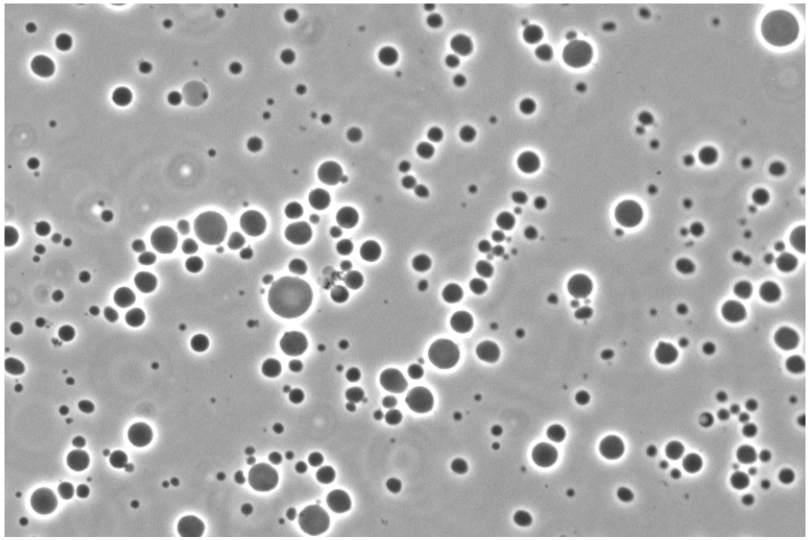
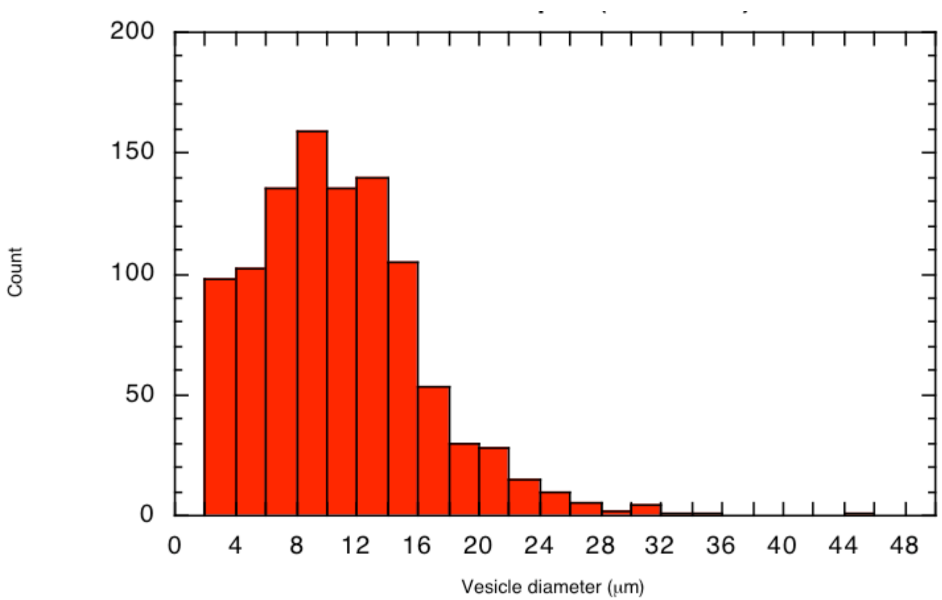
Typical population





Dilution

- Osmotic deswelling in a hyperosmotic solution

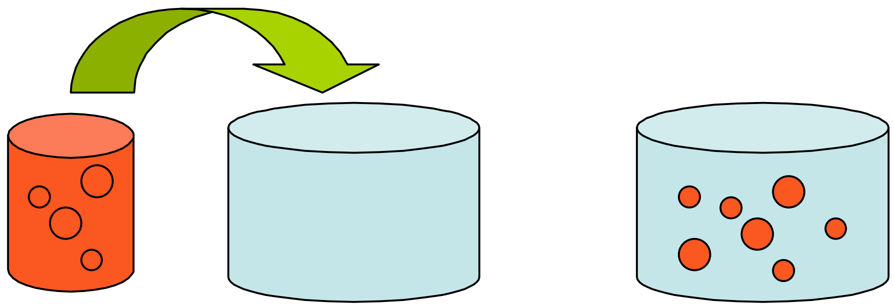
  $0.7 < \nu < 1$

- Refractive index contrast
- Viscosity contrast

  $0.1 < \lambda = \eta_{in}/\eta_{out} < 20$

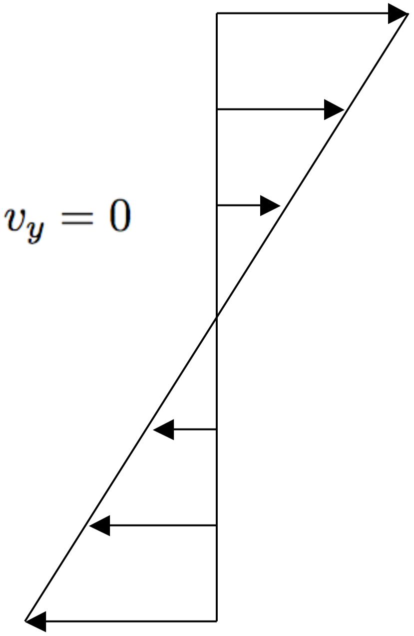
- Density difference

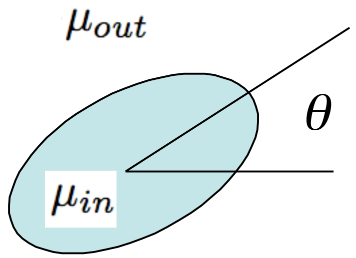
  $0.01 < \Delta\rho < 0.05$ g/cm^3



Dynamics in a simple shear flow

$$v_x = \dot{\gamma}\, y, \quad v_y = 0$$





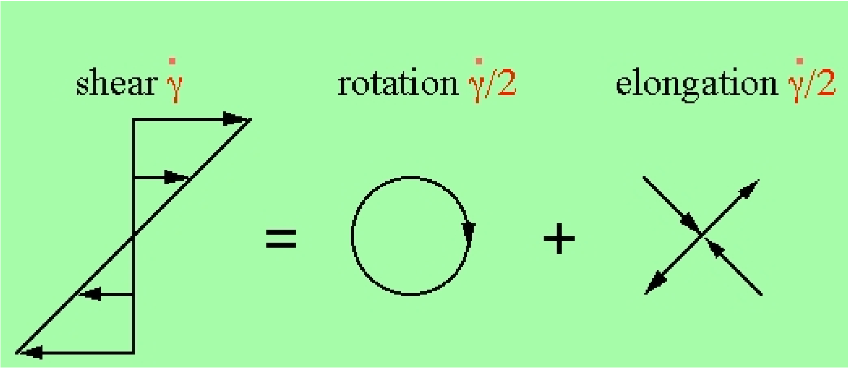
$R_0\,,\ \nu$

Ellipsoidal rigid particles

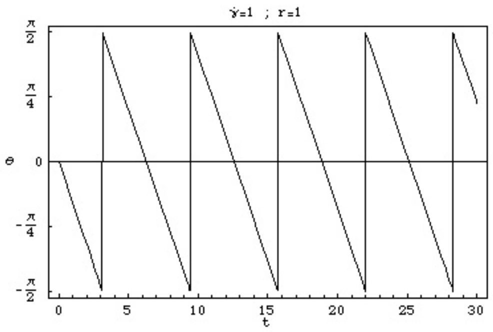
Jeffery, *Proc. Roy. Soc.* **102**, 161 (1922)

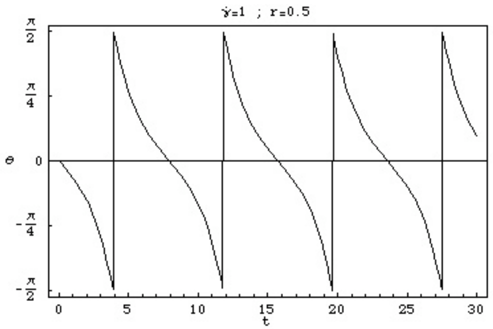
$$\theta(t) = \arctan\left[-r\tan\left(\dot\gamma\frac{r}{1+r^2}(t-t_0)\right)\right]$$

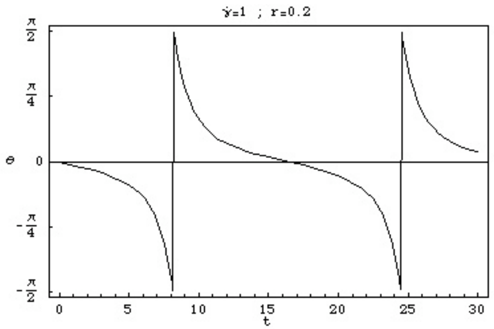
$$\dot\theta(t) = \frac{\dot\gamma}{2}\left(-1+\frac{1-r^2}{1+r^2}\cos 2\theta\right)$$

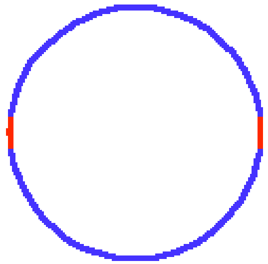


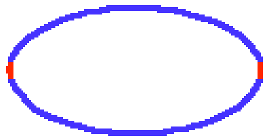
Ellipsoidal rigid particles







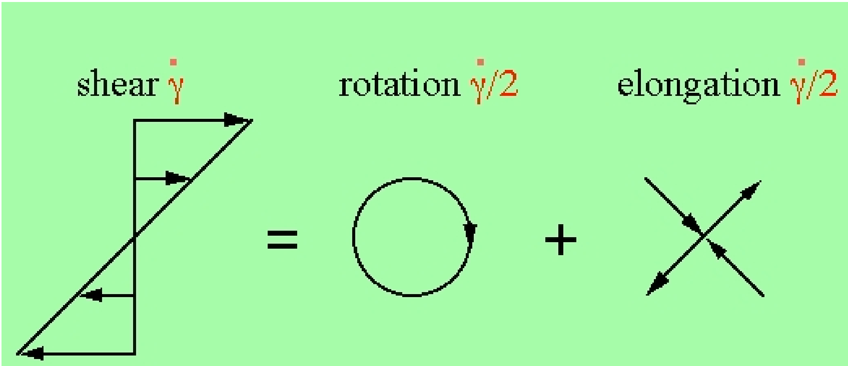




Fluid ellipsoidal particle

Keller and Skalak, J. Fluid Mech. (1982)

- fixed ellipsoidal shape
- Stokes flow inside and outside, Re<<1
- fluid membrane



$$\dot\theta \;=\; A + B\, cos(2\,\theta)$$

$$\dot\phi = \frac{\dot\gamma}{2}\frac{C_1}{C_2-\lambda}\left(r - \frac{1}{r}\right)\cos 2\theta$$

$$A \;=\; -\dot\gamma/2$$

$$B = \dot\gamma\left[\frac{1}{2} + \frac{C_1}{\left(C_2 - \frac{\eta_{in}}{\eta_{out}}\right)}\right] C_3$$

$$C_1 = -\frac{4}{g\left(r^{-4/3}+r^{2/3}\right)\left(r-r^{-1}\right)^2},$$

$$C_2 = \left(1 - \frac{2}{g\left(r^{-4/3}+r^{2/3}\right)}\right),$$

$$C_3 = \frac{1-r^2}{1+r^2},$$

$$g = \int_O^\infty \frac{\mathrm{d}s}{(r^{2/3}+s)^2(r^{-4/3}+s)^{3/2}}$$

Solutions

$$\dot{\theta} = A + B\,cos(2\,\theta)$$

$0 \leq -A/B < 1$, stationary solution :

$$\theta = \frac{1}{2}\arccos\left(-\frac{A}{B}\right)$$

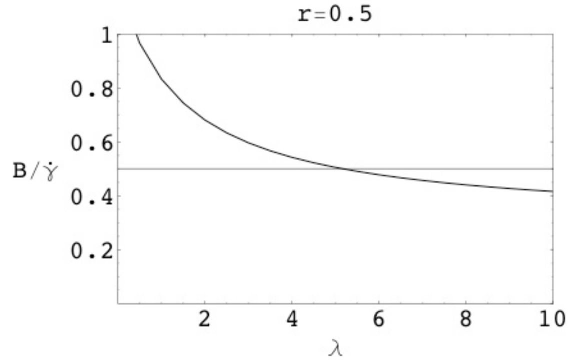
$-A/B > 1$, periodic solution :

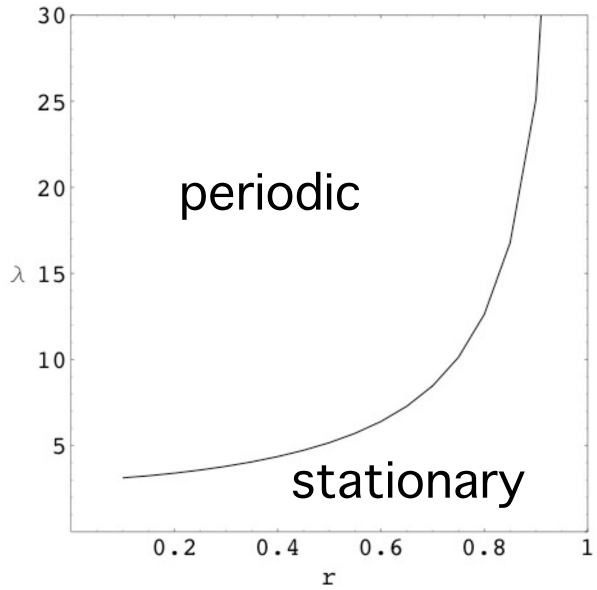
$$\theta(t) = \arctan\left(\frac{A+B}{\sqrt{A^2-B^2}}\tan\left(\sqrt{A^2-B^2}(t-t_0)\right)\right)$$

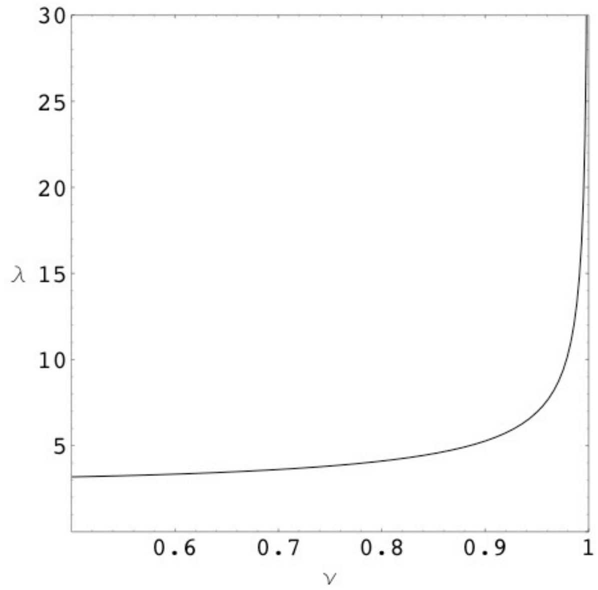
Transition

$$A = -\dot{\gamma}/2$$

$$B = \dot{\gamma}\left[\frac{1}{2} + \frac{C_1}{\left(C_2 - \frac{\eta_{in}}{\eta_{out}}\right)}\right] C_3$$

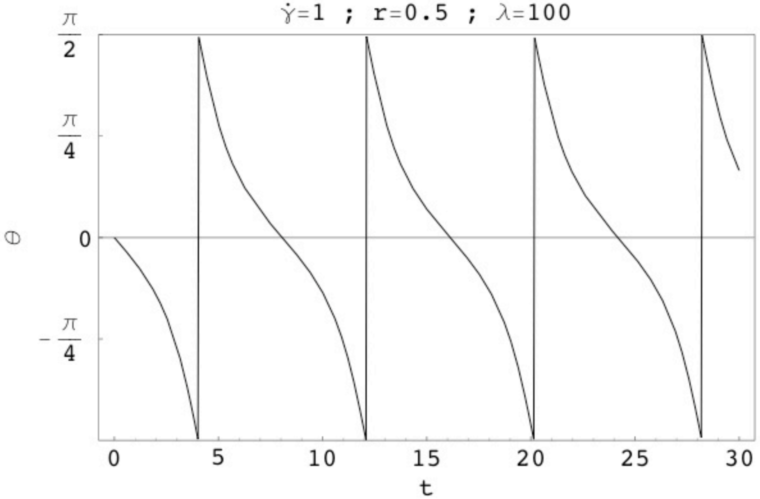


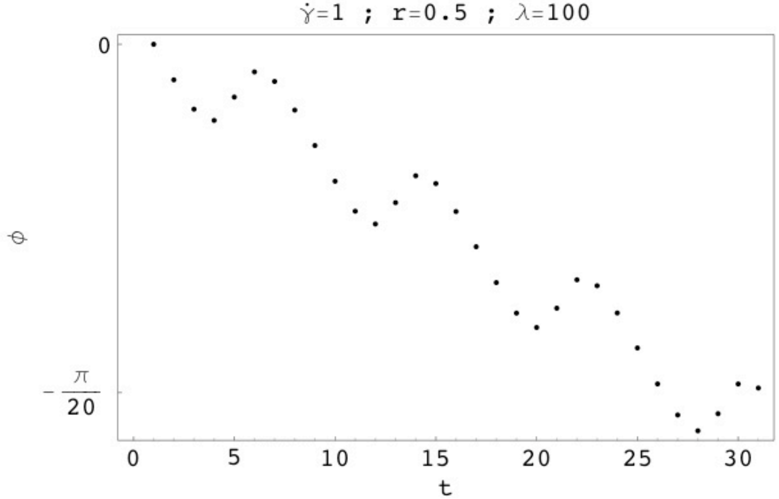


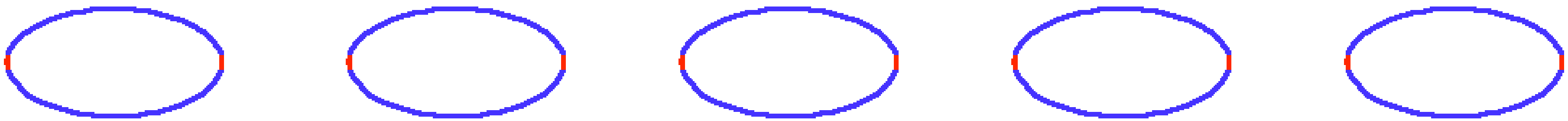


$$\dot{\theta} \;=\; A + B\,cos(2\,\theta)$$

$$\dot{\phi} = \frac{\dot{\gamma}}{2}\frac{C_1}{C_2 - \lambda}\left(r - \frac{1}{r}\right)\cos 2\theta$$

