Vortex dynamics and Compressibility effects in Large-Eddy Simulations (1 of 2)

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Outline:

- Talk 1
  - ▷ Mixing layers (*al.*, J. Silvestrini *et al.*, *Eur. J. Mech. B*, **17**, 1998)
  - ▷ LES in Fourier space
  - ▷ LES in physical space
  - ▷ SGS model assessment
  - ▷ Compressible LES formulations
  - ▷ Air-intake flow (unpublished)

Outline (cont'd):

- Talk 2
  - ▷ spatial discretization
  - ▷ High-order conservation schemes
  - ▷ MILES ENO assessment
  - ▷ example of shock-wave/boundary layer interaction
  - ▷ Cavity flows (Y. Dubief)
  - ▷ Cavity flows (L. Larchevêque *et al.*, *Phys. Fluids*, *Phys. Fluids*, **15**, 2003, *J. Fluid Mech.*, **516**, 2004)
  - ▷ Supersonic compression ramp flows (unpublished)
  - ▷ Solid-propellent rocket flow (unpublished)
  - ▷ Separation control by tangential blowing (preliminary)
  - ▷ Supersonic channel flow (C. Brun *et al.*, ETC5, Toulouse, 2003)
  - ▷ MHD mixing layers and jets (H. Baty *et al.*, *Phys. Plasmas*, **10**, 2003)

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- E. Schwander
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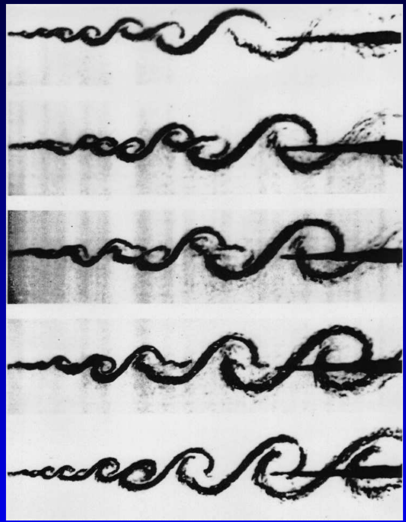
- LEGI Grenoble, M. Lesieur, O. Métais
- ONERA Chatillon
- Observatoire de Strasbourg

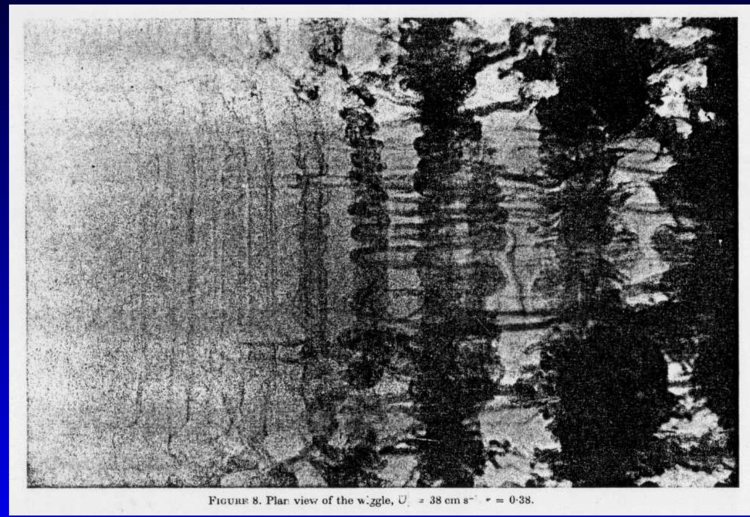
Mixing layers (1/10)

- pairings (Konrad, 1976)



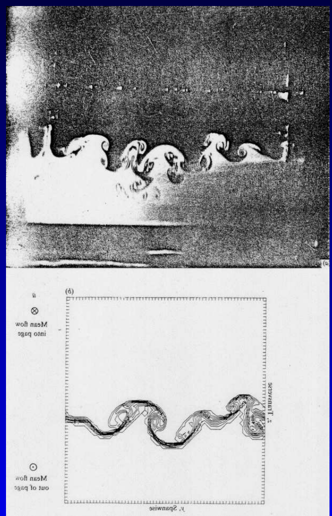
Mixing layers (2/10)

- three-dimensionality (Breidenthal, 1982)



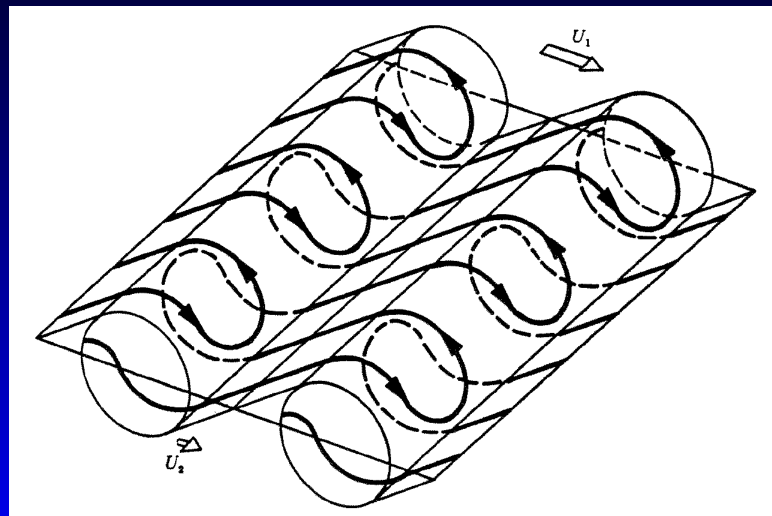
Mixing layers (3/10)

- “rib"-vortices (Bernal *et al.*, 1986; Metcalfe *et al.*, 1986)



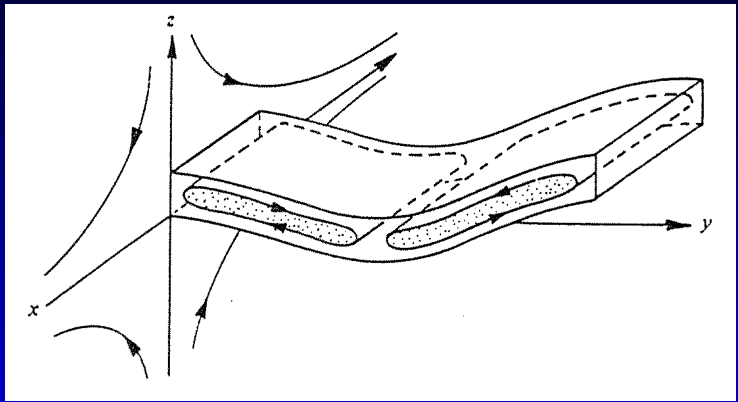
Mixing layers (4/10)

