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Higher order approximations for transport dominated systems

Project proposed by

Boniface NKONGA (Université Bordeaux I & INRIA Futurs)

The accuracy of numerical simulations in computational fluid dynamics (CFD) may be significantly affected by the discretization scheme chosen for the convective terms. Simple central difference methods introduce propagating numerical dispersion terms (odd-order derivatives) which may generate large regions of the flow with unphysical oscillations. Higher order interpolation schemes have been successful in eliminating artificial diffusion, while minimizing numerical dispersion. The QUICK (Quadratic Upstream Interpolation for Convective Kinematics) scheme, exemplified in the steady-state multi-dimensional case, has demonstrated several superior properties in eliminating numerical diffusion and producing low dispersion error. However, the higher-order dispersion terms may still cause overshoots and a few oscillations under highly convective conditions, even in one-dimensional flow.

In certain turbulent flows, turbulent transport variables (such as eddy viscosity) are computed as part of the solution procedure and the overshoots or undershoots may produce negative values, thus resulting in violent non-linear instability. The restriction is more stringent in more complicated, multidimensional flows. These considerations motivate the development of an alternative high-order monotonic scheme for three dimensional environmental flows. The local extremum diminishing (LED) property ensures that the solution satisfies a discrete maximum principle, thereby precluding spurious oscillations. Thus, maxima should not increase and minima should not decrease.

Jameson (1995) developed a systematic procedure for the design of a broad class of essentially local extremum diminishing (ELED) schemes which satisfy monotonicity constraints on both structured and unstructured grids. These schemes can be modified to improve both accuracy and multigrid convergence. The proposed study develops some new ELED schemes for environmental flows. It is known that turbulent mixing in the ocean is highly correlated to the physical diffusion of the flow. Ideally, the convection should not alter the probability density function (pdf) of the density field and is an adiabatic transport process. In realistic ocean simulations, the convective fluxes are not explicitly constrained to preserve adiabaticity, but instead, rely on the convergence of the numerical schemes to a sufficient level of accuracy. Several results indicate that numerical models can manifest unphysically large amounts of mixing due to numerical truncation errors. Smooth solutions from numerical simulations may generate a large amount of numerical dissipation/diffusion without careful control. A very accurate representation of convection is necessary to produce correct results for oceanic applications. In order to overcome thes problems, numerical schemes based on residual distribution and stabilized finite elements approaches will be investigated.