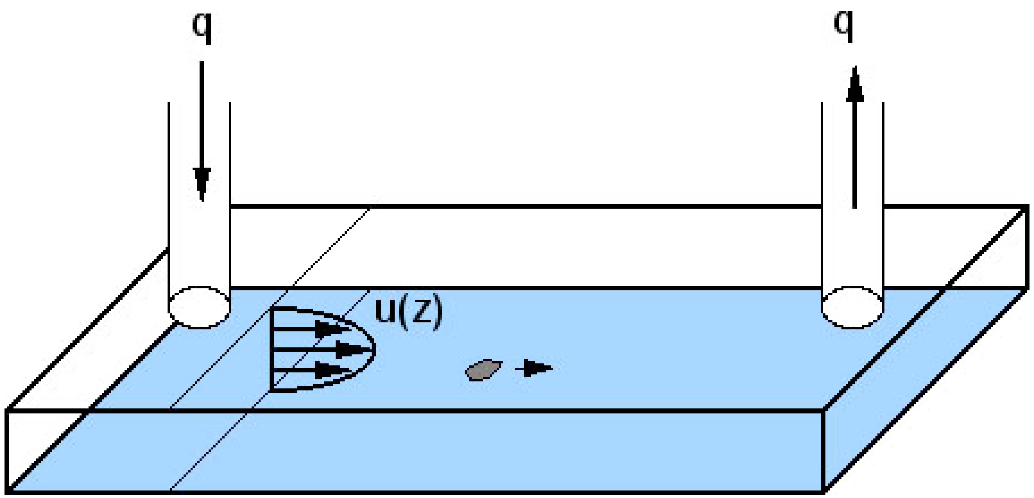


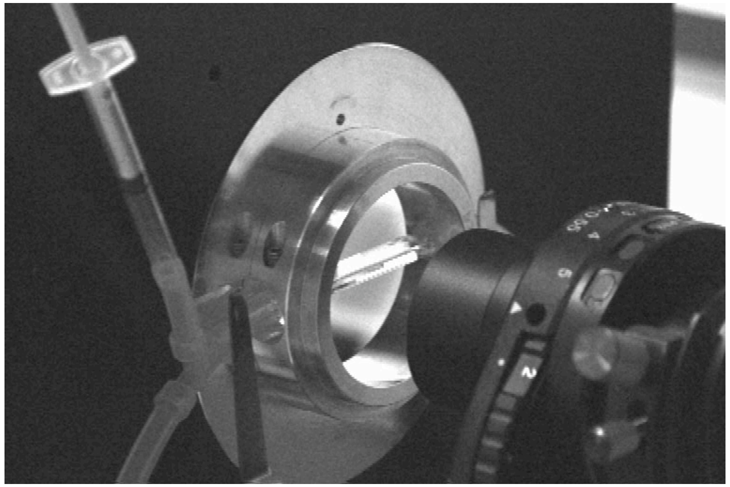




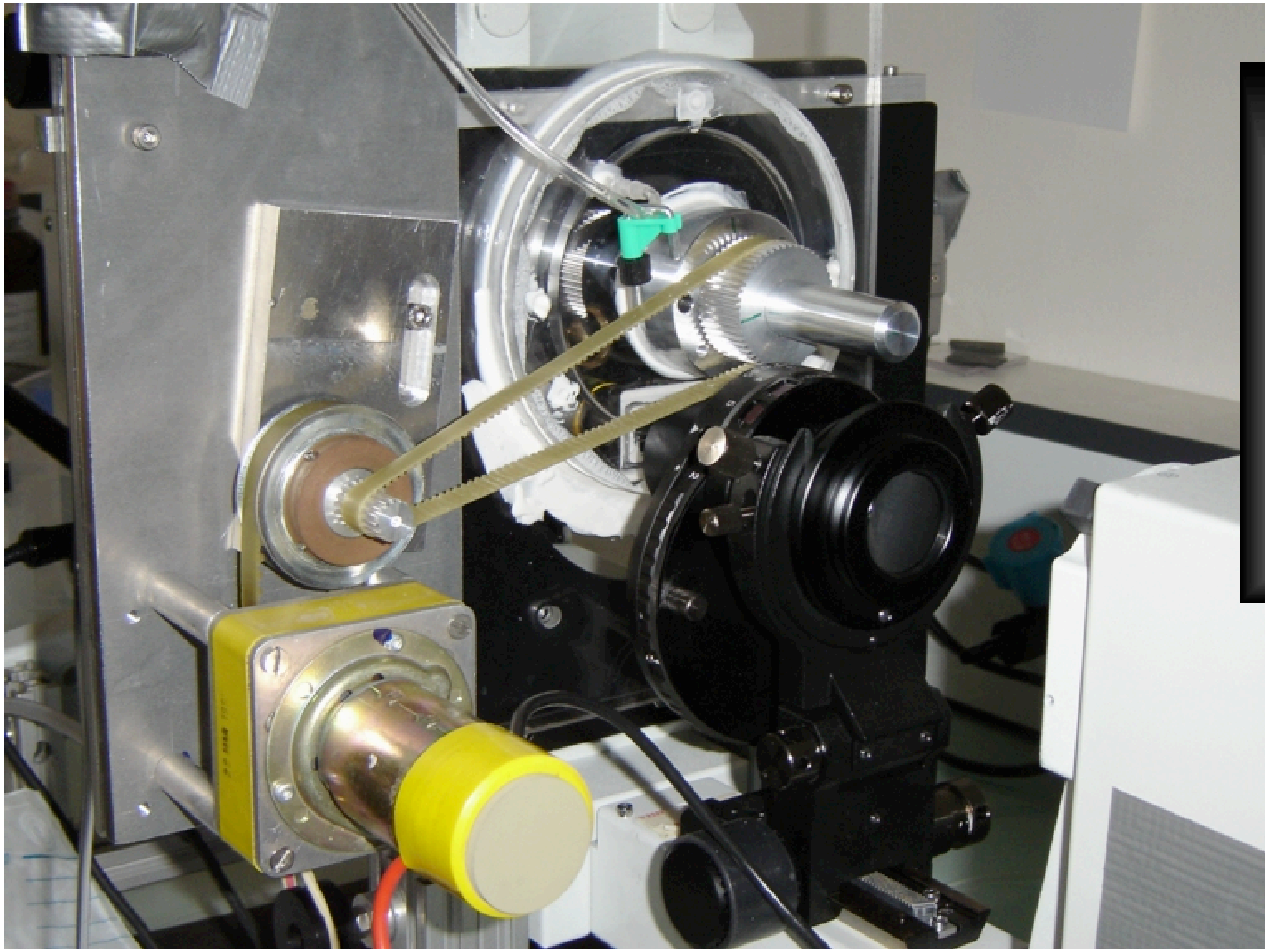
Experimental setup

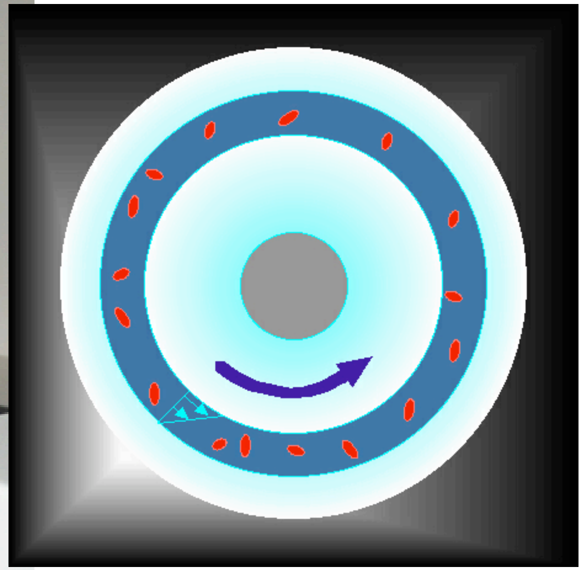
2D channel flow





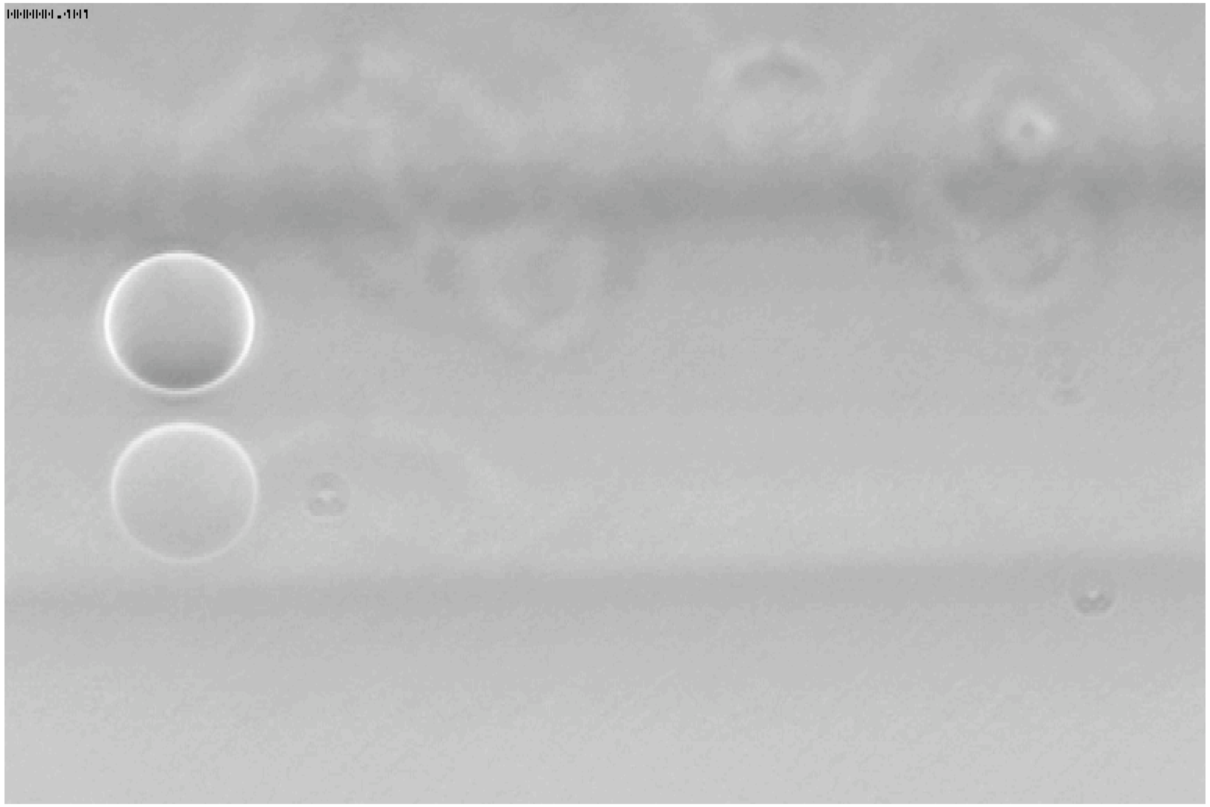
Cylindrical Couette flow



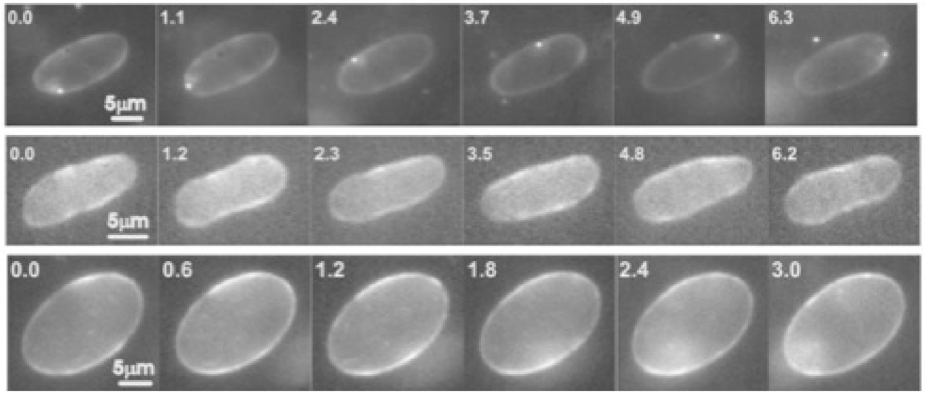


$$1 < \dot{\gamma} < 10\ s^{-1}$$

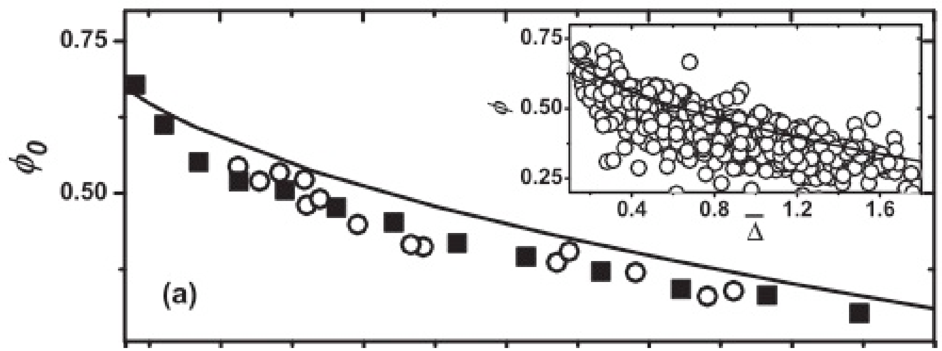
Tank-treading



Tank-treading



Kantsler & Steinberg, *PRL* (2005)



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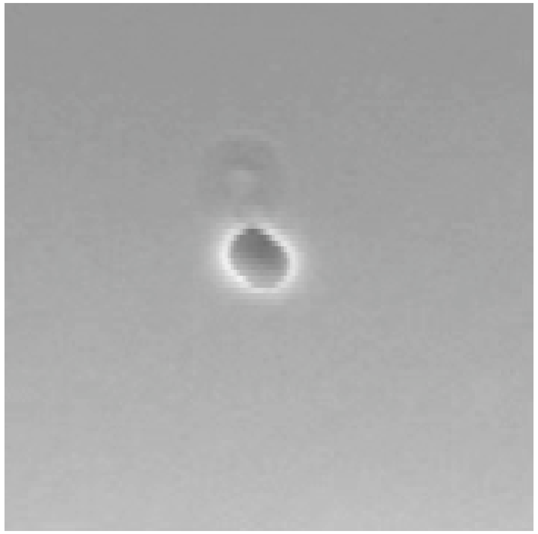
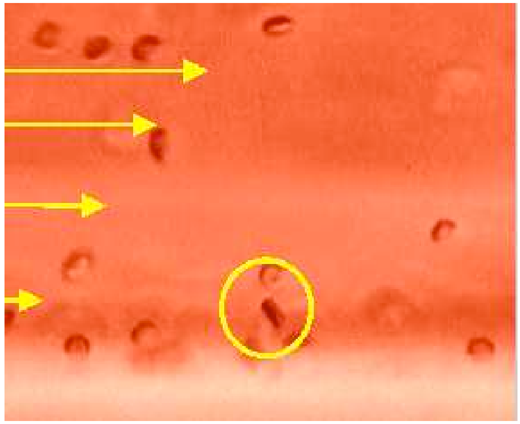
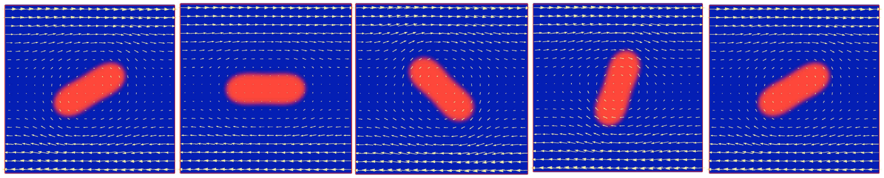
Tumbling



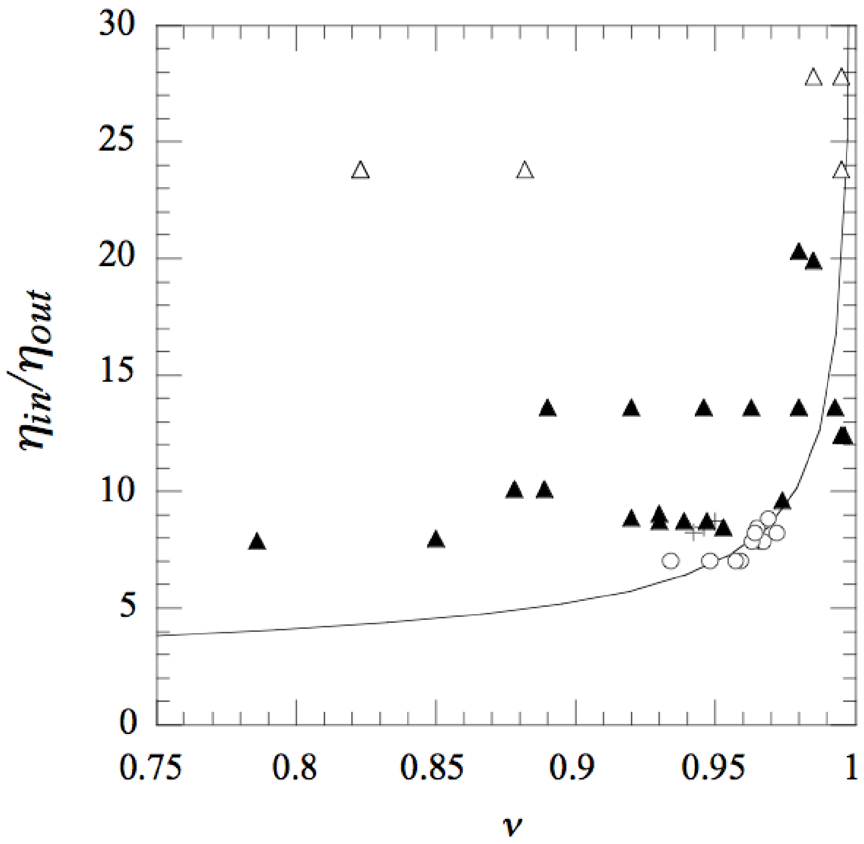


Red blood cells
Faivre, Abkarian, Viallat

Phase field
T. Biben, C. Misbah
Phys. Rev. E **67**, 031908 (2003)

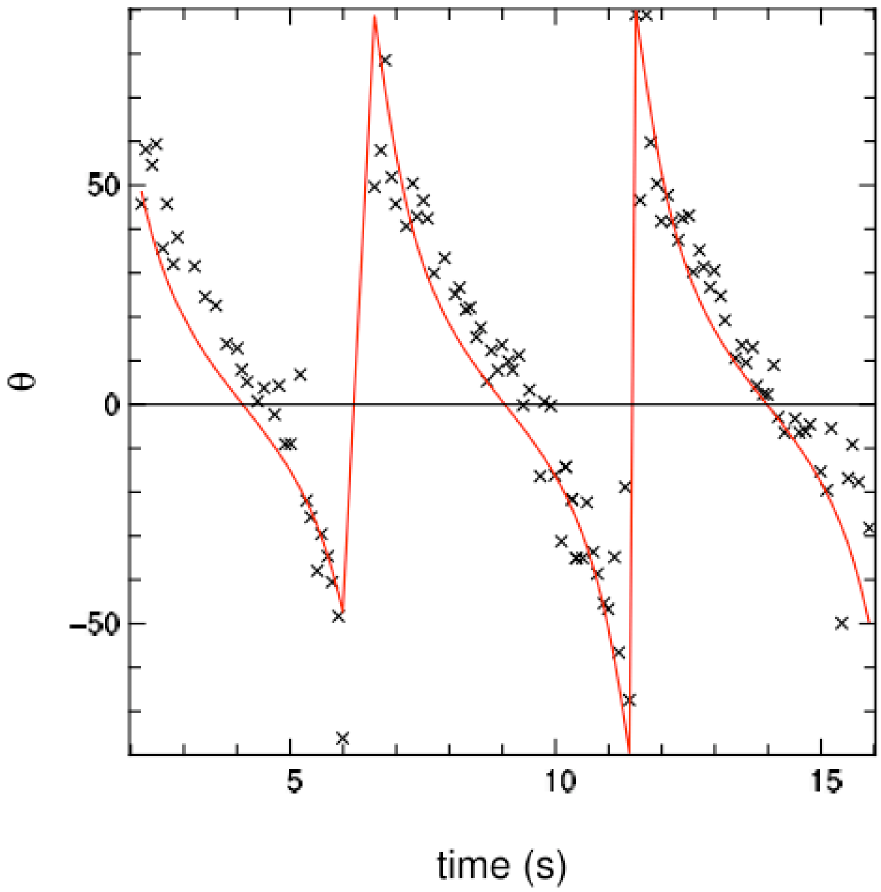


Transition



Kantsler, Steinberg, *PRL* (2005)
Mader, Vitkova, Abkarian, Viallat, Podgorski, *Eur. Phys. J. E* (2006)
Abkarian, Viallat, *Biophys. J.* (2006)

Tumbling



Fit :

$$\dot{\theta} \;=\; A + B\cos(2\,\theta)$$

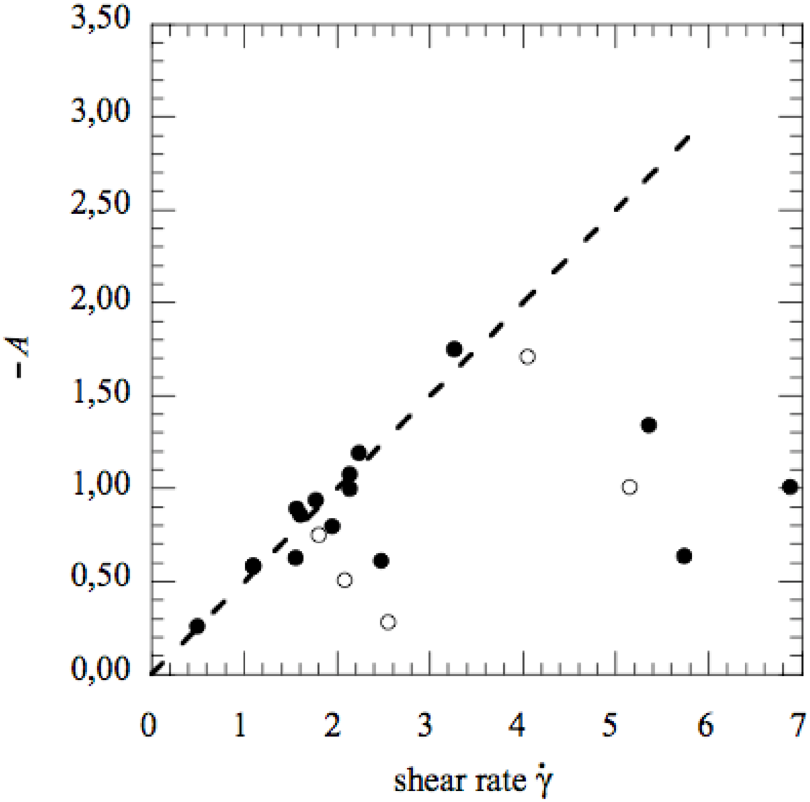
$$\theta(t) = \arctan\left(\frac{A+B}{\sqrt{A^2-B^2}}\tan\left(\sqrt{A^2-B^2}(t-t_0)\right)\right)$$