- conceptual model (Bernal *et al.*, 1986)



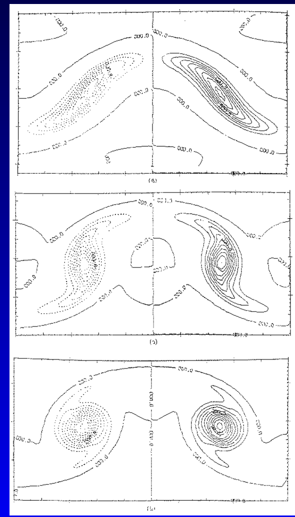
Mixing layers (5/10)

- Hyperbolic instability (Lin & Corcos, 1982)



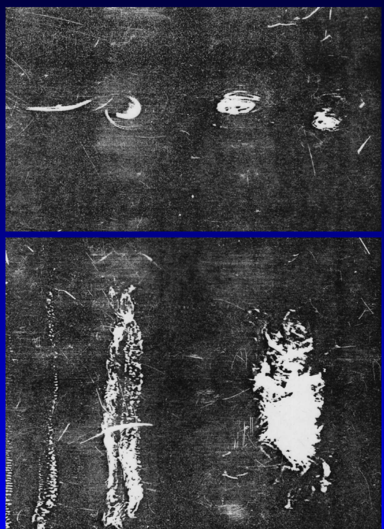
Mixing layers (6/10)

- Hyperbolic instability (Lin & Corcos, 1982)



Mixing layers (7/10)

- Helical pairings (Chandrsuda *et al.*, 1978)

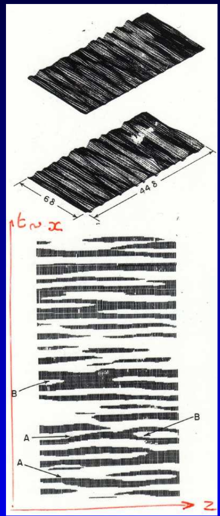


Mixing layers (8/16)

- Helical pairings (Chandrsuda *et al.*, 1978)

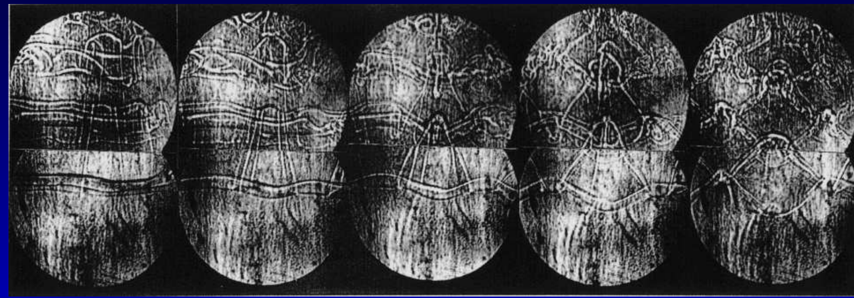
Mixing layers (9/10)

- Dislocations/Branchings/Defects (Browand, 1982)

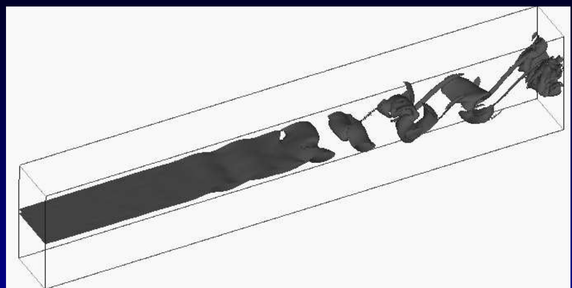


Mixing layers (10/10)

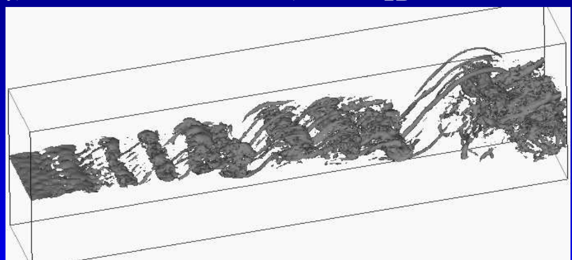
- Helical pairings / Chain-fence-like vortices (Nygaard & Glezer, 1992)



Spatial mixing layers (1/5)



$\|\vec{\omega}\| = 1/3\ \omega_i$, in DNS at $Re = 100$, with $\varepsilon_{2D} = 10^{-4}$ and $\varepsilon_{1D} = 10^{-3}$.



$\|\vec{\omega}\| = 2/3\ \omega_i$, in LES at $\nu = 0$, with $\varepsilon_{2D} = 10^{-5}$ and $\varepsilon_{1D} = 10^{-4}$.

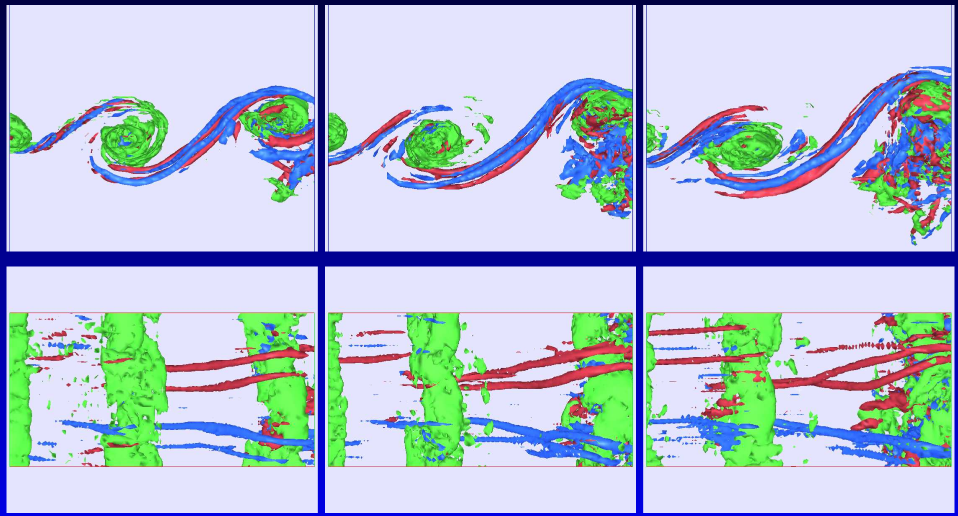
Spatial mixing layers (2/5)

