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Discontinuous Galerkin finite volume schemes for the MHD system

Project proposed by

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The Magneto-Hydro-Dynamics (MHD) equations are a simplified but rich model of conducting fluids. They are encountered for example in stellar physics, geophysics and plasmas physics. With the recent ITER project (International Thermonuclear Experimental Reactor), they are subject to a regain of interest. The goal of the ITER project is to build a big Tokamak reactor in which a very high temperature plasma is flowing. The huge temperature triggers very energetic fusion reactions. The hope is to be able to maintain these reactions in order to produce energy. In a Tokamak, the MHD equations are only an approximation. More precise (but also more CPU consuming) kinetic models are required. Anyway, the MHD model give some useful insights for engineering of preliminary studies.

The Discontinuous Galerkin (DG) method is already used for solving MHD problems (see for example [1]). It has proved to be very efficient and accurate. However, several difficulties are still present:

- in some cases, the MHD first order equations possesses several entropy solutions ([5]). The DG method can be designed in order to satisfy an entropy principle, but it is not clear how it behaves in case of multi solutions;
- the second order terms of MHD equations play a crucial role for selecting physically relevant entropy solutions. Therefore, it is important to implement them in a proper way in a DG scheme;
- finally, the divergence free condition on the magnetic field leads to complications in the implementation of the DG scheme. Recently, Munz et al. [3] then Dedner et al. [2] have proposed and extended MHD system, which relaxes the divergence condition and evacuates in a hyperbolic way the errors of the magnetic field divergence (see also [4]). This approach seems to be simpler to use in a DG scheme and has to be tested.

As a conclusion, this project will permit to address relevant problems in the approximation of the MHD equations by DG schemes. If these problems are solved, the resulting code will permit to simulate useful complex fluids.

References.

[1] T. Barth. On the role of involutions in the discontinous Galerkin discretization of Maxwell and magnetohydrodynamic systems. In Compatible spatial discretizations, volume 142 of IMA Vol. Math. Appl., pages 69–88. Springer, New York, 2006.
[2] A. Dedner, F. Kemm, D. Kröner, C.-D. Munz, T. Schnitzer, and M. Wesenberg. Hyperbolic divergence cleaning for the MHD equations. J. Comput. Phys., 175(2):645–673, 2002.

[3] C.-D. Munz, P. Omnes, R. Schneider, E. Sonnendrücker, and U. Voss. Divergence correction techniques for maxwell solvers based on a hyperbolic model. J. Comput. Phys., 161(2):484–511, 2000.
[4] K. G. Powell. An approximate Riemann solver for magnetohydrodynamics (that works in more than one space dimension. Technical Report NASA/CR-194902 ICASE Report No. 94-24, ICASE-NASA Langley, ICASE, NASA Langley Research Center, April 1994.
[5] M. Torrilhon. Non-uniform convergence of finite volume schemes for Riemann problems of ideal magnetohydrodynamics. J. Comput. Phys., 192(1):73–94, 2003.