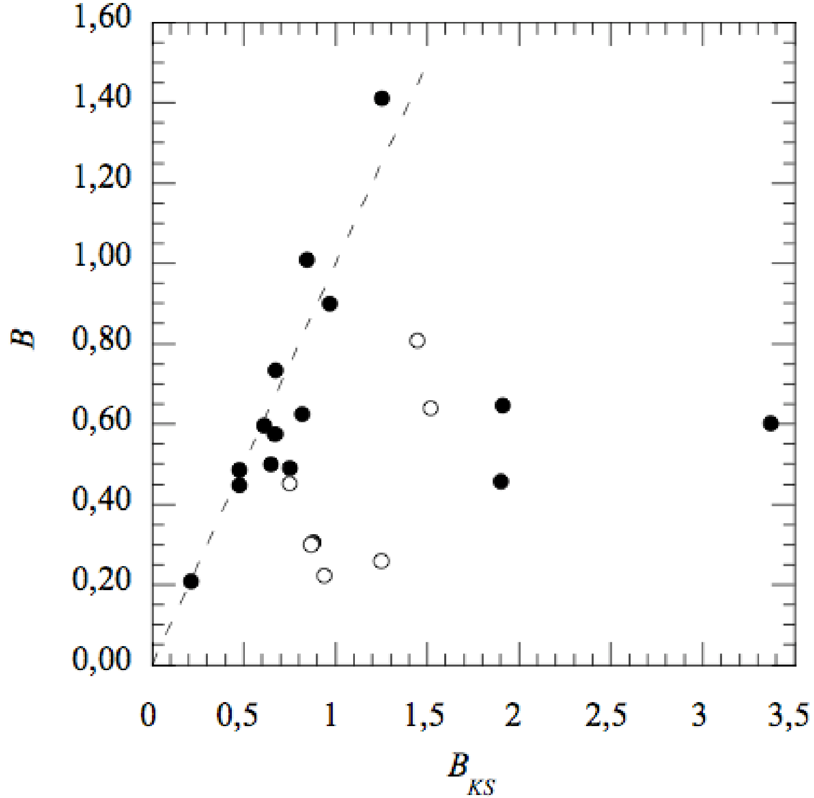
$8 < \eta_{in}/\eta_{out} < 30$

$0.85 < \nu < 1$

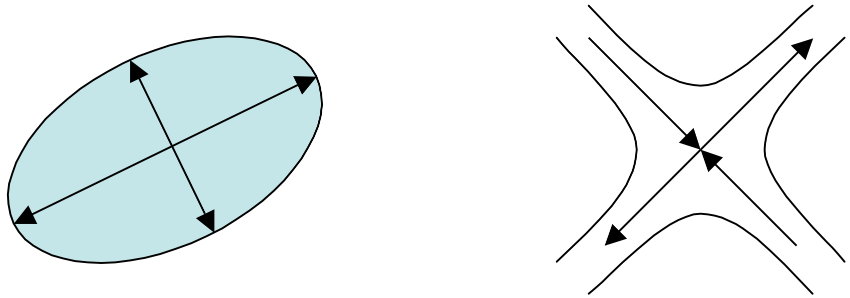
$6 < R_0 < 26\ \mu\text{m}$

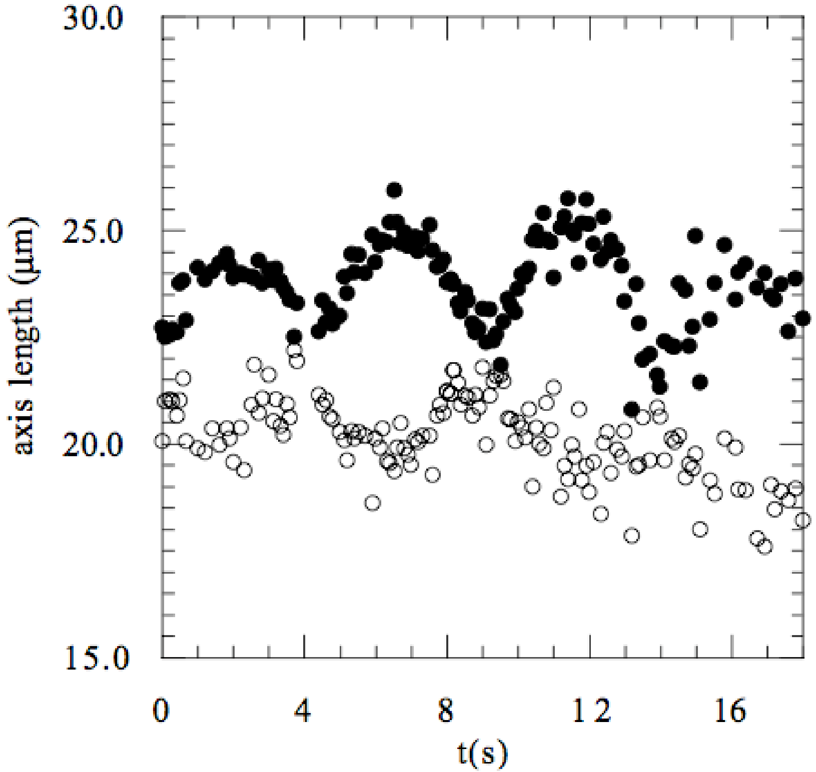
$1 < \gamma < 10$

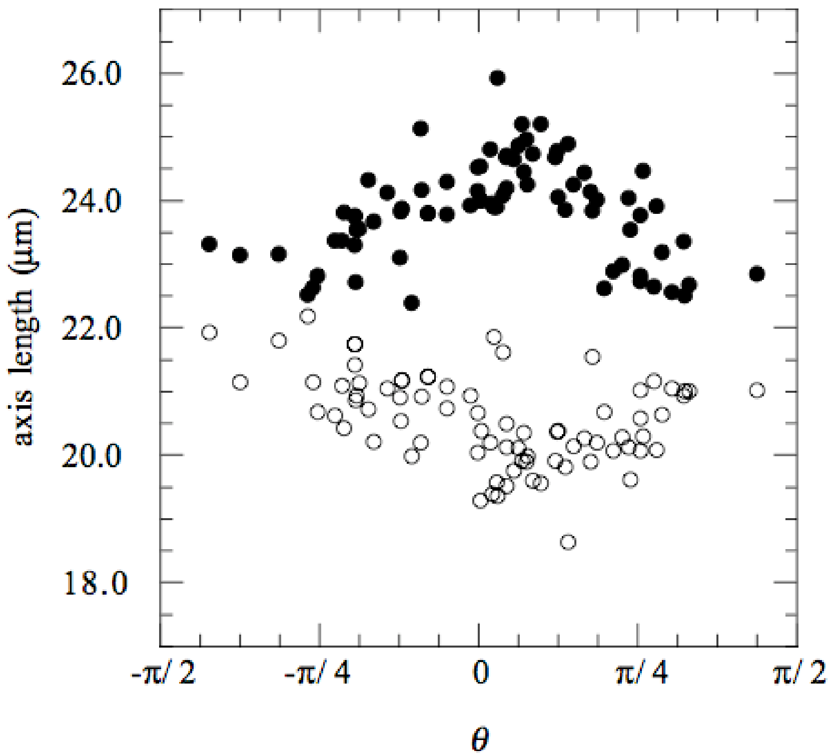




Deformation







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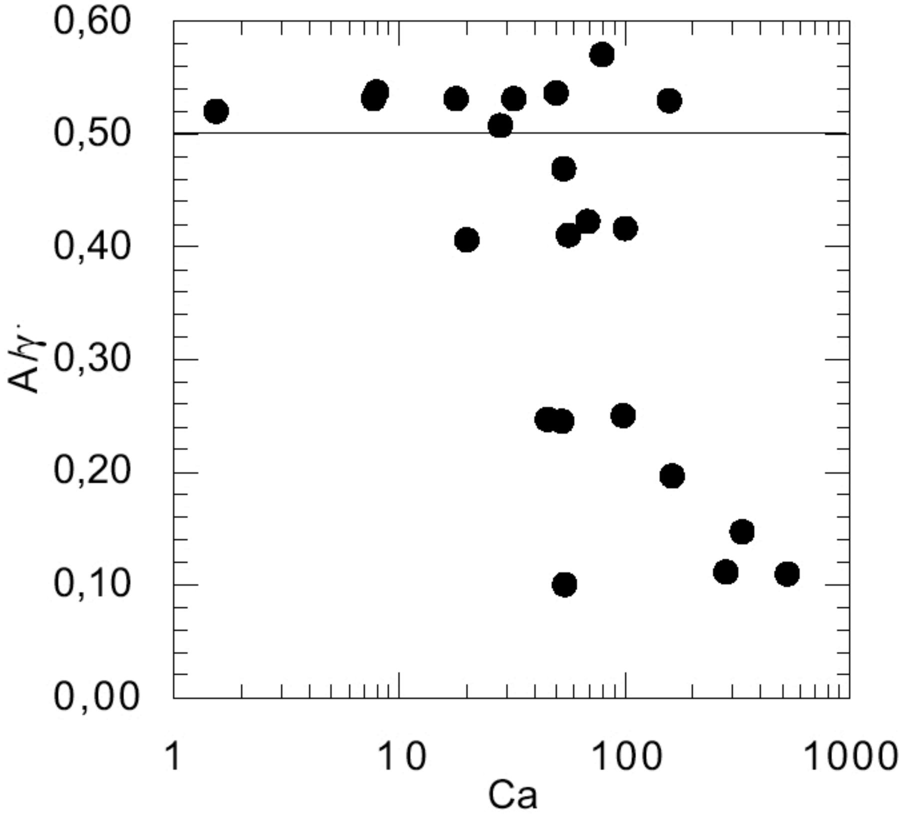
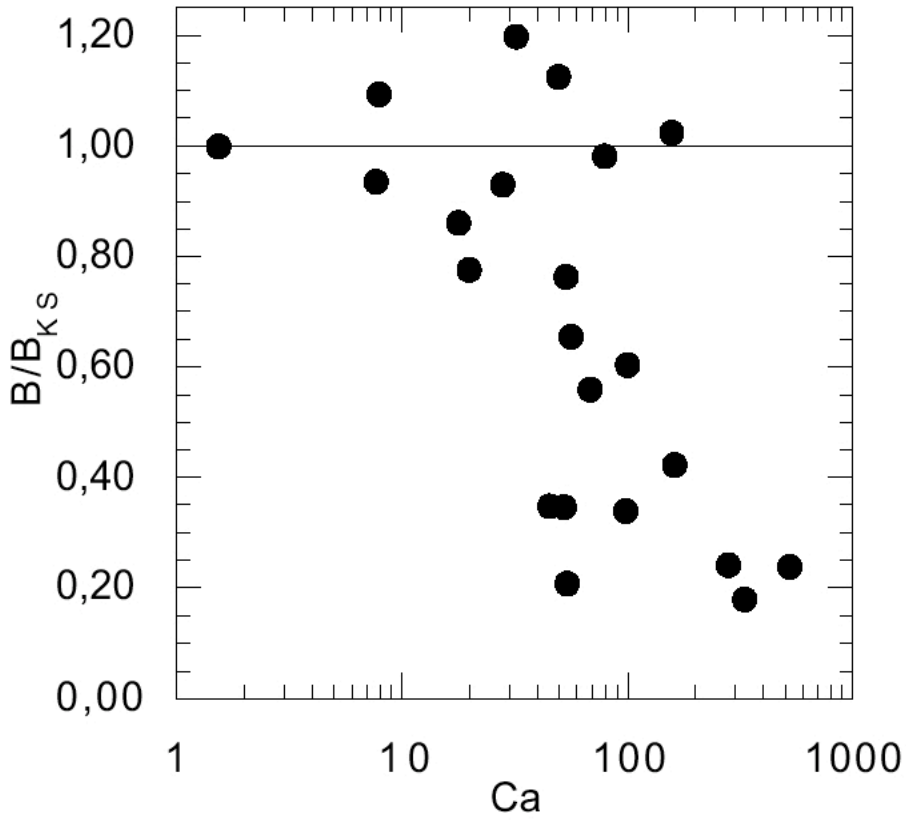
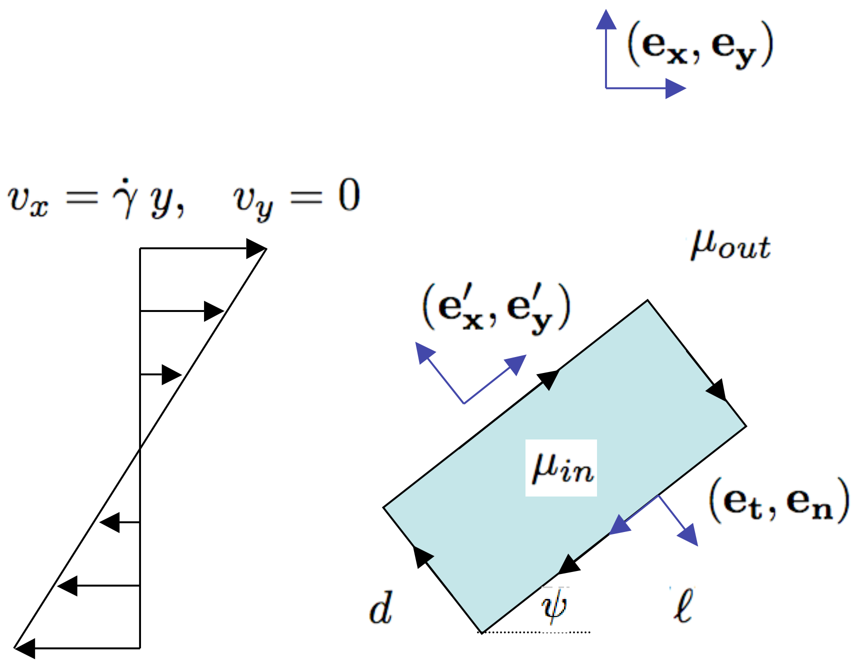
$$Ca = \eta\, \gamma\, R^3 / \kappa_b$$





Simple model: a rectangular vesicle...



Local force ~ relative velocity

$$dF_t = -\mu_{out}\lambda_t V_t dl$$
$$dF_n = -\mu_{out}\lambda_n V_n dl$$

3 components:

- Imposed shear
- Tumbling
- Tank treading

$\Rightarrow$ 3 torques

M. Mader, H. Ez-Zahraouy, C. Misbah, T. Podgorski, Eur. Phys. J. E **22**, 275-280 (2007)

1. Imposed shear

$$V_{shear_t} = -\gamma\, y\mathbf{e_x}.\mathbf{e_t}$$
$$V_{shear_n} = -\gamma\, y\mathbf{e_x}.\mathbf{e_n}$$

$$\mathbf{M_{shear}} = \oint_C \mathbf{r} \times \mathbf{dF_{shear}}$$

$$M_{shear} = -4\mu_{out}\gamma\lambda\left[[\frac{\ell^3}{3} + d\ell^2]\sin^2(\psi) + [\frac{d^3}{3} + d^2\ell]\cos^2(\psi)\right]$$

2. Tumbling

$$\mathbf{V_{tumble}} = \mathbf{w} \times \mathbf{r} \qquad\qquad \mathbf{w} = \frac{d\psi}{dt}\mathbf{e_z}$$

$$V_{tumble_t} = (x' \frac{d\psi}{dt}\mathbf{e_{y'}} - y' \frac{d\psi}{dt}\mathbf{e_{x'}})$$
$$V_{tumble_n} = (+x' \frac{d\psi}{dt}\mathbf{e_{y'}} - y' \frac{d\psi}{dt}\mathbf{e_{x'}})$$

$$\mathbf{M_{tumble}} = \oint_C \mathbf{r} \times \mathbf{dF_{tumble}}.$$

$$M_{tumble} = -4\lambda\mu_{out}\frac{d\psi}{dt}\left[\frac{\ell^3}{3} + \frac{d^3}{3} + d^2\ell + \ell^2 d\right]$$

3. Tank treading

Injected energy = dissipated energy

$$E_{shear} = 4\lambda\mu_{out}V_{tank}\gamma\ell d$$

$$E_{tumble} = 8\lambda\mu_{out}V_{tank}\ell d\frac{d\psi}{dt}$$

$$E_{tank} = -2\lambda\mu_{out}(d+\ell)V_{tank}^2$$

$$v_{x'} = V_{tank}\frac{y'}{\ell}, \quad v_{x'} = -V_{tank}\frac{x'}{\ell}$$

$$\epsilon = 4\mu_{in}V_{tank}^2[\frac{1}{d}+\frac{1}{\ell}]$$

$$V_{tank} = \frac{4\lambda\ell[\dot{\gamma}/2+\dot{\psi}]}{\nu[2(\ell/d+d/\ell)+\lambda(d+\ell)]}$$

$$\nu = \mu_{in}/\mu_{out}$$

$$\mathbf{F_{tank}} = -\mu_{out}\lambda_t V_{tank}\mathbf{e_t}$$

$$\mathbf{M_{tank}} = \oint_C \mathbf{r}\times\mathbf{dF_{tank}}$$

$$M_{tank} = 8\lambda\mu_{out}V_{tank}\ell d$$

Equations of motion

$$\frac{d\psi}{dt} = A + B\cos(2\psi)$$

$$A = -\frac{1}{2}, \quad B = \frac{(\bar{\ell}-1)(\bar{\ell}^2+4\bar{\ell}+1)}{2\left[(\bar{\ell}+1)^3 - \frac{12\bar{\lambda}\bar{\ell}^3}{(\nu(\bar{\ell}-1)^2+\bar{\lambda}\bar{\ell}(\bar{\ell}+1))}\right]}$$

$$\bar{\ell} = \ell/d$$

$$\bar{\lambda} = d\lambda$$

$$\frac{d\bar{\ell}}{dt} = -\frac{\bar{\ell}-\bar{\ell}_{eq}}{\tau} + \zeta\bar{\ell}\dot{\gamma}\sin(2\psi)$$

linear spring forcing

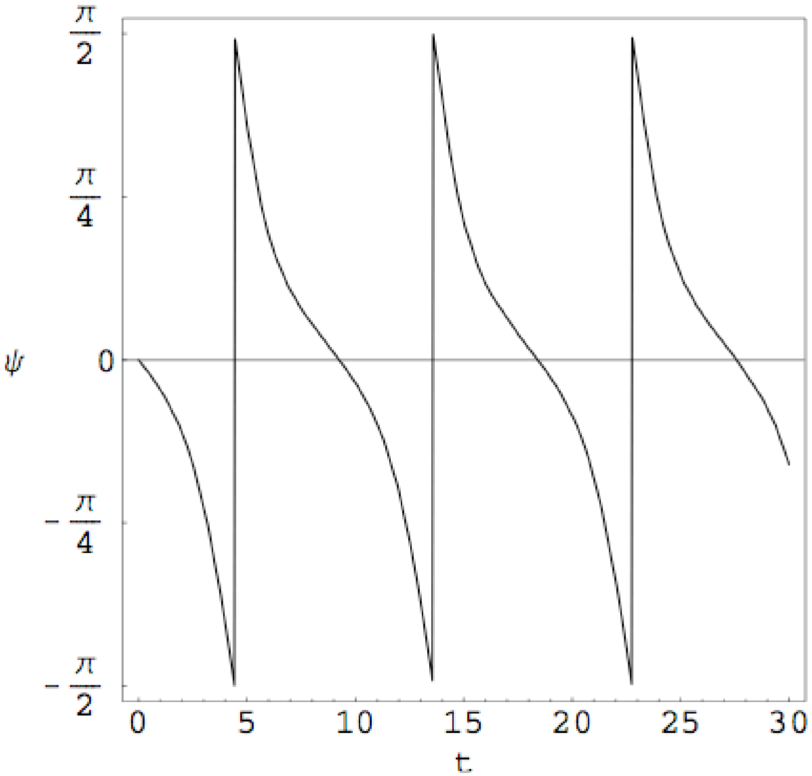
Capillary number

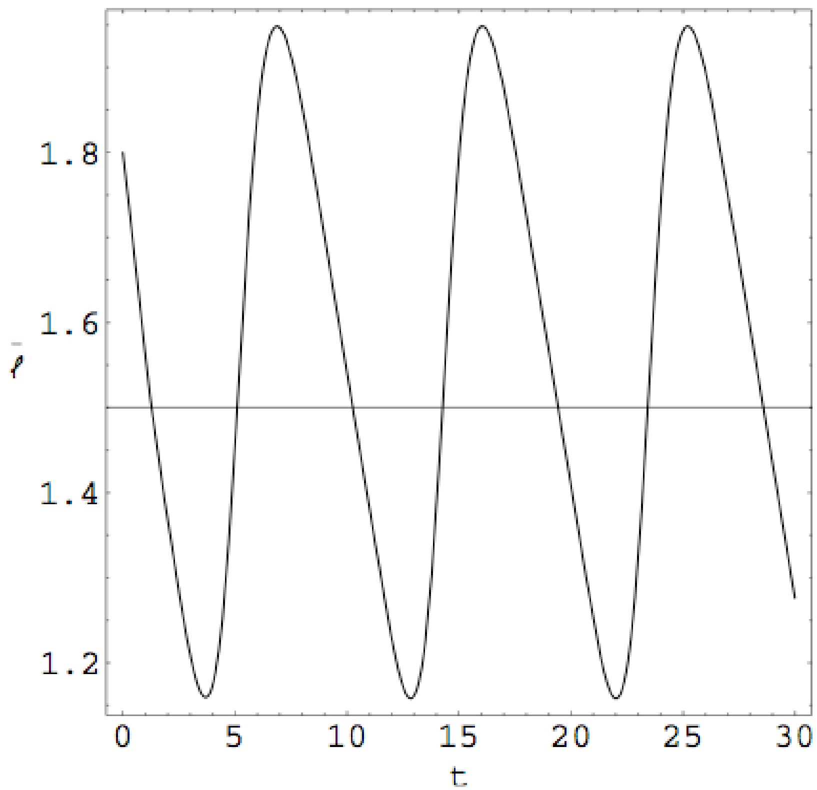
$$\chi = \tau\dot{\gamma}$$

$l_{eq} = 1.5$ $\nu = 10$

$\zeta = 0.35$ $\lambda = 3;$

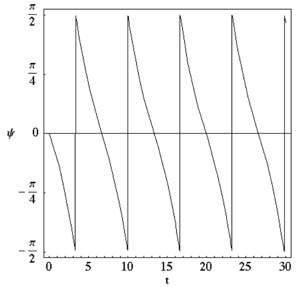
$\chi = 1$



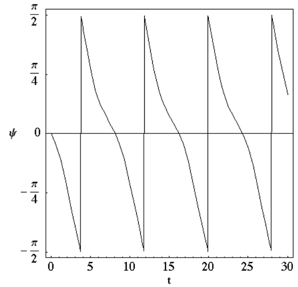


Fit

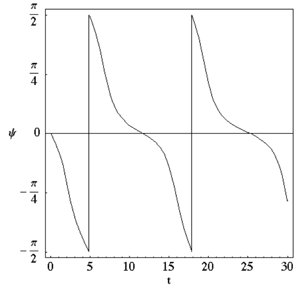
$$\frac{d\psi}{dt} = A_{eff} + B_{eff}\cos(2\psi)$$