LES $(L_x, L_y) = (16\lambda_i, 4\lambda_i),\ (N_x, N_y) = (384, 96)$

- narrow domain: $L_z = 2\ \lambda_i$, $N_z = 48$
  - side view
  - pressure
  - vorticity
- wider domain: $L_z = 4\ \lambda_i$, $N_z = 96$
  - pressure
  - vorticity

Spatial mixing layers (3/5)

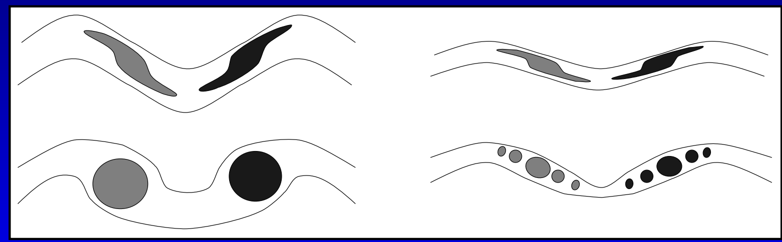
- Multiple-stage roll-up & pairing



Spatial mixing layers (4/5)

- as conjectured by Lin & Corcos, *J. Fluid Mech.*, **141**, 139–178 (1978).

. . . In a layer where the sign of the vorticity alternates (in the direction along which strain is absent), each portion of the layer that contains vorticity of a given sign eventually contributes that vorticity to a single vortex. This may occur in a single stage if the initial layer thickness is not excessively small next to the spanwise extent of vorticity of a given sign or, otherwise, in a succession of stages involving local roll-up and pairing.



Spatial mixing layers (5/5)

- DNS, $Re = 100$
- $(L_x, L_y) = (16\lambda_i, 4\lambda_i),\ (N_x, N_y) = (480, 96)$
- narrow domain: $L_z = 2\ \lambda_i$, $N_z = 48$
- forcing amplitude: $0.05U$
- $\|\vec{\omega}\| = 3/4\ \omega_i$

LES in Fourier space (1/8)

- Navier-Stokes in Fourier space (statistical homogeneity)

$$\hat{u}_i(\underline{k}, t) = \left(\frac{1}{2\pi}\right)^3 \int \mathrm{e}^{-i\underline{k}.\underline{x}}\ \ u_i(\underline{x}, t) d\underline{x}$$

$$\frac{\partial}{\partial t}\hat{u}_i(\underline{k}, t) + \nu k^2 \hat{u}_i(\underline{k}, t) = $$
$$- ik_m \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \int_{\underline{p}+\underline{q}=\underline{k}} \hat{u}_j(\underline{p}, t)\hat{u}_m(\underline{q}, t) d\underline{p}$$

LES in Fourier space (2/8)

- Passive scalar

$$\hat{T}(\underline{k},t) = \left(\frac{1}{2\pi}\right)^3 \int \mathrm{e}^{-i\underline{k}\cdot\underline{x}} \; T(\underline{x},t)d\underline{x}$$

$$\frac{\partial}{\partial t}\hat{T}(\underline{k},t) + \kappa k^2 \hat{T}(\underline{k},t) = - ik_j \int_{\underline{p}+\underline{q}=\underline{k}} \hat{u}_j(\underline{p},t)\hat{T}(\underline{q},t)d\underline{p}$$

LES in Fourier space (3/8)

- Low-pass filter (sharp filter):

$$\bar{\hat{f}} = \hat{f} \;\text{ for }\; |\underline{k}| < k_C = \pi/\Delta x, \bar{\hat{f}} = 0 \;\text{ for }\; |\underline{k}| > k_C$$

- Spectral eddy viscosity (Heisenberg, Kraichnan . . .):

$$\frac{\partial}{\partial t}\hat{u}_i(\underline{k},t) + [\nu + \nu_t(k|k_C)]k^2 \hat{u}_i(\underline{k},t) = - ik_m \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \int_{\underline{p}+\underline{q}=\underline{k}}^{|\underline{p}|,|\underline{q}|<k_C} \hat{u}_j(\underline{p},t)\hat{u}_m(\underline{q},t)d\underline{p}$$

with

LES in Fourier space (4/8)

- Spectral eddy viscosity $\nu_t(k|k_C)$:

$$\nu_t(k|k_C)\; k^2 \hat{u}_i(\underline{k},t) = ik_m(\delta_{ij} - \frac{k_i k_j}{k^2}) \int_{\underline{p}+\underline{q}=\underline{k}}^{|\underline{p}|\text{or}|\underline{q}|>k_C} \hat{u}_j(\underline{p},t)\hat{u}_m(\underline{q},t)d\underline{p}$$

- Spectral eddy diffusivity $\kappa_t(k|k_C)$:

$$\kappa_t(k|k_C)\; k^2 \hat{T}(\underline{k},t) = ik_j \int_{\underline{p}+\underline{q}=\underline{k}}^{|\underline{p}|\text{or}|\underline{q}|>k_C} \hat{u}_j(\underline{p},t)\hat{T}(\underline{q},t)d\underline{p}$$

LES in Fourier space (5/8)

- Spectral eddy diffusivity $\kappa_t(k|k_C)$ satisfies:

$$\frac{\partial}{\partial t}\hat{T}(\underline{k},t) + [\kappa + \kappa_t(k|k_C)]k^2 \hat{T}(\underline{k},t) = - ik_j \int_{\underline{p}+\underline{q}=\underline{k}}^{|\underline{p}|,|\underline{q}|<k_C} \hat{u}_j(\underline{p},t)\hat{T}(\underline{q},t)d\underline{p}$$

- Two-point stochastic closures (EDQNM, TFM, LHDIA . . .) provide model expressions for $\nu_t(k|k_C)$ and $\kappa_t(k|k_C)$

LES in Fourier space (6/8)

- Spectral-peak eddy coefficients: EDQNM $\longrightarrow$

$$\nu_t(k|k_C) = \left[\frac{E(k_C)}{k_C}\right]^{1/2} \nu_t^+ \left(\frac{k}{k_C}\right)$$

assuming $E(k) \sim k^{-5/3}$ for $k \gtrsim k_C$

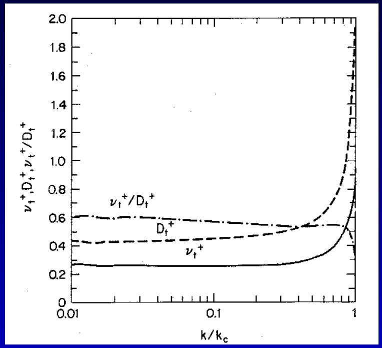
- Asymptotics: $\nu_t^+ \left(\frac{k}{k_C}\right) \longrightarrow 0.441\ {C_K}^{-3/2} \sim 0.28$ when $\frac{k}{k_C} \longrightarrow 0$
- reminder: (isotropic) energy spectrum $E(k)$:

$$E(k,t) = 2\pi k^2 \langle \underline{\hat{u}}(\underline{k},t).\underline{\hat{u}}^*(\underline{k},t)\rangle_{||\underline{k}||=k}$$

$$\frac{1}{2}\langle \underline{u}.\underline{u}\rangle = \int E(k)dk$$

LES in Fourier space (7/8)