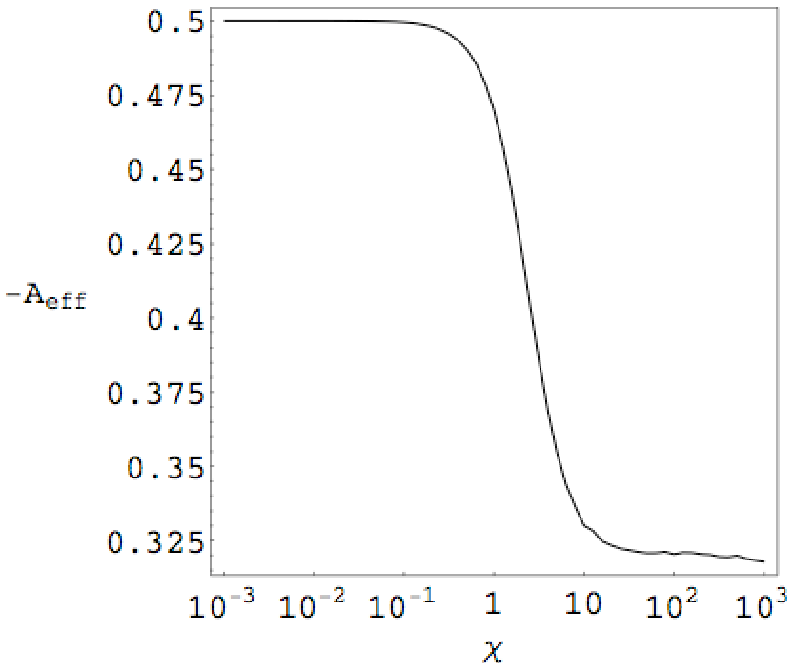
$\chi = 0.1$

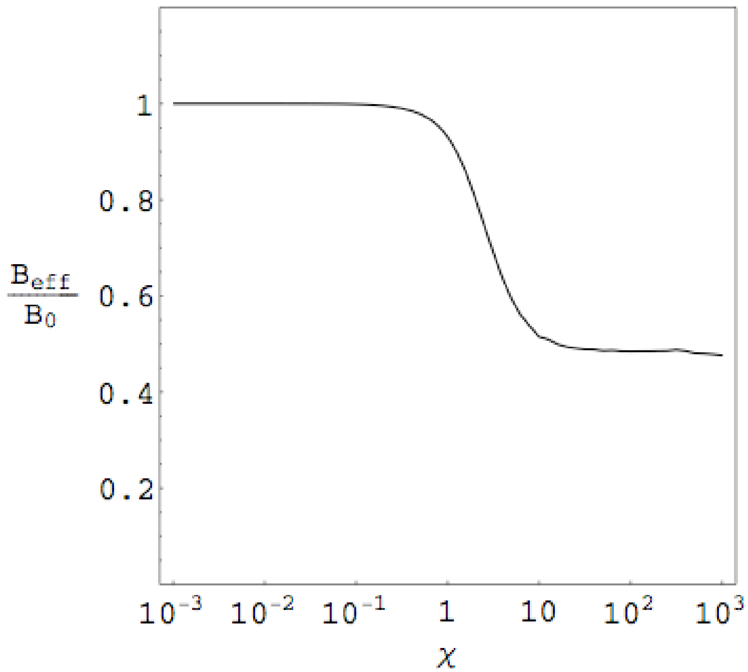


$\chi = 1$

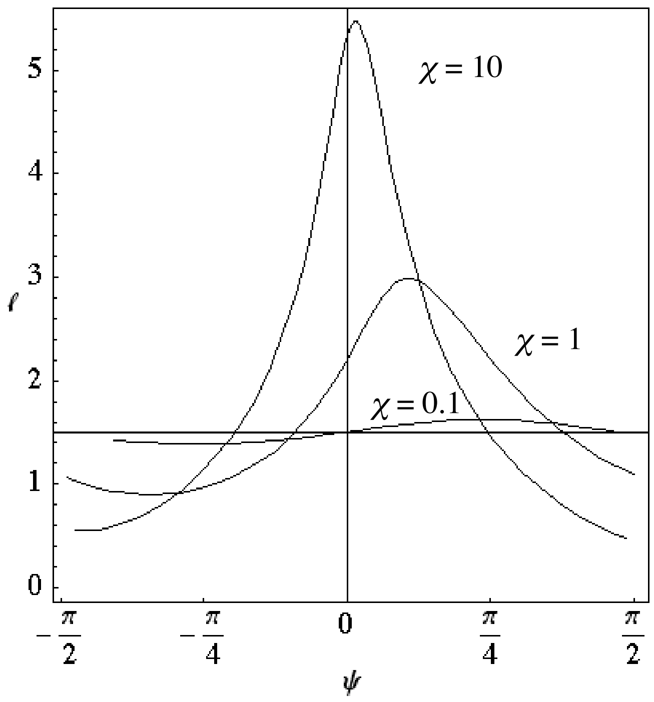


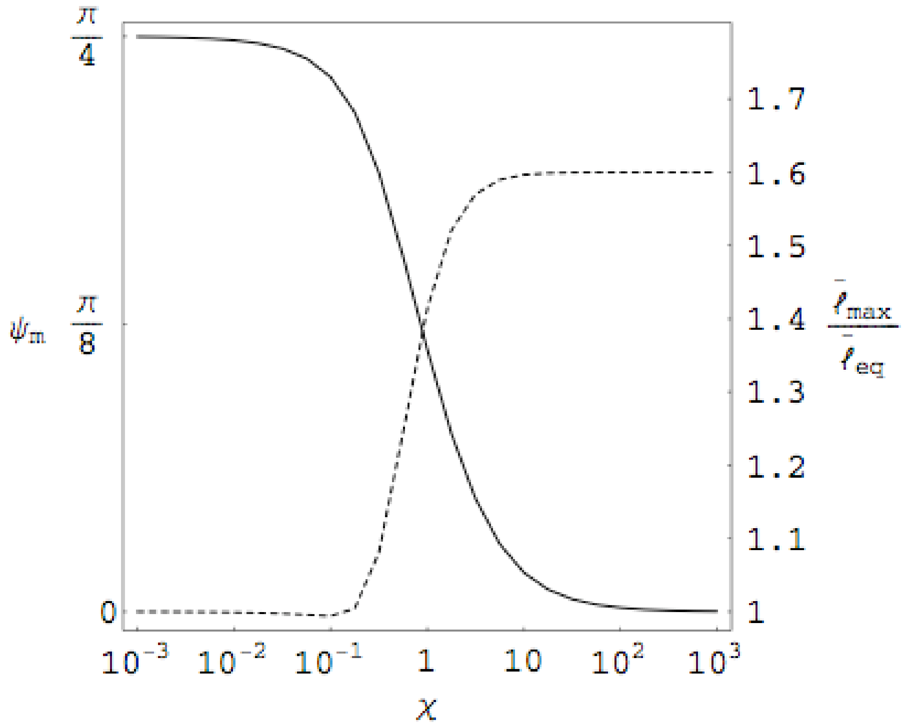
$\chi = 10$





Phase shift rotation - deformation





Ellipsoidal vesicle

- Keller & Skalak for the dynamics of the angle
- Phenomenological model for deformation:

$$\eta_{in} \frac{1}{r_2} \partial_t r_2 = -\frac{\kappa_c}{R^3} \frac{\partial U}{\partial r_2} - \eta_{out} \dot{\gamma} \sin 2\psi$$

$$\partial_t r_2 = -\frac{r_2}{\chi \lambda} \frac{\partial U}{\partial r_2} - \frac{r_2}{\lambda} \sin 2\psi$$

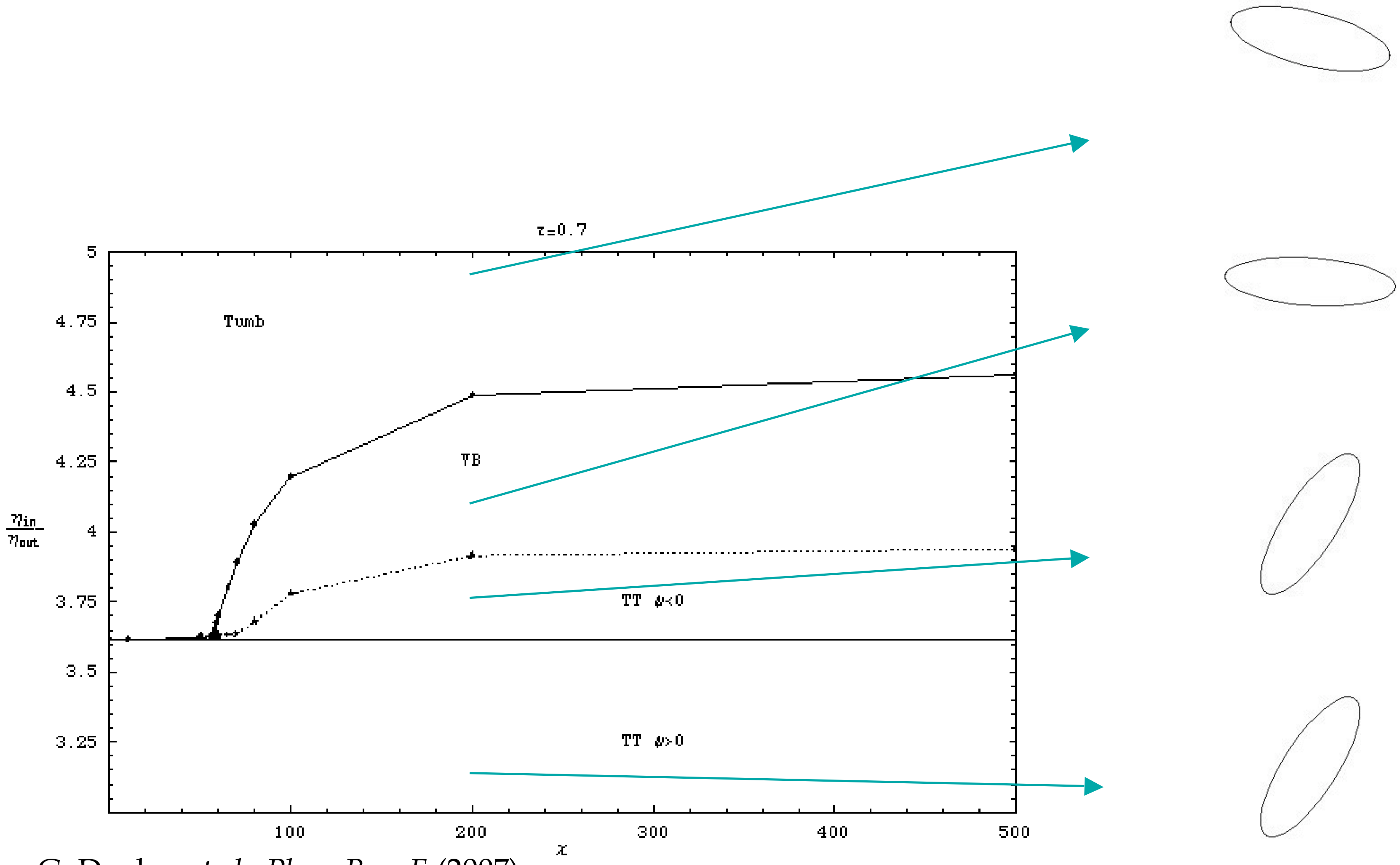
$$\chi = \frac{\eta_{out} \dot{\gamma} R^3}{\kappa_c}$$

$$U = \kappa_c \int H^2 ds$$

(evaluated numerically)

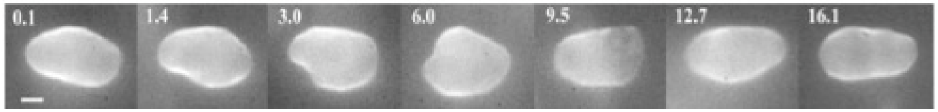
$$\boxed{\begin{aligned} \partial_t \psi &= -\frac{1}{2} + B(r_2, r_3, \lambda) \cos 2\psi \\ \partial_t r_2 &= -\frac{r_2}{\chi \lambda} \frac{\partial U}{\partial r_2} - \frac{r_2}{\lambda} \sin 2\psi \end{aligned}}$$

Phase diagram

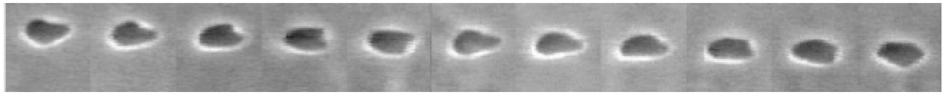


G. Danker *et al.*, *Phys. Rev. E* (2007)

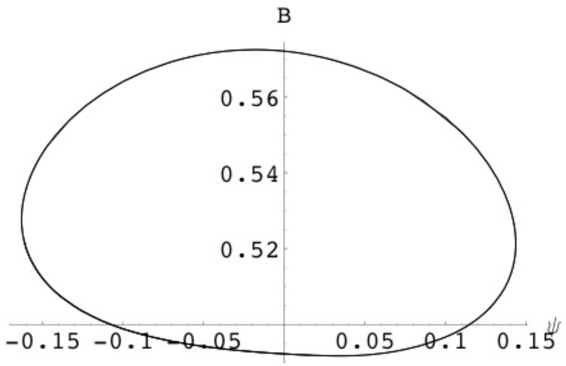
Vacillating-breathing (vb)

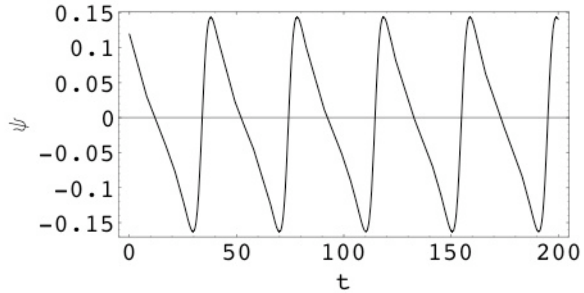


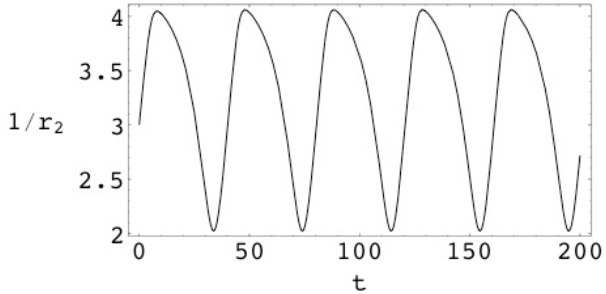
Kantsler & Steinberg, *PRL* (2006)

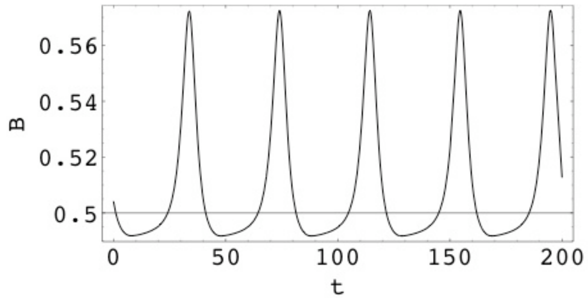


Mader et al., *Eur. Phys. J.* E (2006)









First order small deformation theory

C. Misbah, *Phys. Rev. Lett.* **96**, 028104 (2006)

$$\epsilon \partial_t \psi = -\frac{1}{2} + \frac{h}{2R} \cos(2\psi)$$

$$\epsilon \partial_t R = h\left[1 - 4\frac{R^2}{\Delta}\right] \sin(2\psi)$$

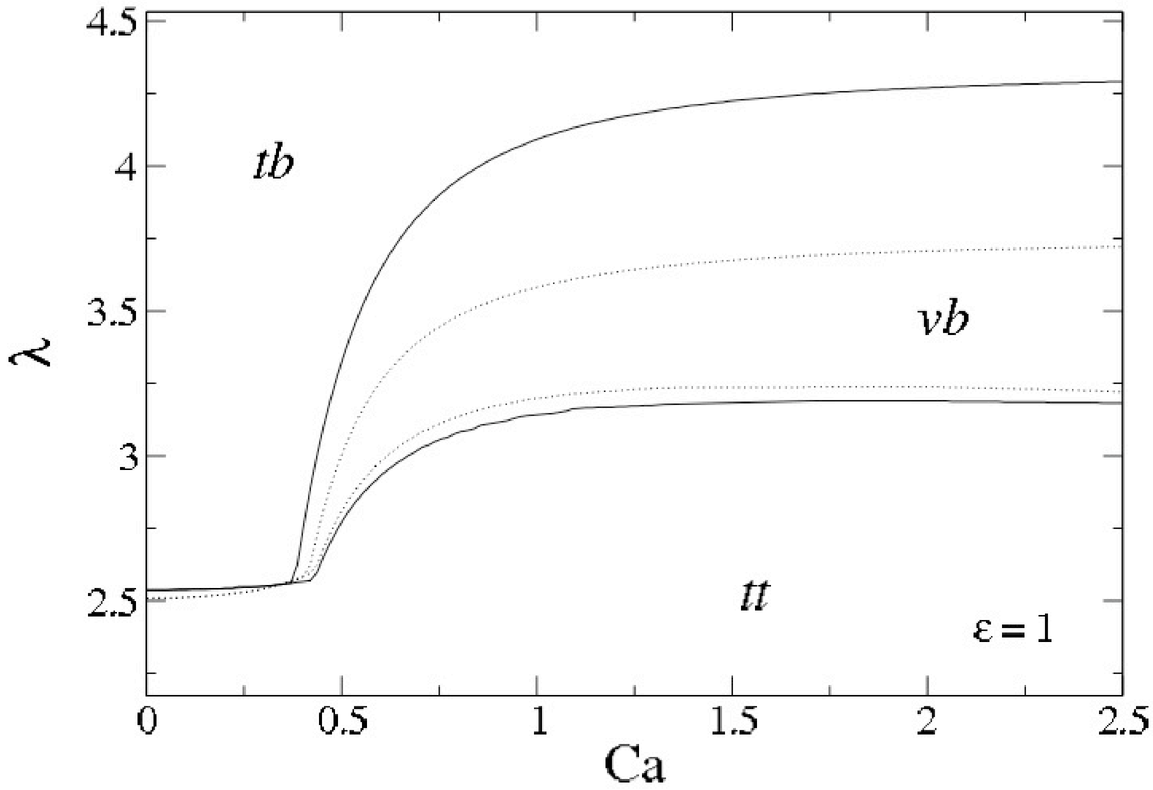
$$2R/2\sqrt{\Delta} = \cos(\alpha)$$

$$h = 60\sqrt{2\pi/15}/(32 + 23\lambda)$$

High Ca limit
Coexistence of *tu* and *vb*

Higher order small deformation theory

$$\begin{aligned} T\partial_t\alpha &= -S\sin\alpha\sin 2\psi + \cos 3\alpha \\ &\quad + \Lambda_1\left(\cos 3\alpha + 2\cos\alpha\right) + \epsilon\Lambda_2\, S\cos 2\alpha\sin 2\psi, \\ T\partial_t\psi &= \frac{S}{2}\left\{\frac{\cos 2\psi}{\cos\alpha}\left[1+\epsilon\Lambda_2\sin\alpha\right] - \Lambda\right\}, \end{aligned}$$



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Rheology: viscosity of a dilute suspension

Rigid spheres (Einstein 1906/1911)

$$[\eta] = \frac{\eta - \eta_0}{\eta_0 \phi} = \frac{5}{2}$$

Drops (Taylor 1934)

$$[\eta] = \frac{5\lambda/2 + 1}{\lambda + 1}$$

Vesicles - tt (Misbah, 2006)

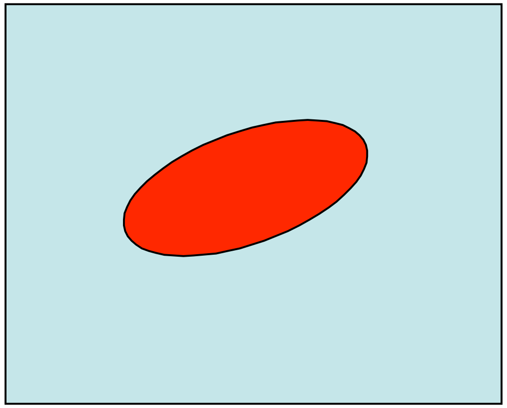
$$[\eta]_{tt} = \frac{5}{2} - \Delta \frac{23\lambda + 32}{16\pi}$$

Vesicles - tumbling (Danker & Misbah, 2007)

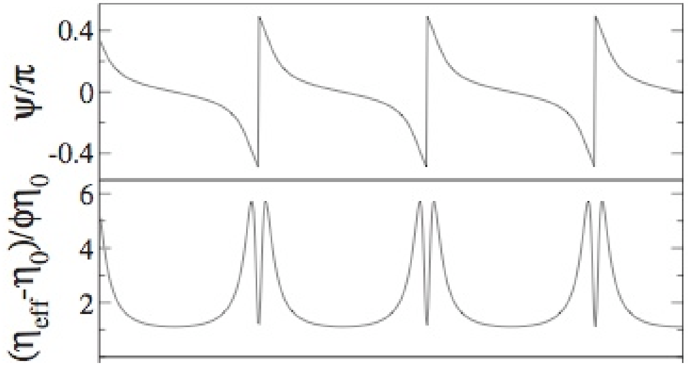
$$[\eta]_{tb} = \frac{5}{2} + \sqrt{\frac{30}{\pi}} \left[\frac{\sqrt{\Delta - 4h^2}}{\sqrt{\Delta + 4h^2} + \sqrt{\Delta}} - h \right]$$

$$h = 60\sqrt{2\pi/15}/(32 + 23\lambda)$$

Average stress in volume



$\eta = \eta_s (1 + [\eta]\, \phi)$



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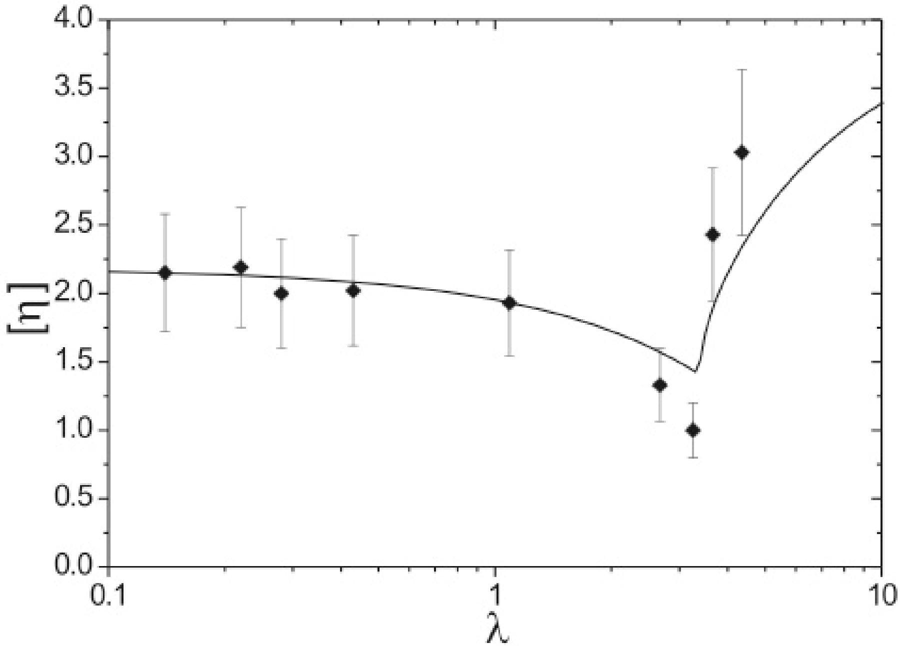
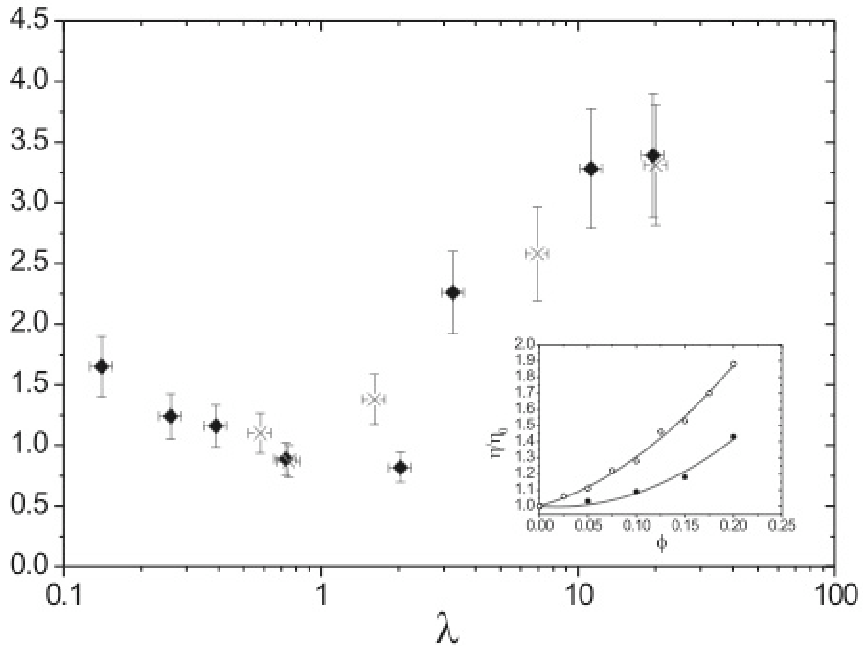
Rheology of dilute suspensions

Vesicles



Red blood cells



$$\eta = \eta_s (1 + [\eta]\, \phi)$$

V. Vitkova *et al., Biophys. J.* (2008)

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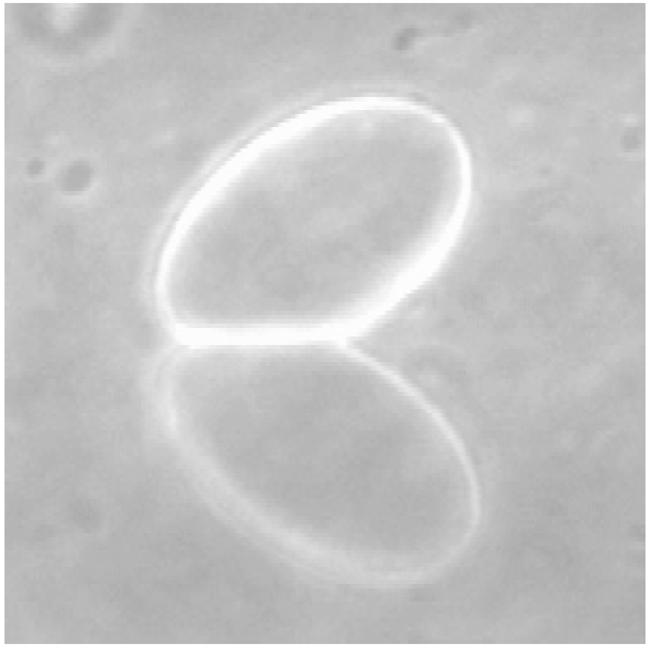
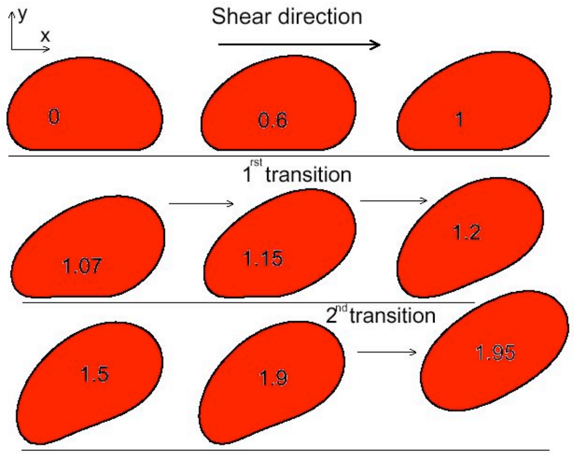
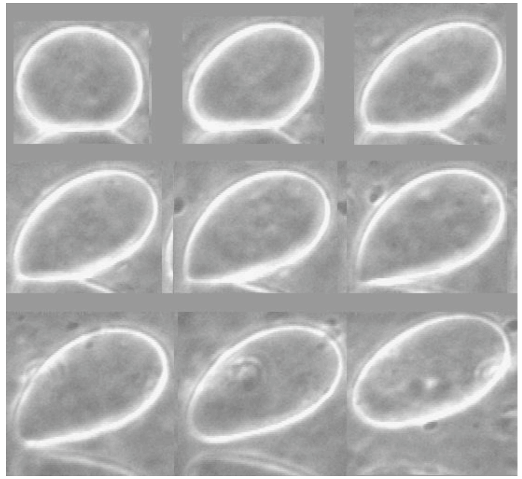
Interactions with the substrate: lift

In the tank-treading regime:

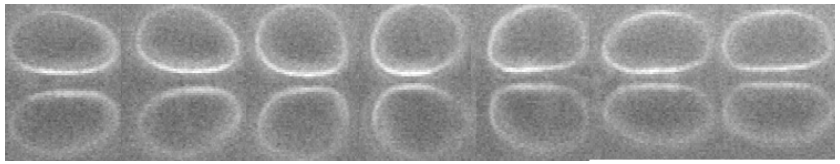


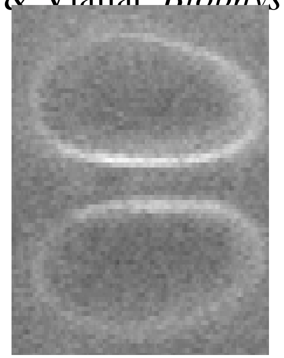


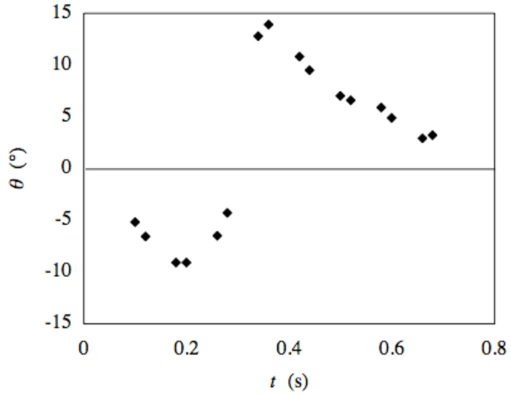


I. Cantat & C. Misbah, *Phys. Rev. Lett (1999)*
M. Abkarian, Lartigue C. & Viallat A., *Phys. Rev. Lett.* (2002)
Abkarian & Viallat, *Biophys. J.* (2005)

In the tumbling/vb regime:







M. Mader, C. Misbah, T. Podgorski, *Micrograv. Sci. Tech.* (2006)

Olla (1997)

$$\frac{\mathrm{d}z}{\mathrm{d}t} = U(\lambda, r_2, r_3)\dot{\gamma}\frac{R^3}{z^2}$$

Experiments by Abkarian (2002)

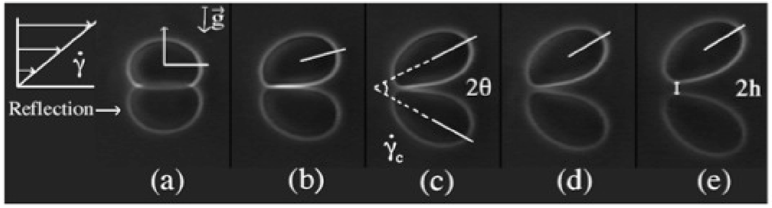
Determination of the lift force from
a balance with gravity:

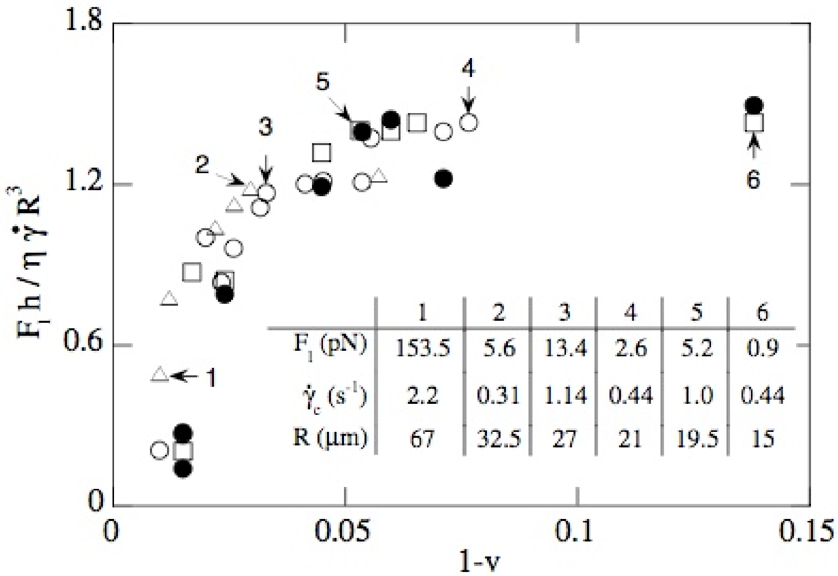
$$F_l \sim \eta\dot{\gamma}\frac{R^3}{h}$$

Stokes drag

$$F_d \sim \eta(\mathrm{d}z/\mathrm{d}t)R$$

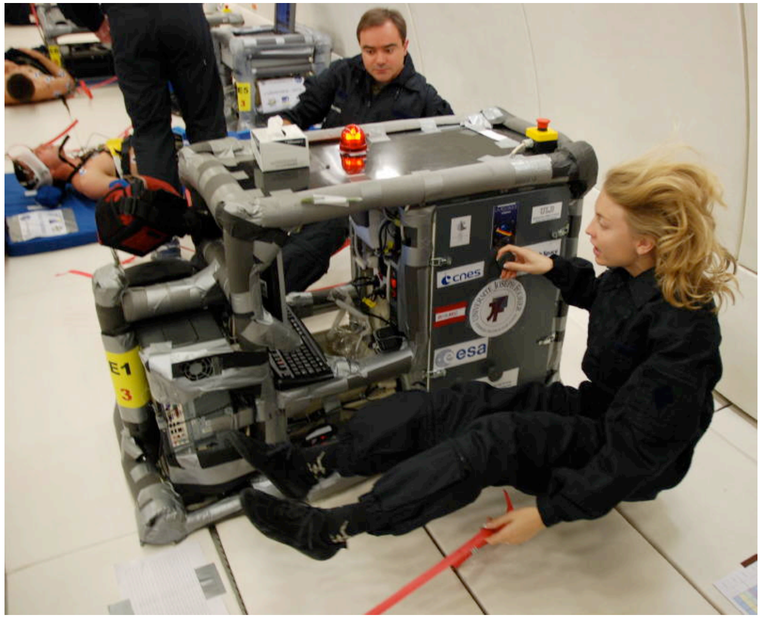
Drift velocity scales like $\dot{\gamma}R^2/h$





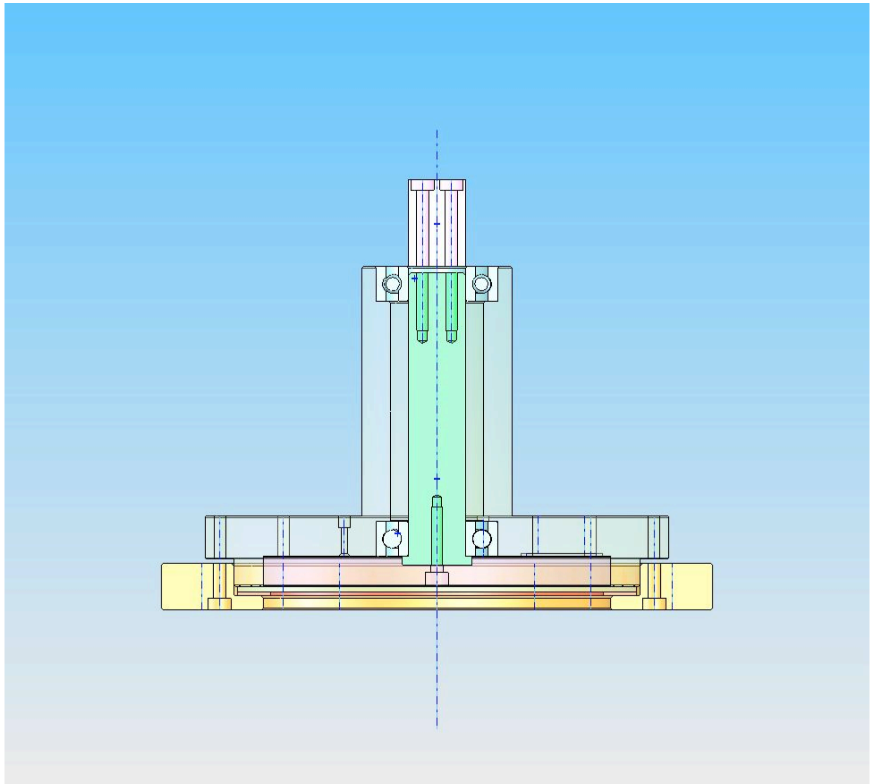
Abkarian, *PRL* (2002)

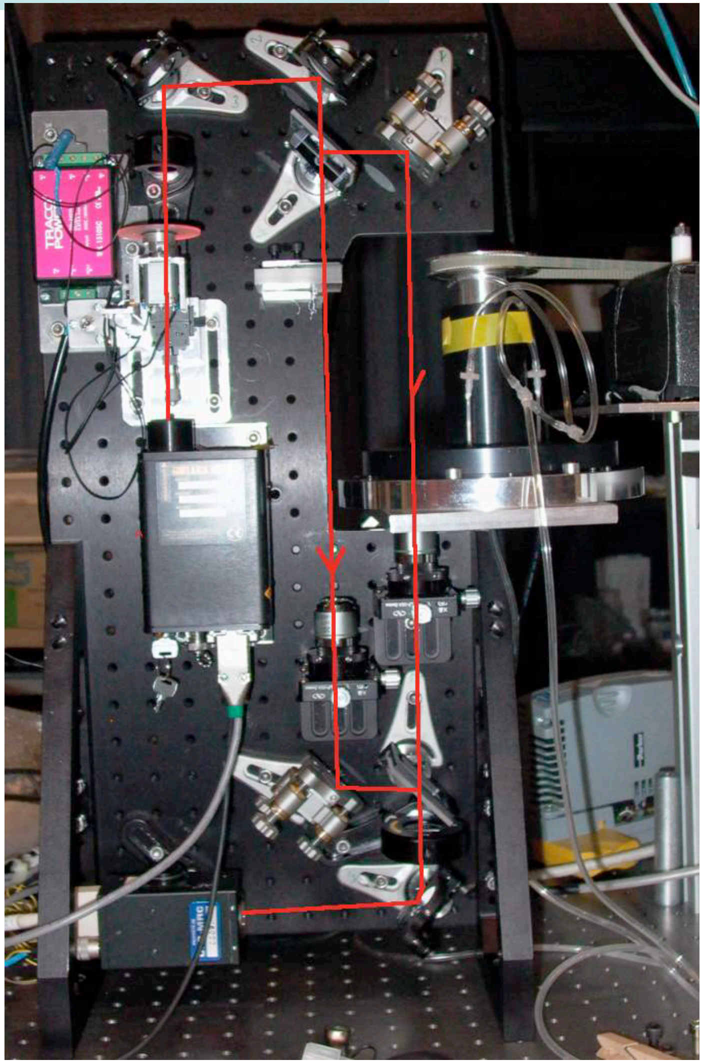
Microgravity experiments



Parabolic flights (with CNES and ESA)

Shear flow between discs
Digital Holographic Microscopy
(coll. F. Dubois, N. Callens (ULB Brussels)