- EDQNM non-dimensional eddy coefficients: ν_t^+ , $\quad \kappa_t^+ = \frac{\nu_t^+}{Pr_t}$



- $\longrightarrow Pr_t \approx 0.6$

LES in Fourier space (8/8)

- Spectral-dynamic model (Lesieur-Métais-Lamballais, 1996): for $E(k) \sim k^{-m}$ at k_C. The value of the plateau is recomputed using EDQNM non-local expansions, the peak is unchanged $\longrightarrow$

$$\nu_t(k|k_C) = 0.31\ \frac{5-m}{m+1}\sqrt{3-m}\ {C_K}^{-3/2} \left[\frac{E(k_C)}{k_C}\right]^{1/2} \nu_t^+ \left(\frac{k}{k_C}\right)$$

$$Pr_t = 0.18\ (5-m)$$

whenever $m \leq 3$, otherwise $\nu_t = 0$.

LES in physical space (1/10)

- Physical space (*finite-differences methods, or finite-volume...*), ρ uniform, grid of mesh Δx
- low-pass spatial filter $G_{\Delta x}$, cut-off scale Δx

$$\bar{f}(\underline{x},t) = f * G_{\Delta x} = \int f(\underline{y},t) G_{\Delta x}(\underline{x}-\underline{y}) d\underline{y}\ .$$

- filter commutes with space and time derivatives (if mesh uniform).

LES in physical space (2/10)

- Navier-Stokes equations

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j) = -\frac{1}{\rho_0}\,\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}(2\nu\, S_{ij})$$

with $S_{ij} = (1/2)(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$, strain-rate tensor

- filtered equations :

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j) = -\frac{1}{\rho_0}\,\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(2\nu \bar{S}_{ij} + T_{ij})$$

with $T_{ij} = \bar{u}_i\bar{u}_j - \overline{u_i u_j}$, SubGrid-Stress tensor

LES in physical space (3/10)

- eddy-viscosity assumption (Boussinesq):

$$T_{ij} = 2\nu_t(\underline{x}, t)\ \bar{S}_{ij} + \frac{1}{3}T_{ll}\ \delta_{ij}$$

- LES momentum equations

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j) = -\frac{1}{\rho_0}\,\frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j}[2(\nu + \nu_t)\bar{S}_{ij}]$$

- continuity: $\partial \bar{u}_j/\partial x_j = 0$,
- macro pressure $\bar{P} = \bar{p} - (1/3)\rho_0 T_{ll}$.
- models: Smagorinsky, structure-function, dynamic Smagorinsky …

LES in physical space (4/10)

- Smagorinsky model
  - ▷ *a la* Prandtl mixing length argument: $\nu_t \sim \Delta x\ v_{\Delta x}$
  - ▷ $v_{\Delta x} \sim \frac{\partial v}{\partial x}\Delta x$
  - ▷ $v_{\Delta x} = \Delta x|\bar{S}|$, with $|\bar{S}| = \sqrt{2\bar{S}_{ij}\bar{S}_{ij}}$.
  - ▷ $\longrightarrow$

$$\boxed{\nu_t = (C_S\Delta x)^2|\bar{S}|}$$

  - ▷ inertial arguments $\longrightarrow C_S \approx \frac{1}{\pi}\left(\frac{3C_K}{2}\right)^{-3/4} \longrightarrow C_S \approx 0.18$ for $C_K = 1.4$

LES in physical space (5/10)

- Structure-function model (Métais-Lesieur, *J. Fluid Mech.*, 1992)

$$\nu_t^{SF}(\underline{x}, \Delta x, t) = 0.105\ C_K^{-3/2}\ \Delta x\ [\bar{F}_2(\underline{x}, \Delta x, t)]^{1/2}$$

with the 2nd-order velocity structure-function at scale Δx

$$\bar{F}_2(\underline{x}, \Delta x, t) = \left\langle \|\underline{\bar{u}}(\underline{x}, t) - \underline{\bar{u}}(\underline{x} + \underline{r}, t)\|^2 \right\rangle_{\|\underline{r}\| = \Delta x}$$

  - ▷ consistent with the spectral peak model thru (Batchelor)

$$\langle \bar{F}_2(\underline{x}, \Delta x, t)\rangle_{\underline{x}} = 4\int_0^{k_c} E(k,t)\left(1 - \frac{\sin(k\Delta x)}{k\Delta x}\right) dk \ .$$

  - ▷ In the limit of $\Delta x \to 0$

$$\nu_t^{SF} \approx 0.777\ (C_S\Delta x)^2\sqrt{2\bar{S}_{ij}\bar{S}_{ij} + \bar{\omega}_i\bar{\omega}_i}$$

LES in physical space (6/10)

- Filtered Structure Function model (Ducros *et al.*, *J. Fluid Mech.*, **326**, 1-36, 1996)

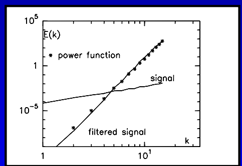
$$\nu_t^{FSF} = 0.0014\, C_K^{-\frac{3}{2}} \Delta x \left[\breve{F}_{2_{\Delta x}}(\underline{x}, \Delta x) \right]^{\frac{1}{2}}$$

  - ▹ density-weighted filtered variables: $\widetilde{\underline{u}} = \dfrac{\overline{\rho u}}{\overline{\rho}}$

$$\breve{F}_{2_{\Delta x}}(\underline{x}, t) = \left\langle \| \breve{\underline{u}}(\underline{x} + \underline{r}, t) - \breve{\underline{u}}(\underline{x}, t) \|^2 \right\rangle_{\|\underline{r}\| = \Delta x}$$

  - ▹ $\breve{\underline{u}}$: convolution of $\widetilde{\underline{u}}$ by 2nd-order centered finite-difference Laplacian filter, iterated 3 times

$$\frac{\tilde{E}(k)}{E(k)} \approx 40^3 \left(\frac{k}{k_C} \right)^9$$



  - ▹ $\sim$ hyperviscosity ; spectral-peak ; ADM

LES in physical space (7/10)