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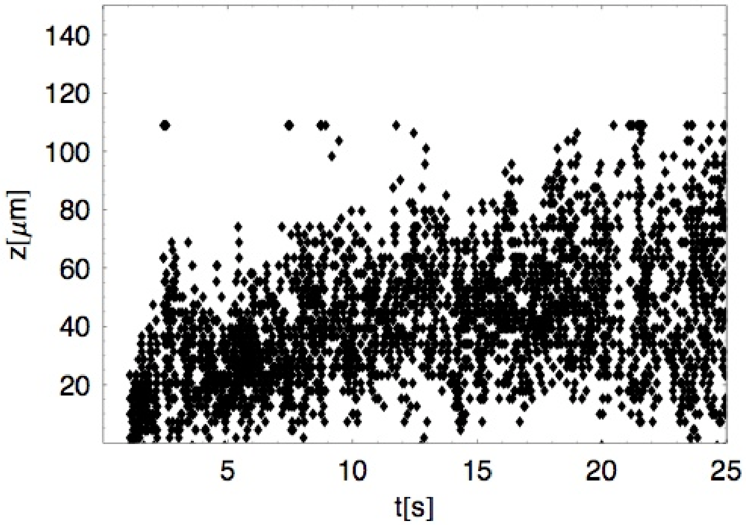
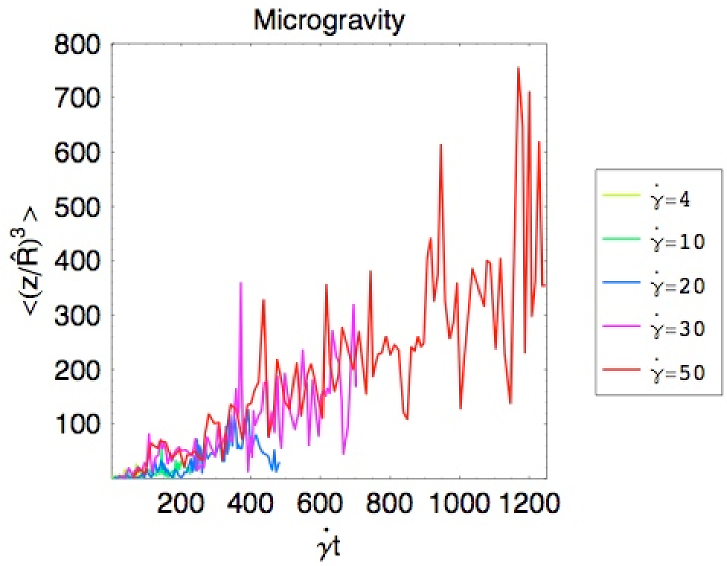
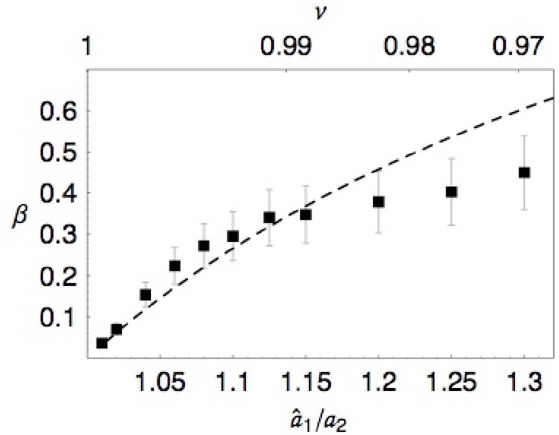
Lift on vesicles in μg





$$\frac{\mathrm{d}z}{\mathrm{d}t} = U(\lambda, r_2, r_3)\dot{\gamma}\frac{R^3}{z^2}$$

$$\left(\frac{z}{R}\right)^3 = 3U\dot{\gamma}t.$$



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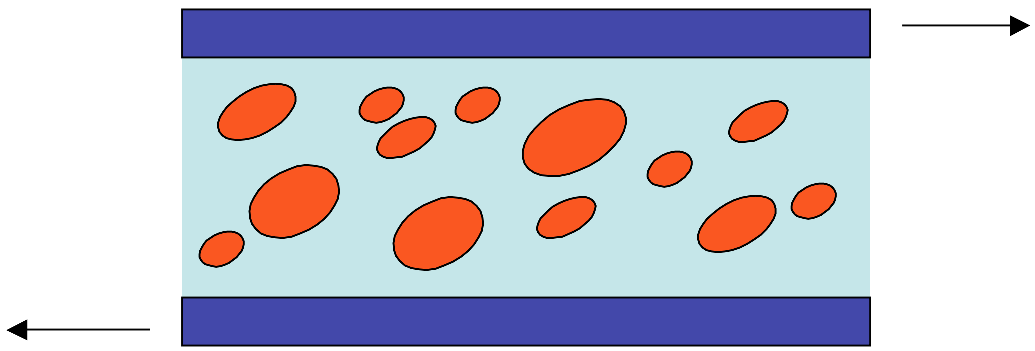
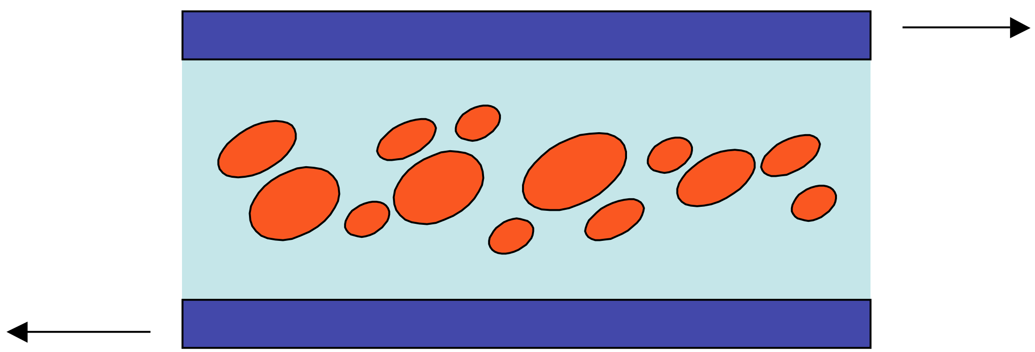
Equilibrium distribution of a suspension





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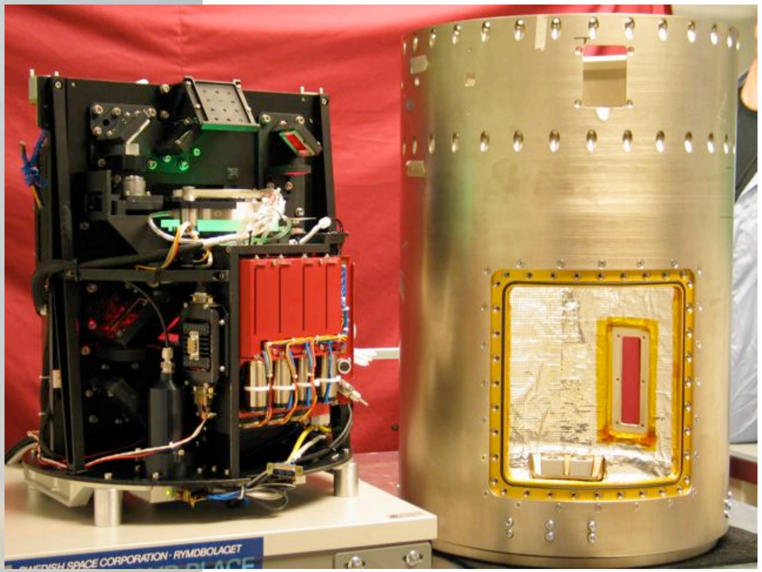
BIOMICS experiment
Maser 11 (May 2008)

esa

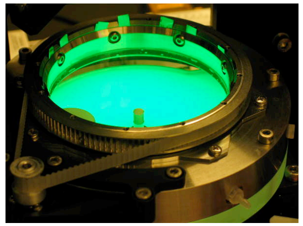
MRC

spectro



Swedish Space Corporation

lambda-x sprl



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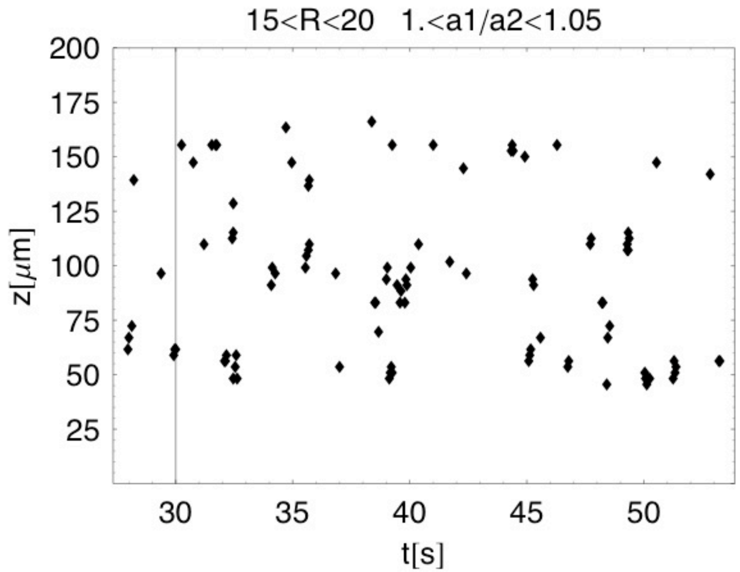
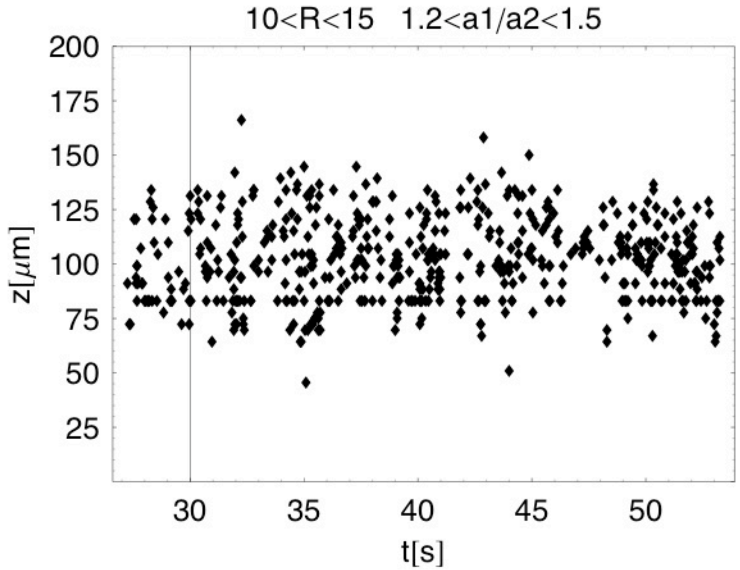
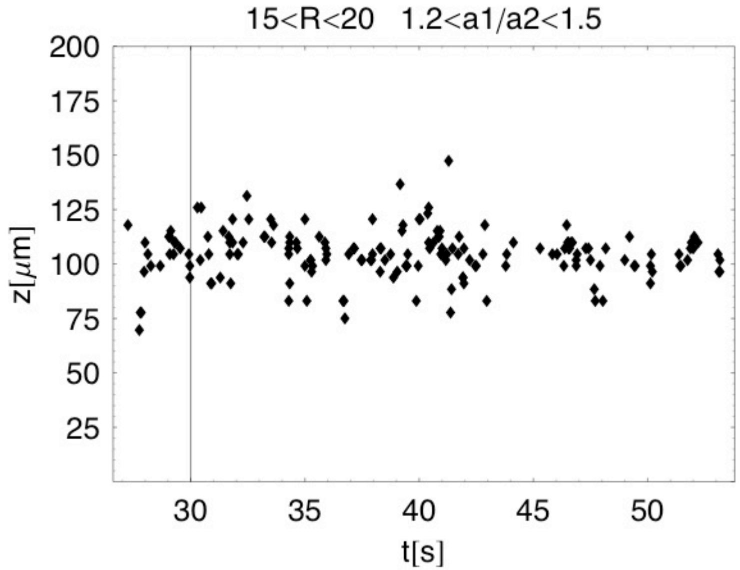
• Spatial distribution strongly dependent on size and reduced volume of vesicles







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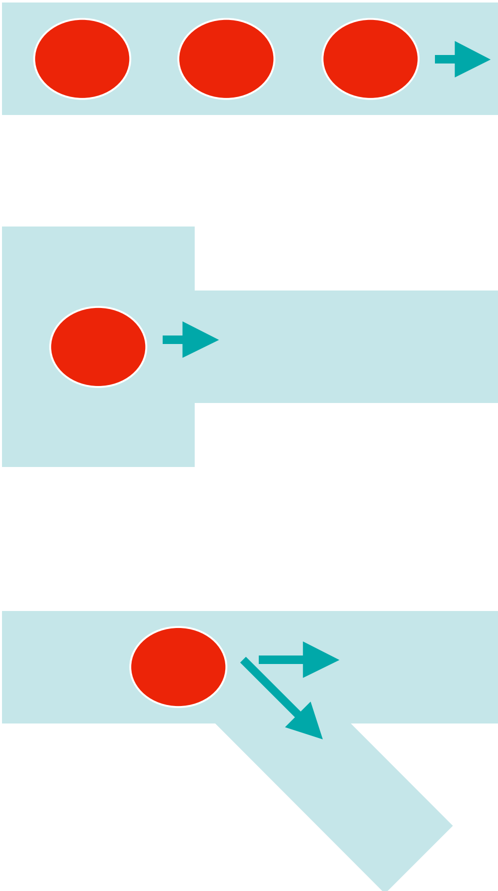
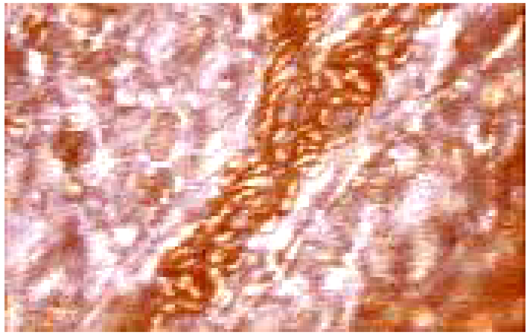
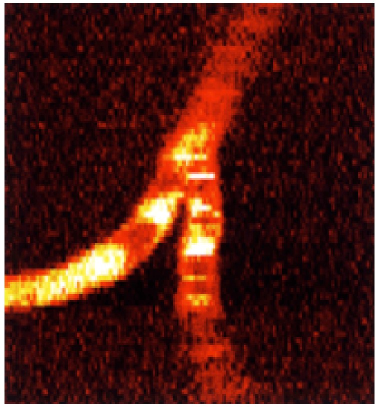
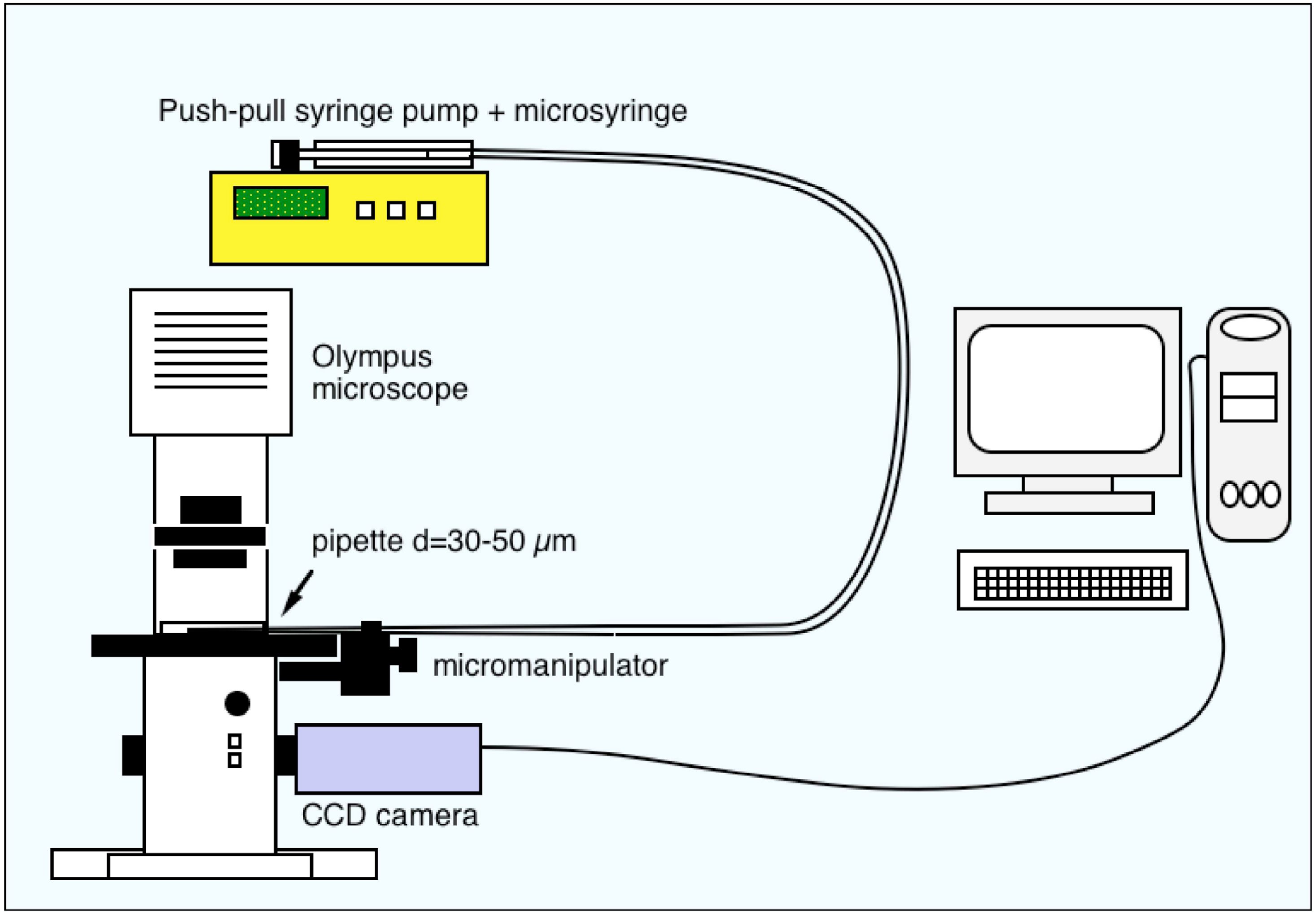
Confined flows

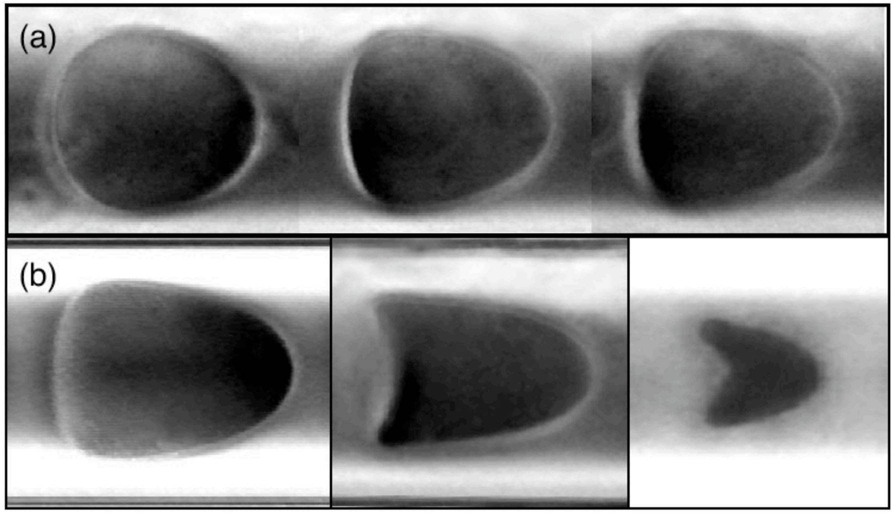




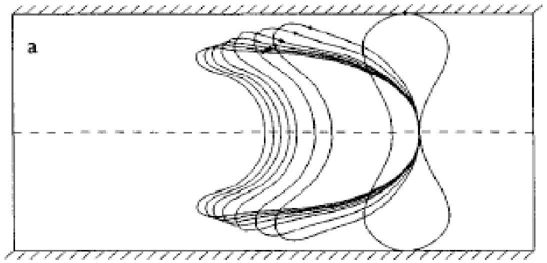


Flow in pipettes

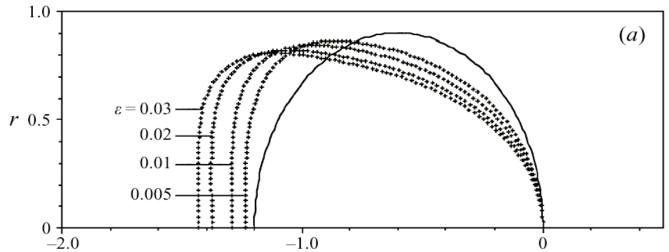




V. Vitkova, M. Mader & T. Podgorski, Europhys. Lett. (2004)

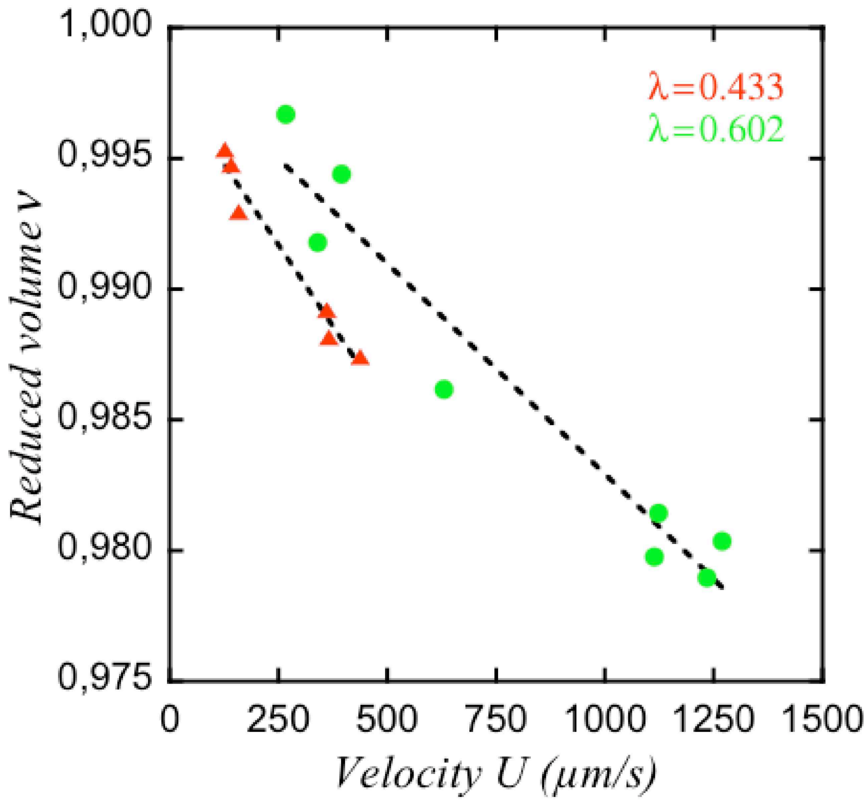


Secomb *et al.*, *J. Fluid Mech.* **163**, 405 (1986)



Quéguiner & Barthès-Biesel, *J. Fluid Mech.* **348**, 349 (1997)

Volume variations (deswelling)



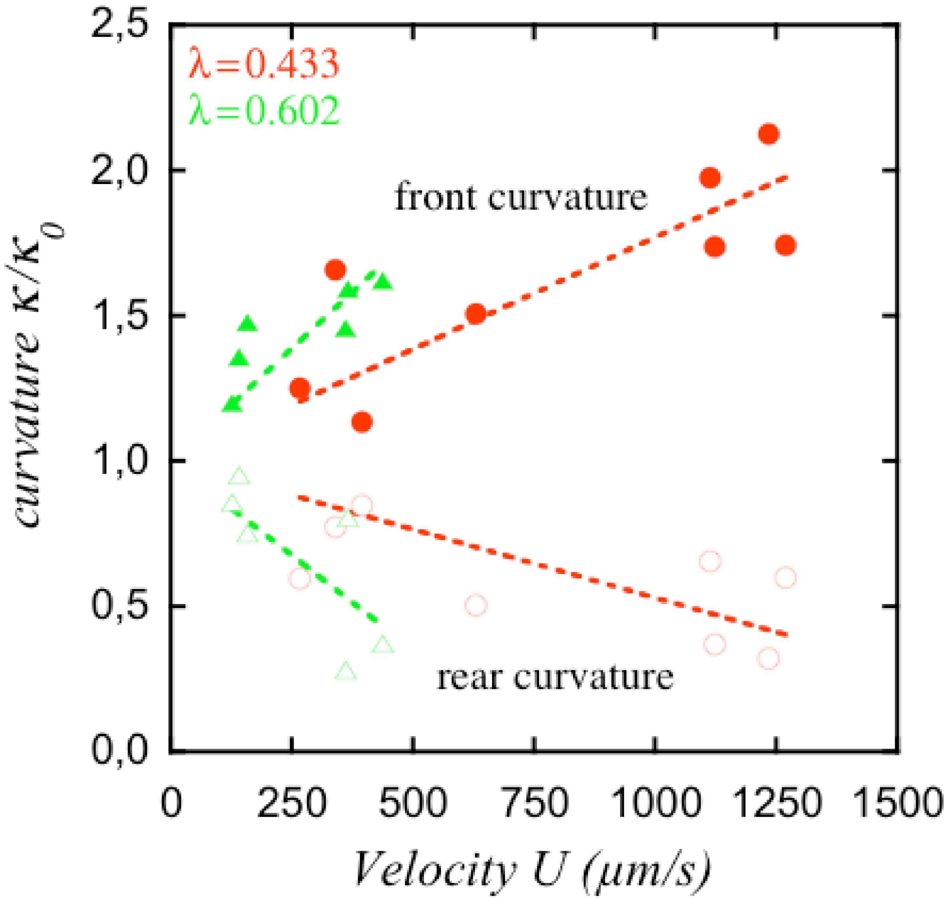
$$\Delta\Pi = R_g T \Delta c$$

$$\Delta c \sim -c_0 \Delta\nu / \nu$$

$$\Delta P \sim \frac{\eta U R}{h^2} \sim \frac{\eta U}{R_p} \frac{\lambda}{(1-\lambda)^2}$$

$$\nu \sim \nu_0 - \frac{\eta U}{R_p} \frac{\lambda}{(1-\lambda)^2} R_g T$$

Deformation



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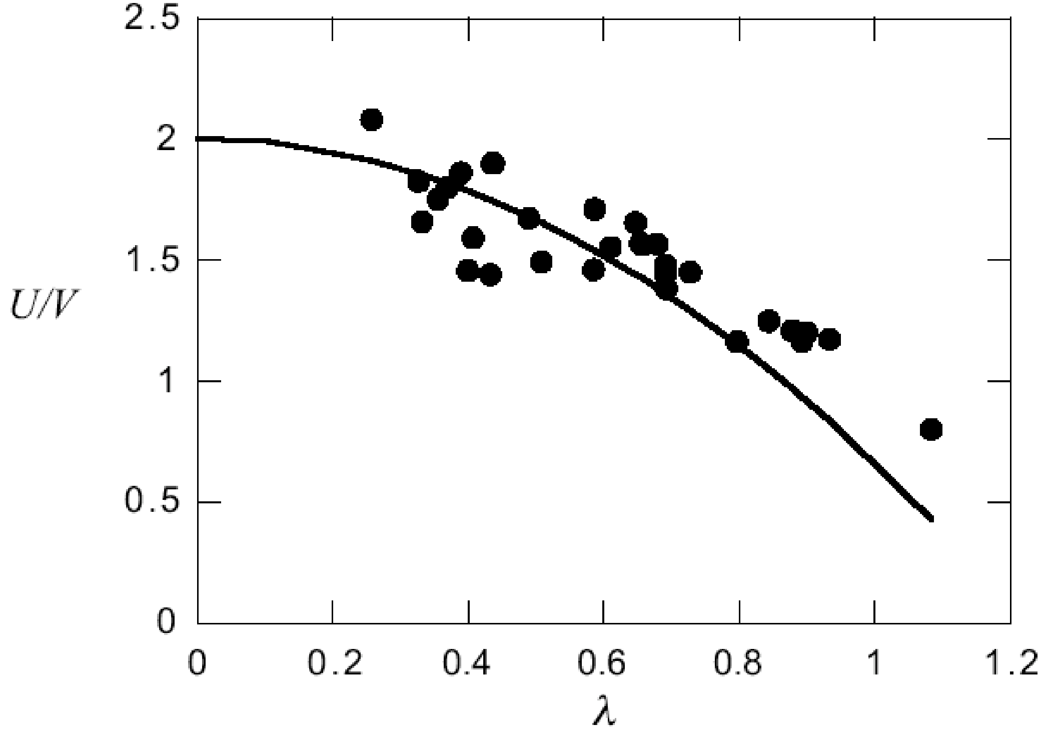
Mobility

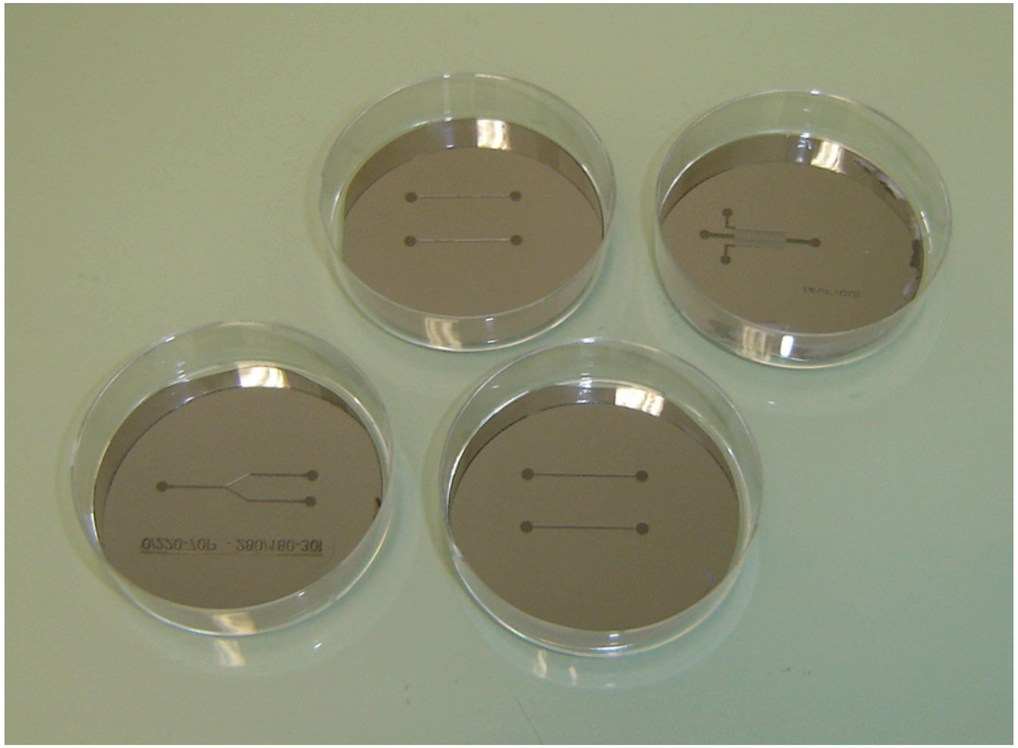


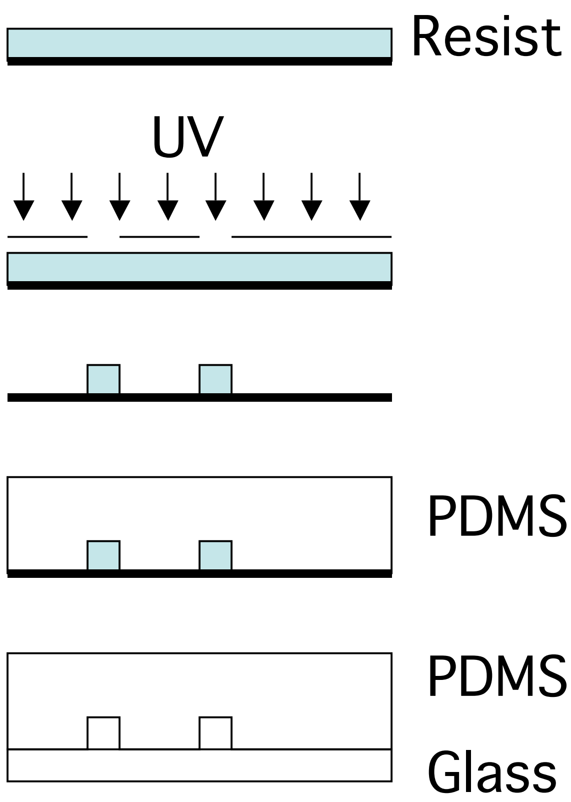
Rigid spheres

$$\frac{U}{V} = 2 - \frac{4}{3}\lambda^2 + O(\lambda^2)$$

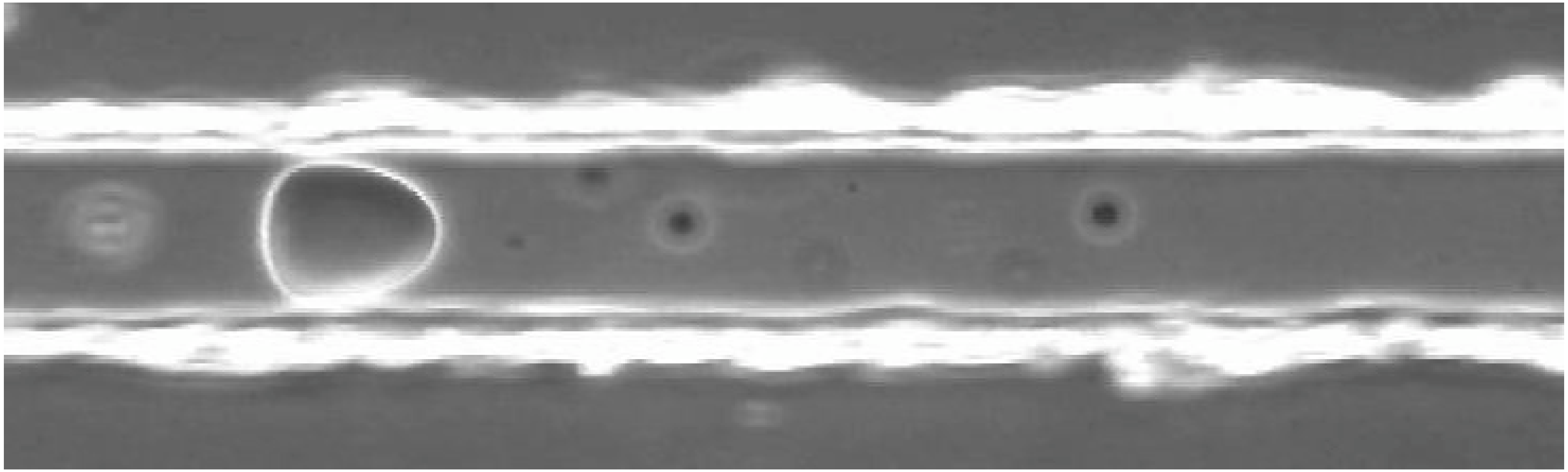
Hetsroni *et al.*, *J. Fluid Mech.* (1970)

Microfluidics

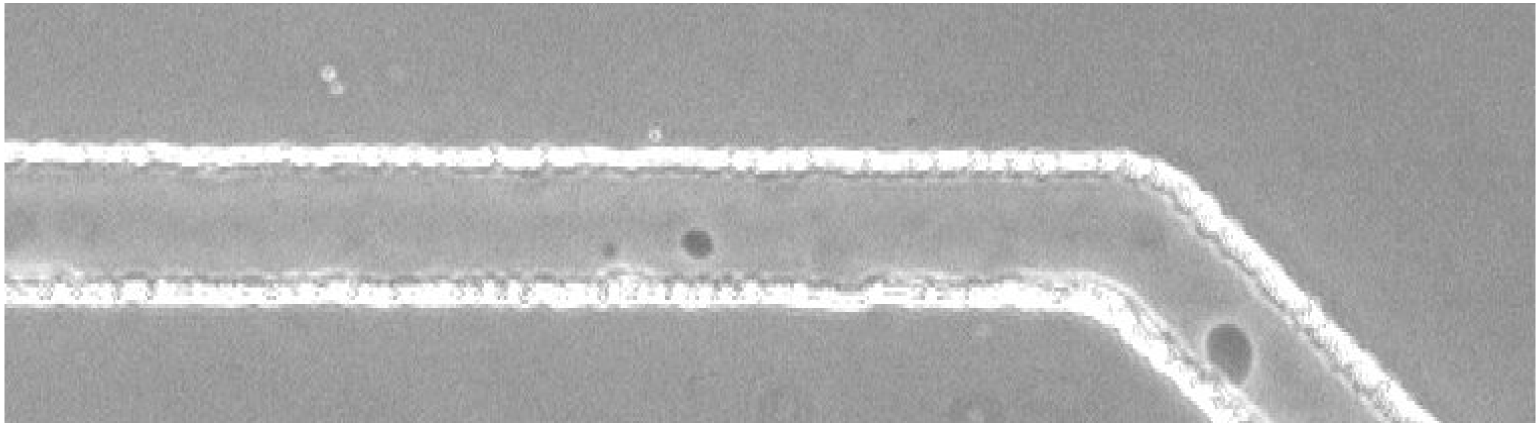




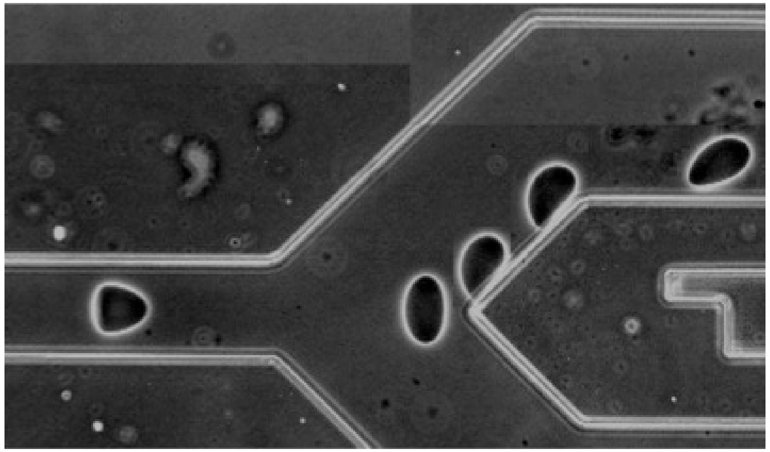
Flow in microchannels



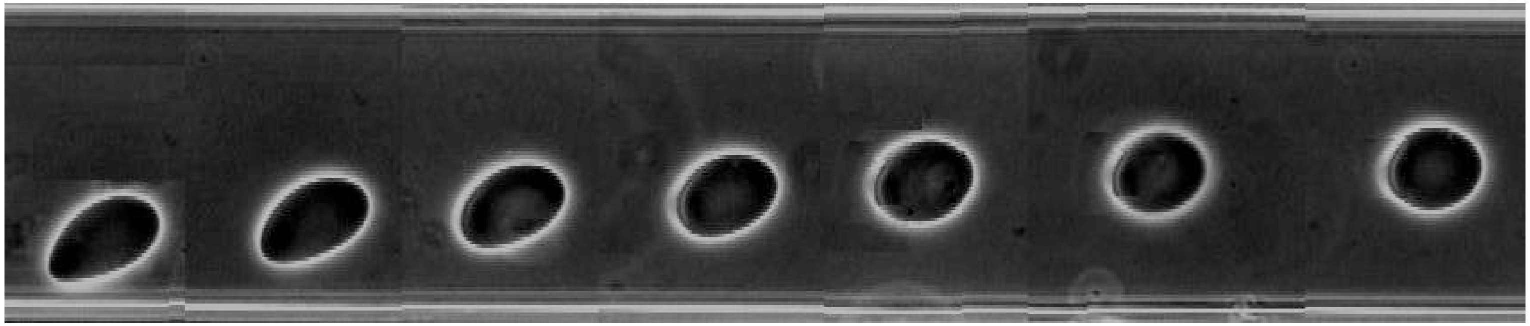
Lateral migration



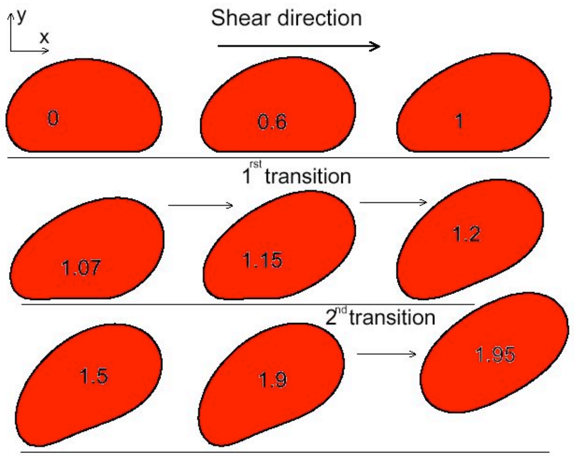
Gwennou Coupier



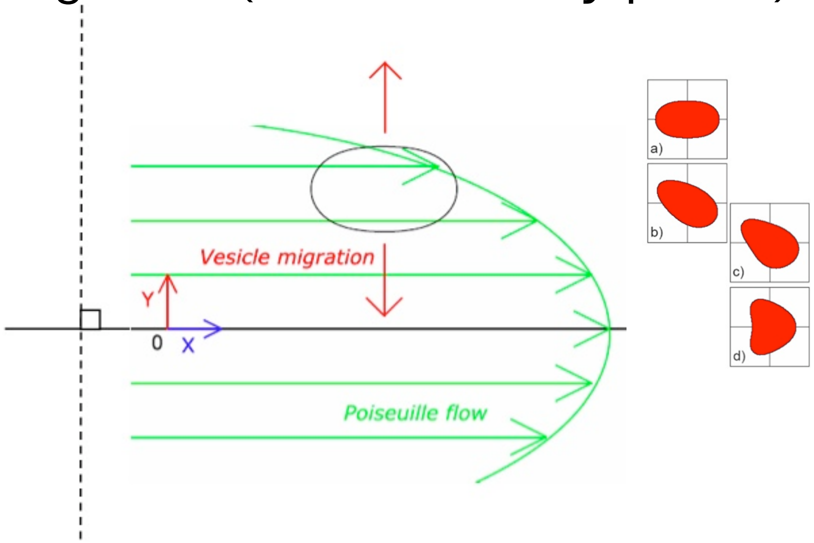
Two mechanisms



Lift (interaction with wall)



Migration (curved velocity profile)



B. Kaoui et al. Phys. Rev E **77**, 021903 (2008)

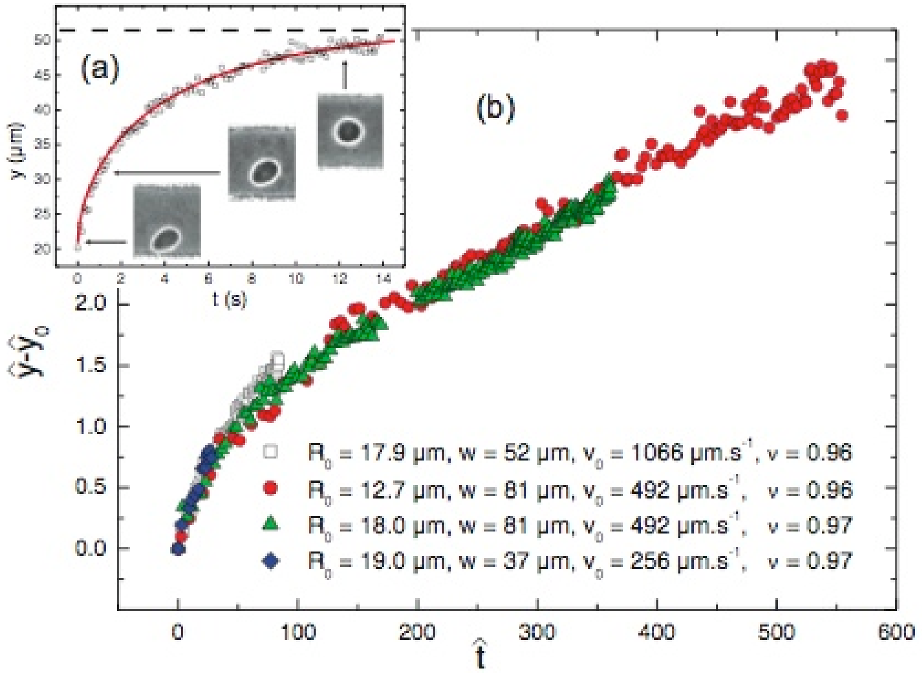
Experiments

$$\hat{y} = y/R_0$$

$$\hat{w} = w/R_0$$

$$\hat{t} = \int_0^t \dot{\gamma}(y)d\tau = c\int_0^t \left[w - y(\tau)\right]d\tau$$