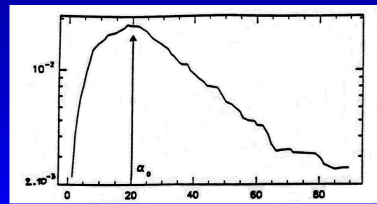
- Selective Structure-Function model David, 1992)

$$\nu_t^{SSF} = 0.16\, \Phi_{20^\circ}(\underline{x}, t)\, C_K^{-\frac{3}{2}} \Delta x \left[\overline{F}_{2_{\Delta x}}(\underline{x}, \Delta x) \right]^{\frac{1}{2}}$$

$$\Phi_{\alpha_0}(\underline{x}, t) = \begin{cases} 1 & \text{if} \quad (\underline{\omega}, \breve{\underline{\omega}}) \geq 20^\circ \\ 0 & \text{otherwise} \end{cases} .$$

with

$$\breve{\underline{\omega}}(\underline{x}, t) = \langle \underline{\omega}(\underline{x} + \underline{r}, t) \rangle_{\|\underline{r}\| \leq \Delta x} .$$



LES in physical space (8/10)

- Mixed Scale model (Sagaut, Ta Phuoc)

$$\nu_t^{MS} = C_m(\alpha) |\widetilde{S}|^{\alpha} \left(q_c^2 \right)^{\frac{(1-\alpha)}{2}} \Delta^{(1+\alpha)} \quad (1)$$

$${q_c}^2 = \frac{1}{2} (\widetilde{u}_k - \widehat{\widetilde{u}}_k)^2 \quad (2)$$

  - ▹ Gaussian test filter

$$\widehat{\widetilde{u}}_i = \frac{1}{4} \left[\widetilde{u}_{i-1} + 2\widetilde{u}_i + \widetilde{u}_{i+1} \right] \quad (3)$$

  - ▹ $\alpha = 1 \longrightarrow$ Smagorinsky's model
  - ▹ $\alpha = 0 \longrightarrow$ Bardina's TKE model
  - ▹ inertial arguments yield $C_m(\alpha) = 0.06$ for $\alpha = 1/2$.

LES in physical space (9/10)

- Selective Mixed Scale model (Sagaut et al.)

$$\nu_t^{SMS} = 0.06 f_{\theta_0} |\widetilde{S}|^{1/2} \left(q_c^2 \right)^{1/4} \Delta^{3/2} \quad , \quad (4)$$

  - ▹ Mixed Scale model with the selection function

$$f_{\theta_0}(\theta) = \begin{cases} 1 & \text{if} \quad \theta \geq \theta_0 \\ r(\theta)^n & \text{otherwise} \end{cases} \quad (5)$$

  in which

$$r(\theta) = \frac{\tan^2(\theta/2)}{\tan^2(\theta_0/2)} \quad ; \quad n = 2 \quad (6)$$

  instead of David's

$$f_{\theta_0}(\theta) = \begin{cases} 1 & \text{if} \quad \theta \geq \theta_0 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

  still with $\theta_0 = 20^\circ$.

LES in physical space (10/10)

- Hybrid models (Sagaut et al.)
  - ▷ Scale similarity model

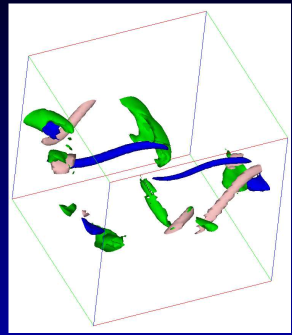
$$\frac{\mathcal{T}_{ij}}{\overline{\rho}} = \widetilde{u}_i\widetilde{u}_j - \widetilde{u_i u_j} \equiv L^m{}_{ij} = \widehat{\widetilde{u}}_i\widehat{\widetilde{u}}_j - \widehat{\widetilde{u}_i\widetilde{u}_j} \qquad (8)$$

  - ▷ $L^m{}_{ij}$: *Resolved Subgrid Stress Tensor*
  - ▷ a la Bardina if filters $\widetilde{}$ and $\widehat{}$ are both defined at grid level Δ
    (even if the Gaussian filter $\widehat{\widetilde{u}}_i = \frac{1}{4}\left[\widetilde{u}_{i-1} + 2\widetilde{u}_i + \widetilde{u}_{i+1}\right]$ is wider than the grid filter (box filter)
  - ▷ a la Germano, if $\widehat{}$ is at scale 2Δ
- Hybridation with an eddy-viscosity model

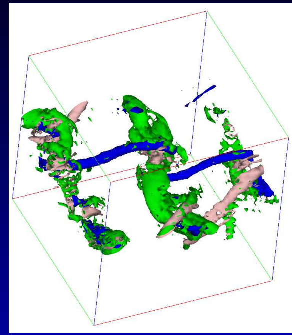
$$\mathcal{T}_{ij} = \frac{1}{2}\,\overline{\rho}\,\left(L^m{}_{ij} + \nu_t\widetilde{S_{ij}}\right) \qquad (9)$$

- See Lenormand *et al.*, *AIAAJ*, **38**, 8, pp. 1340-1350 for assessement in channel flow at Mach 0.5 and 1.5.

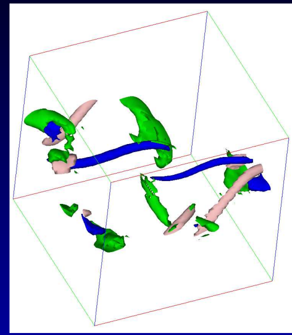
SGS models assessment:



Smagorinsky model: $\max|\omega_1| = 2.92\ \omega_i$

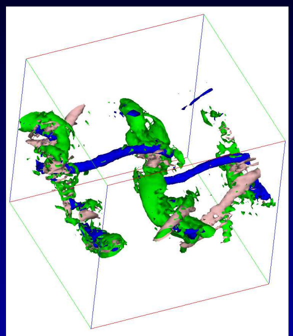


Spectral-Cusp model: $\max|\omega_1| = 4.75\ \omega_i$

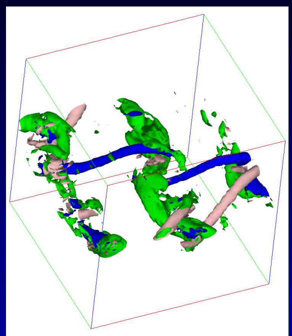


Structure-Function model : $\max|\omega_1| = 2.86\ \omega_i$

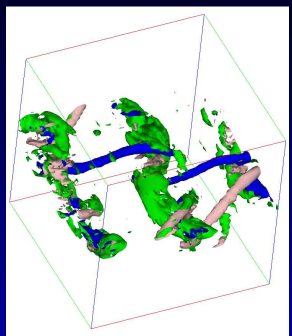
"smarter" SGS models



Spectral-Cusp model: $\max|\omega_1| = 4.75\ \omega_i$



Filtered Structure-Function model : $\max|\omega_1| = 4.83\ \omega_i$

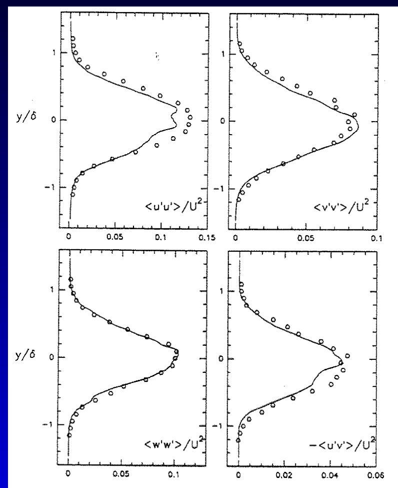


Selective Structure-Function model : $\max|\omega_1| = 5.42\ \omega_i$

Further reading: LESIEUR, M. & MÉTAIS, O. (1996) 'New trends in large-eddy simulations of turbulence', *Ann. Rev. Fluid Mech.*, **28**, 45-82.

SGS models assessment (2/3):

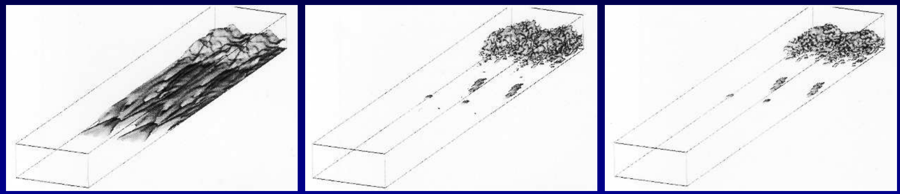
- Reynolds stresses



Lines: LES, spectral-dynamic model.
Symbols: exp. Bell & Mehta (1990)

SGS models assessment (3/3):

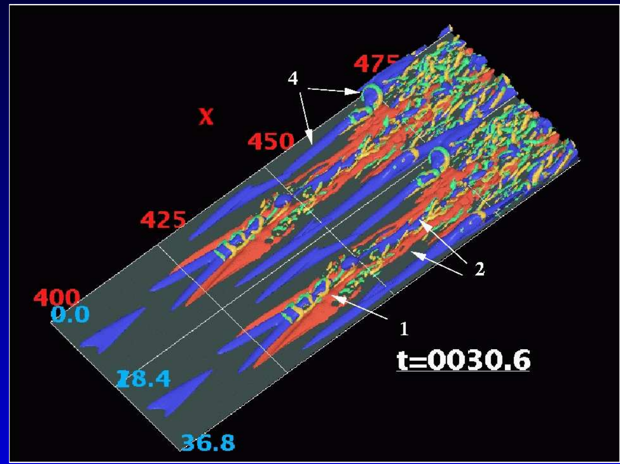
- Transitional boundary layer (simulated with FSF model (Ducros *et al.* *J. Fluid Mech.*, **336**, *1996*)): $\nu_t = 2/3\,\nu$



SF FSF SSF

Boundary layers (1/4)

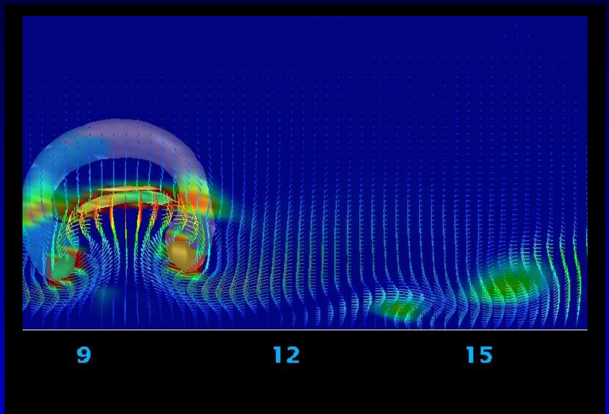
- K-type transition, grid 2



$u' = +0.18\,U_\infty$ (red), $u' = -0.18\,U_\infty$ (blue),
$Q = \frac{1}{2}(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij}) = \frac{1}{2\rho}\nabla^2 P = 0.1\,U_\infty^2/\delta_1^2$ ($\omega_x > 0$, $\omega_x < 0$)

Boundary layers (2/4)

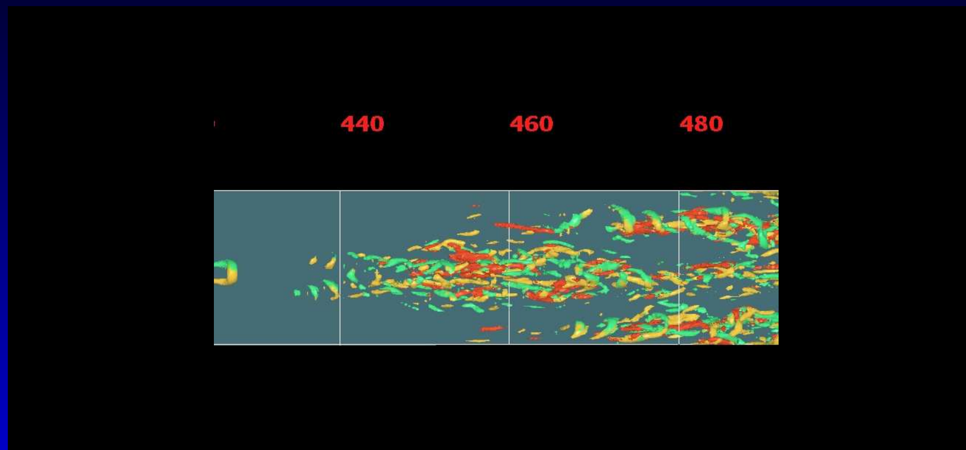
- H-type transition, grid 1



$u' = +0.18\,U_\infty$ (red), $u' = -0.18\,U_\infty$ (blue),
$Q = 0.1\,U_\infty^2/\delta_1^2$ ($\omega_x > 0$, $\omega_x < 0$).

Boundary layers (3/4)

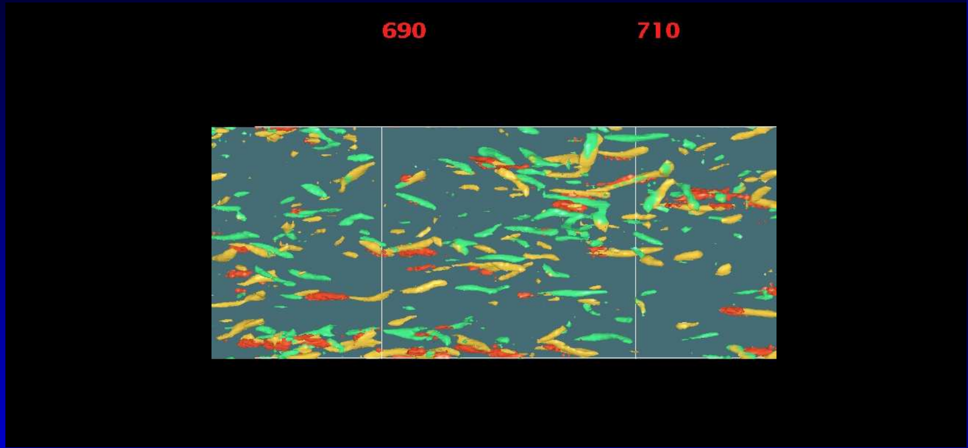
- Is the SGS model intelligent ?



Transitional portion:
$\nu_t = 0.5\nu$ (red);
$Q = 0.1\,U_\infty^2/\delta_1^2$ ($\omega_x > 0$, $\omega_x < 0$).

Boundary layers (4/4)

- Is the SGS model intelligent ?



Turbulent portion:
$\nu_t = 0.5\nu$ (red);
$Q = 0.1\ U_\infty^2/\delta_1^2$ ($\omega_x > 0$, $\omega_x < 0$).

Compressible LES Formulations (1/19)

- Filtering of direct application of conservation principles:

$$\frac{\partial \overline{\rho}}{\partial t} + \text{div}\ (\overline{\rho \underline{u}}) = 0 \qquad (10)$$

$$\frac{\partial \overline{\rho \underline{u}}}{\partial t} + \underline{\text{div}}\ \left(\overline{\rho \underline{u} \otimes \underline{u}} + \overline{p}\underline{\underline{I}} - \overline{\underline{\underline{\sigma}}}\right) = 0 \qquad (11)$$

$$\frac{\partial \overline{\rho E}}{\partial t} + \text{div}\ \left[\overline{(\rho E + p)\underline{u}} + \overline{\underline{q}} - \overline{\underline{\underline{\sigma}}.\underline{u}}\right] = 0 \qquad (12)$$

Compressible LES Formulations (2/19)

- Newton and Fourier laws:

$$\underline{\underline{\sigma}} = 2\mu(T)\underline{\underline{S_0}} + \mu_v \text{div}\ \underline{u} \quad , \quad \underline{q} = -k(T)\ \underline{\text{grad}}\ T$$

$$\underline{\underline{S_0}} = \frac{1}{2}\left(\underline{\underline{\text{grad}}}\ \underline{u} + {}^t\ \underline{\underline{\text{grad}}}\ \underline{u}\right) - \frac{1}{3}\ \text{div}\ \underline{u}$$

- Filtered ideal-gas equations of state (13)

$$\overline{p} = R\ \overline{\rho\ T}\ , \quad \overline{\rho E} = C_v \overline{\rho T} + \frac{1}{2}\overline{\rho \underline{u}.\underline{u}} = \overline{p}/(\gamma - 1) + \frac{1}{2}\overline{\rho \underline{u}.\underline{u}}\ ,$$

correct up to about $600K$ in air, with $\gamma = C_p/C_v = 1.4$.

Compressible LES Formulations (3/19)

- Beyond $600K$, $\gamma \nearrow$ (vibrations of polyatomic molecules).
- μ_v never zero in polyatomic molecules, and can be $\gg \mu$ across shocks (Smits & Dussauge, 1996).
- $\longrightarrow$ Stokes hypothesis ($\underline{\underline{\sigma}}$ trace-free) also excludes shocks.
- in monoatomic gases, helium or argon (no vibration nor rotation), $\gamma = 5/3$ until ionization and $\mu_v = 0$.
- Sutherland's law for μ valid between $100K$ and $1900K$. Constant $Pr = 0.7$ valid in air, even beyond 600K.

Compressible LES Formulations (4/19)

- specificity of filtered conservative equations : triple correlation ($\frac{1}{2}\overline{\rho \underline{u}.\underline{u}}$) involved in a time derivative.
- 2 approaches:
  - ▷ Reynolds filtering
  - ▷ Favre filtering

Compressible LES Formulations (5/19)

- Non-density-weighted variables (e.g. Boersma & Lele, 1999, CTR Briefs, 365-377).
  - ▷ resolved variables $(\overline{\rho}, \overline{\underline{u}}, \overline{p}, \overline{T})$
  - ▷ continuity equation (46a) becomes

$$\frac{\partial \overline{\rho}}{\partial t} + \text{div}\ (\overline{\rho}\ \overline{\underline{u}}) = -\text{div}\ (\overline{\rho \underline{u}} - \overline{\rho}\ \overline{\underline{u}}) \qquad (13)$$

  - ▷ exact pointwise mass preservation lost, but r.h.s. is conservative and $\int_\Omega r.h.s$ can be zero, with appropriate flux corrections (in 3D FV or conservative FD).

Compressible LES Formulations (6/19)

- Non-density-weighted variables (cont'd)
  - ▷ weakly-dissipative model of r.h.s. could increase robustness drastically (as in A.D.M., Leonard, Adams, Stoltz).
  - ▷ closure of $\frac{1}{2}\overline{\rho \underline{u}.\underline{u}}$: secondary issue

Compressible LES Formulations (7/19)

- Density-weighted variables: $\widetilde{\phi} = \dfrac{\overline{\rho\ \phi}}{\overline{\rho}}$,

  $\forall \phi \notin [\rho, p]$
  - ▷ resolved variables $(\overline{\rho}, \widetilde{\underline{u}}, \overline{p}, \widetilde{T})$
  - ▷ $\overline{\underline{u}}$, $\overline{T}$ not computable (but molecular terms $\overline{\underline{\underline{\sigma}}}$, $\overline{\underline{q}}$ and $\overline{\underline{\underline{\sigma}}.\underline{u}}$ are non-linear and thus non-computable anyway).