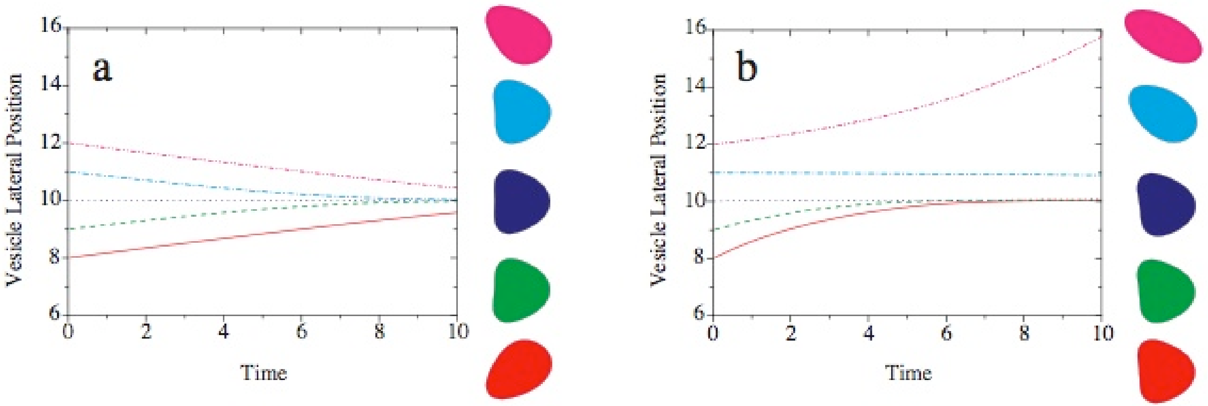
$$v_m = \xi c R_0^2 \frac{\hat{w}-\hat{y}}{(\hat{y}-\hat{y}_0)^\delta} = \xi \frac{R_0\dot{\gamma}(y)}{(\hat{y}-\hat{y}_0)^\delta}$$



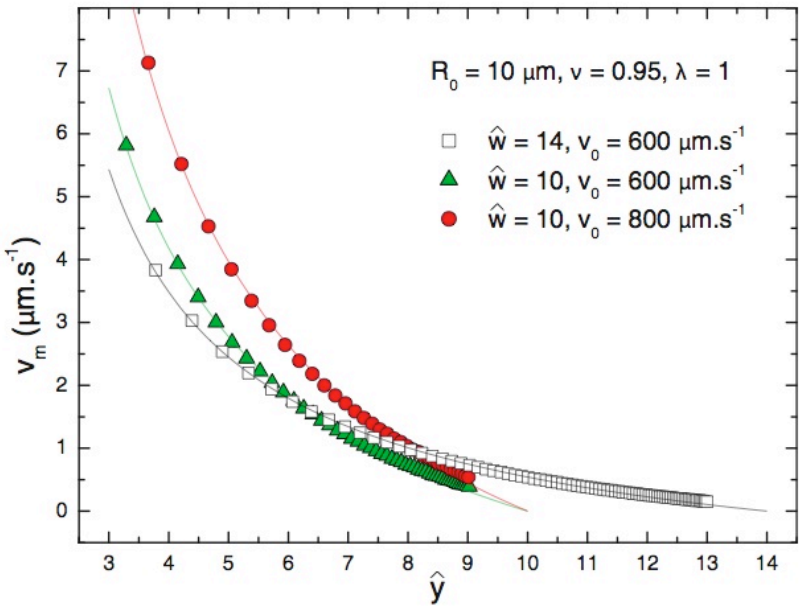
G. Coupier, B. Kaoui, T. Podgorski and C. Misbah, submitted to Phys. Fluids. (2008)

Boundary integral simulations

unbounded

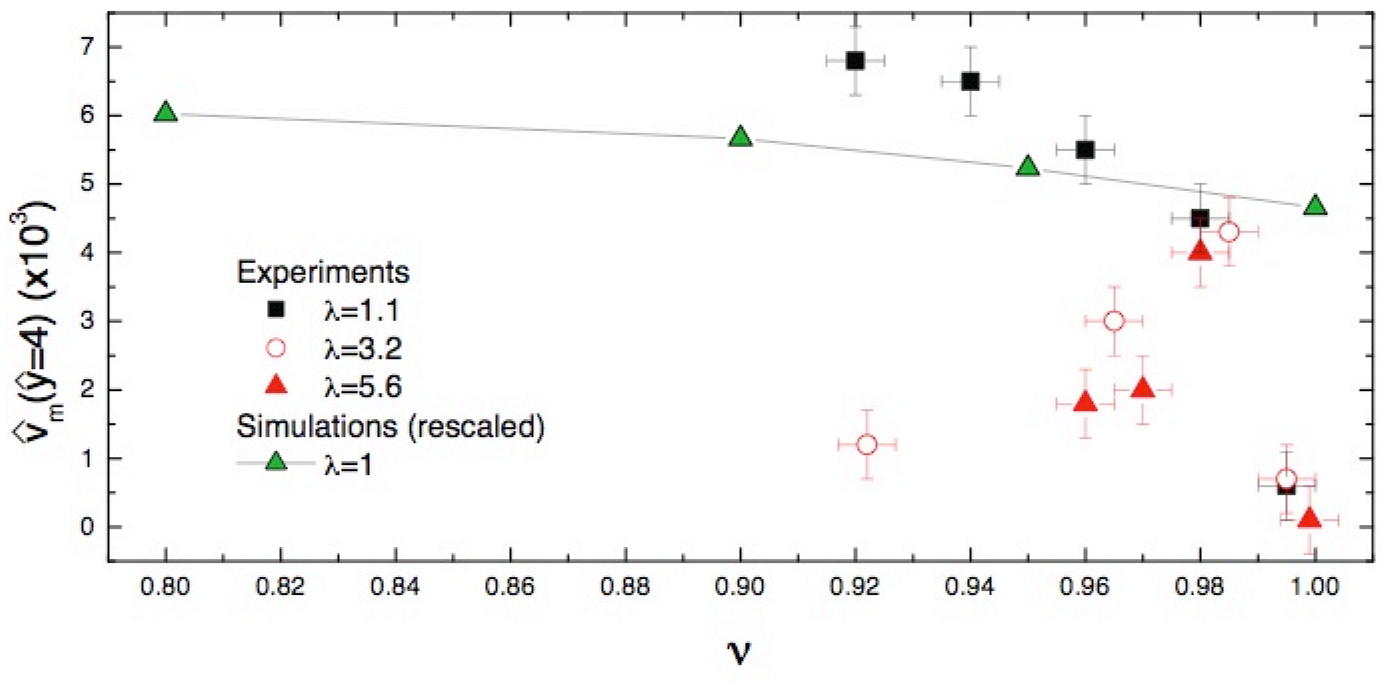


With wall at y=0



Badr Kaoui

Migration velocity



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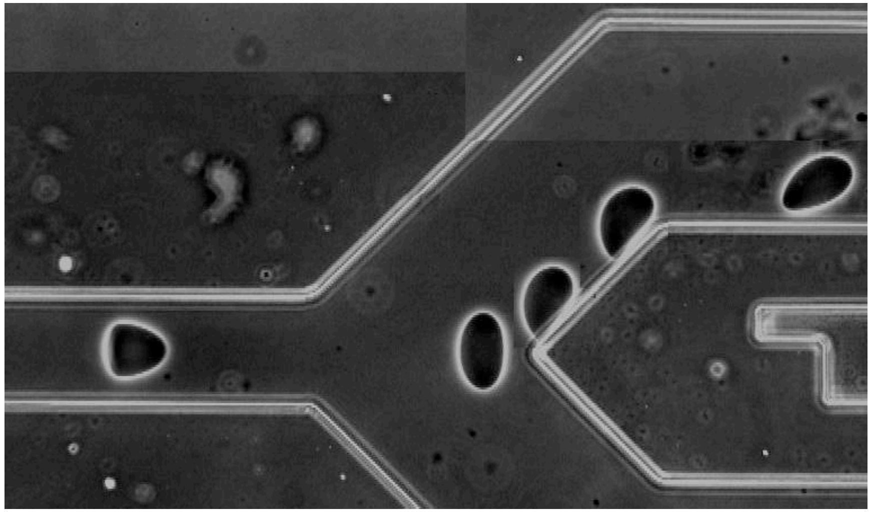
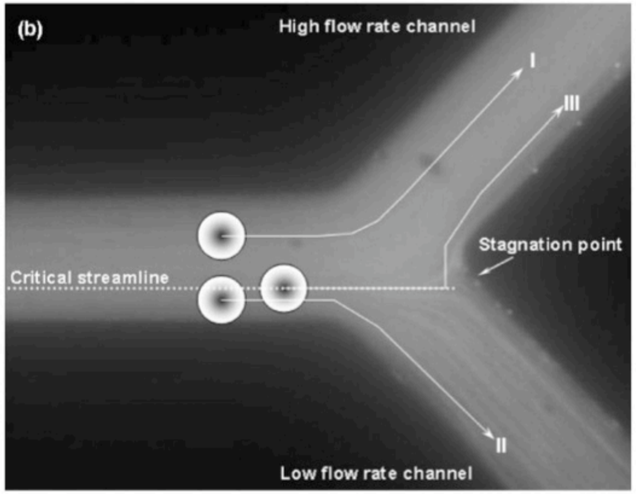
Some perspectives

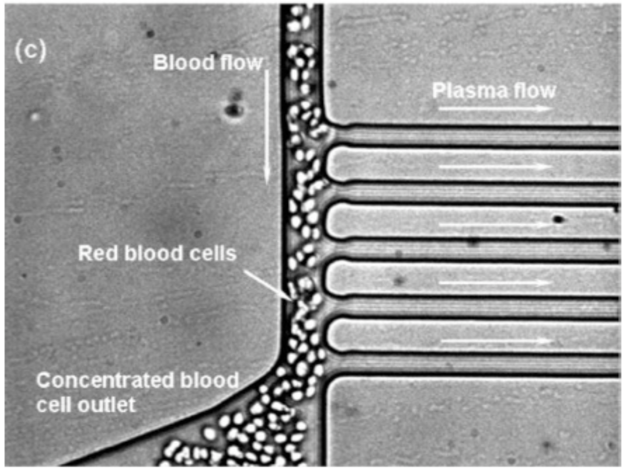
And ideas for simulations...

Zweifach-Fung effect

And behavior at bifurcations







S. Yang, A. Undar & J. Zahn

Lab on a Chip **6**, 871-880 (2006)

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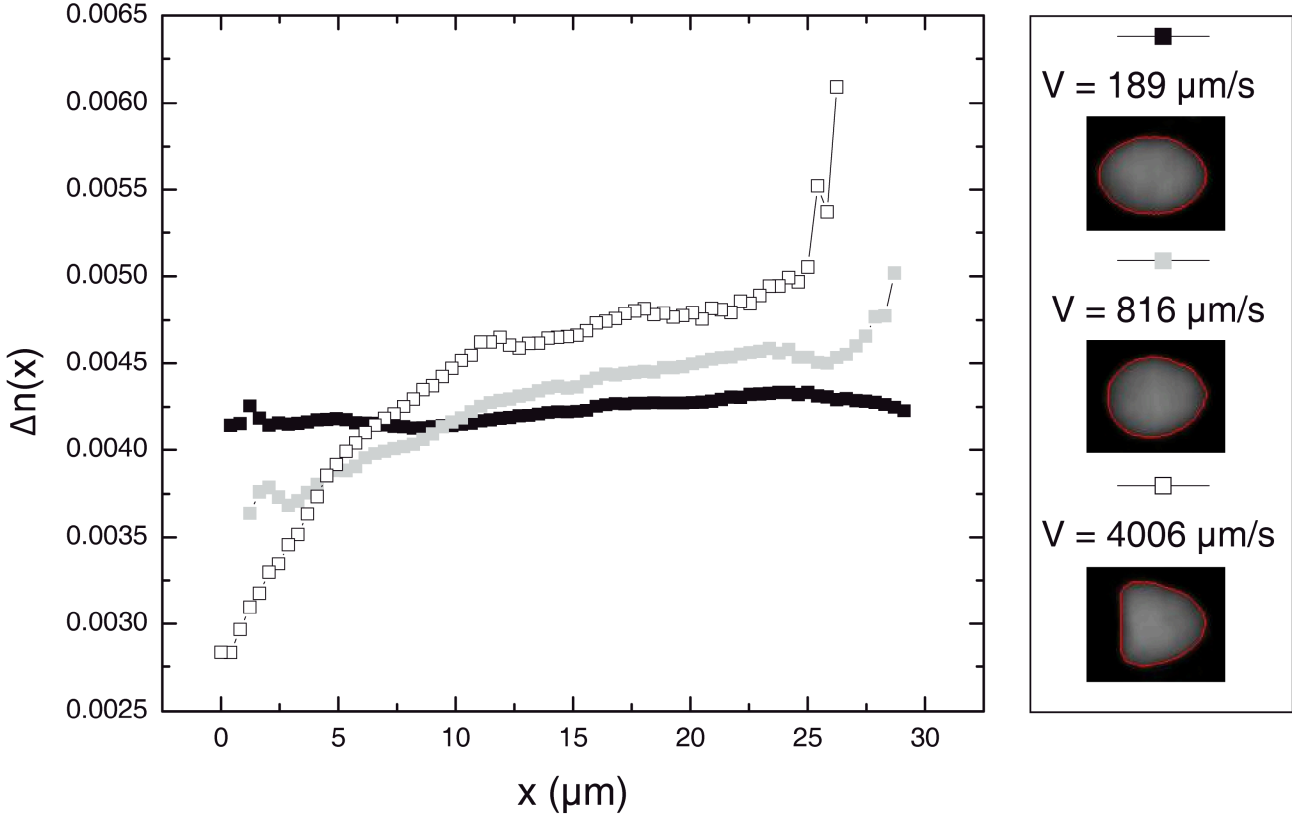
Concentration profiles inside vesicles



C. Minetti, N. Callens, G. Coupier, T. Podgorski, F. Dubois, sumitted to *Applied Optics* (2008)

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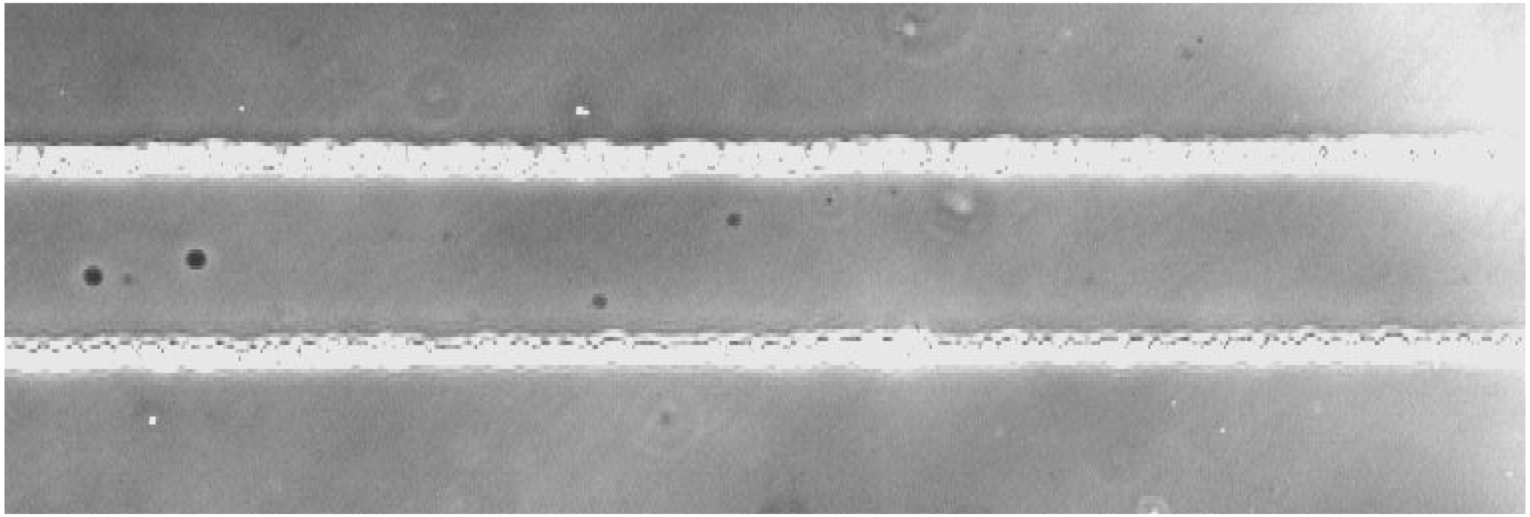
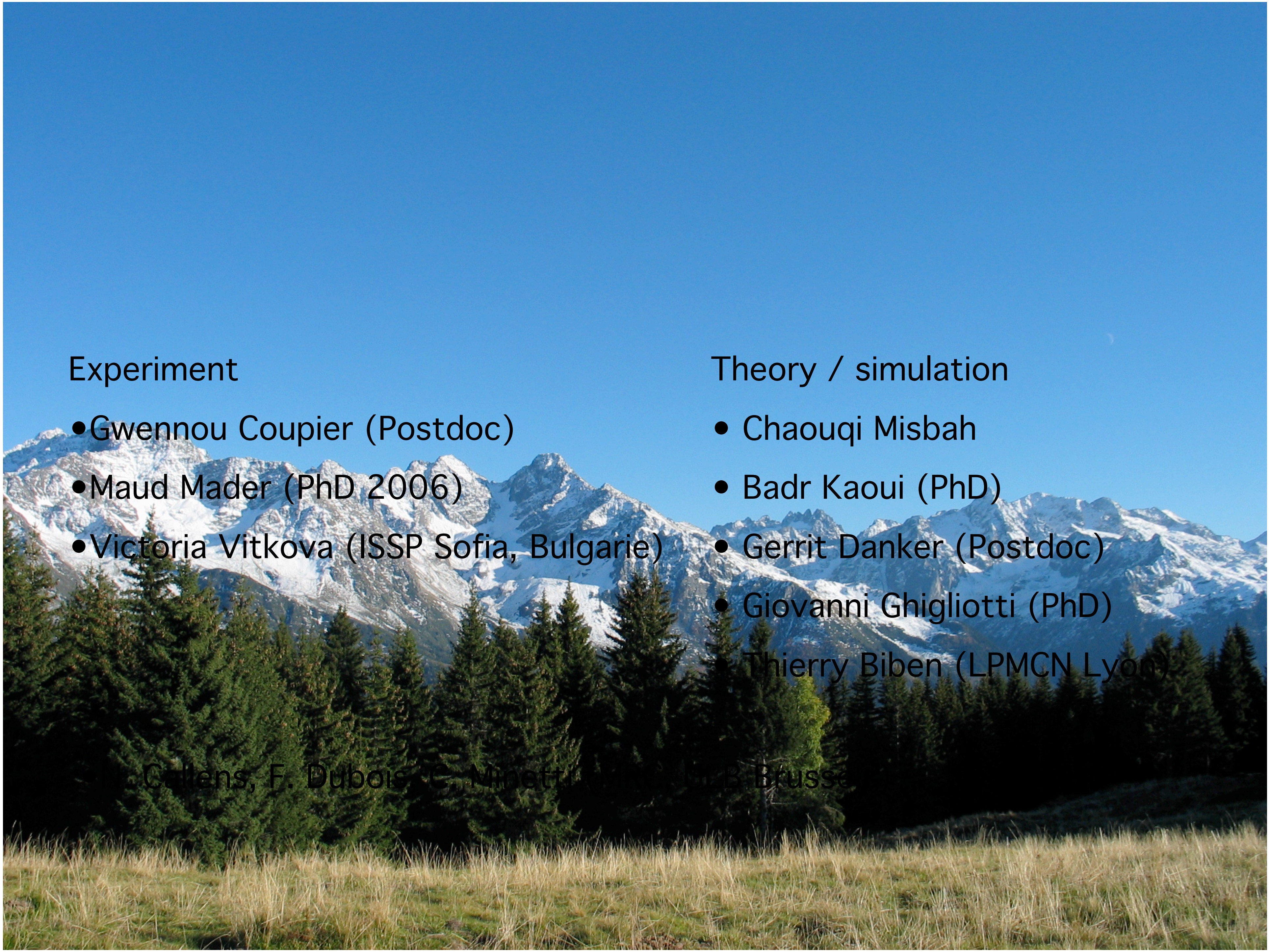
Structuration of the flow



G. Coupier

- Homogeneous distribution at the entrance
- Interactions lead to cluster formation


Experiment
•Gwennou Coupier (Postdoc)
•Maud Mader (PhD 2006)
•Victoria Vitkova (ISSP Sofia, Bulgarie)
Theory / simulation
• Chaouqi Misbah
• Badr Kaoui (PhD)
• Gerrit Danker (Postdoc)
• Giovanni Ghigliotti (PhD)
• Thierry Biben (LPMCN Lyon)
Callens, F. Dubois, C. Minetti