Compressible LES Formulations (8/19)

- Density-weighted variables (cont'd)
  - ▷ (pointwise) exact mass preservation ensured: continuity equation (46a) becomes

$$\frac{\partial \overline{\rho}}{\partial t} + \text{div } (\overline{\rho}\, \underline{\widetilde{u}}) = 0 \tag{14}$$

  - ▷ subgrid-stress tensor

$$\begin{aligned} \underline{\underline{\tau}} &= -\overline{\rho \underline{u} \otimes \underline{u}} + \overline{\rho}\, \underline{\widetilde{u}} \otimes \underline{\widetilde{u}} \\ &= -\overline{\rho}\, (\widetilde{\underline{u} \otimes \underline{u}} - \underline{\widetilde{u}} \otimes \underline{\widetilde{u}}) \end{aligned} \tag{15}$$

Compressible LES Formulations (9/19)

- Density-weighted variables (cont'd)
  - ▷ filtered total (or stagnation) energy

$$\begin{aligned} \overline{\rho E} &= \overline{\rho} C_v \widetilde{T} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}} - \tfrac{1}{2} tr(\underline{\underline{\tau}}) \\ &= \frac{\overline{p}}{\gamma - 1} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}} - \tfrac{1}{2} tr(\underline{\underline{\tau}}) \end{aligned} \tag{16}$$

  - ▷ weakly-compressible two-scale DIA expansions (Yoshizawa, 1986, *Phys. Fluids*, **29**, 2152.) suggest model for $-\frac{1}{2}tr(\underline{\underline{\tau}})$
  - ▷ adequation to more compressible situations questioned by Speziale *et al.* (1988, *Phys Fluids*, **31** (4), 940-942.).
  - ▷ Three ways out

Compressible LES Formulations (10/19)

- Density-weighted variables, 3 ways out (cont'd)
  1. Replace (46c) by non-conservative

$$\frac{\partial \overline{\rho e}}{\partial t} + \text{div } [\overline{\rho e \underline{u}} + \overline{\underline{q}}] = -\overline{p \text{ div } \underline{u}} + \overline{\underline{\underline{\sigma}} : \underline{\underline{\text{grad }} \underline{u}}} \tag{17}$$

(Moin *et al.*, 1991 *Phys. Fluids A*, **3** (11)) or (Erlebacher *et al.*, 1992, *J. Fluid Mech.*, **238**)

$$\frac{\partial \overline{\rho h}}{\partial t} + \text{div } [\overline{\rho e \underline{u}} + \overline{\underline{q}}] = \frac{\partial \overline{p}}{\partial t} - \overline{p \text{ div } \underline{u}} + \overline{\underline{\underline{\sigma}} : \underline{\underline{\text{grad }} \underline{u}}} \tag{18}$$

Compressible LES Formulations (11/19)

- Density-weighted variables, 3 ways out (cont'd)
  1. (cont'd), with internal energy $\rho e = C_v \rho T = \frac{p}{\gamma - 1}$ or (static) enthalpy $\rho h = \rho e + p = C_p \rho T = \frac{\gamma p}{\gamma - 1}$.
  2. add transport equation of resolved kinetic energy (RKE), i.e. $\frac{1}{2}\, \overline{\rho}\, \underline{\widetilde{u}}.\underline{\widetilde{u}}$ to (17) or (18) (Lee, 1992, Kuerten *et al.*, 1992, Vreman *et al.*, 1995, System I)
     - ▷ non-conservative terms $-\overline{p \text{ div } \underline{u}}$ and $+\overline{\underline{\underline{\sigma}} : \underline{\underline{\text{grad }} \underline{u}}}$ remain, along with RKE's contribution $\underline{\widetilde{u}}.\underline{\text{div }} (\underline{\underline{\tau}})$
     - ▷ succesful, e.g. in channel flow $M = 1.5$, $Re_\tau = 222$ (Lenormand *et al.*, 2000, *AIAA J.*, **38**, 8)

Compressible LES Formulations (12/19)

- Density-weighted variables, 3 ways out (cont'd):
  3. keep fully conservative (46c) and lump $tr(\underline{\tau})$ with the filtered internal energy. $\longrightarrow$ modified (and computable) pressure and temperature $\check{p}$ and $\check{T}$: (Vreman *et al.*, 1995, System II)

$$\overline{\rho E} = \overline{\rho} C_v \check{T} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}} = \frac{\check{p}}{\gamma - 1} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}} \quad , \qquad (19)$$

$$\boxed{\check{p} = \overline{p} - \tfrac{\gamma-1}{2} tr(\underline{\tau}) \quad , \quad \check{T} = \check{p}/(\overline{\rho} R)} \quad . \qquad (20)$$

  - ▷ counterpart of *macro-pressure* $\overline{p} - \frac{1}{3}tr(\underline{\tau})$ in incompressible LES with eddy-viscosity assumption $\underline{\underline{\tau_D}} \simeq 2\overline{\rho}\nu_t \underline{\underline{S_0}}$, with $\underline{\underline{\tau_D}} = \underline{\tau} - \frac{1}{3}tr(\underline{\tau})$.

Compressible LES Formulations (13/19)

- 3rd way out: macro-temperature closure (cont'd):
  - ▷ Filtered momentum eq. (46b) becomes

$$\frac{\partial \overline{\rho}\underline{\widetilde{u}}}{\partial t} + \underline{\text{div}} \left[\overline{\rho}\underline{\widetilde{u}} \otimes \underline{\widetilde{u}} + \left(\check{p} - \tfrac{5-3\gamma}{6} tr(\underline{\tau})\right) \underline{I} - \underline{\underline{\tau_D}} - \underline{\overline{\sigma}} \right] = 0$$

  - ▷ $\frac{5-3\gamma}{6}\, tr(\underline{\tau}) = 0$ in monoatomic gases $(\gamma = 5/3)$.
  - ▷ $\frac{5-3\gamma}{6} tr(\underline{\tau})/\check{p} = \frac{5-3\gamma}{3}\gamma M_{sgs}^2$. with $M_{sgs}^2 = \frac{1}{2}|tr(\underline{\tau})|/\overline{\rho}\check{c}^2 = \frac{1}{2}|tr(\underline{\tau})|/(\gamma\check{p})$.

Compressible LES Formulations (14/19)

- 3rd way out: macro-temperature closure (cont'd):
  - ▷ neglecting it in air is 3.75 less stringent than approximation $\gamma M_{sgs}^2 \ll 1$ required to neglect $-\frac{1}{2}tr(\underline{\tau})$ with respect to $\overline{p}$ (see Erlebacher *et al.*, 1992, in a non-conservative context).

Compressible LES Formulations (15/19)

- Density-weighted variables (cont'd): closure of total enthalpy flux $\overline{(\rho E + p)\underline{u}}$
  - ▷ resolved pressure: $\breve{p} = \check{p}$ or $\overline{p}$
  - ▷ at least three levels of decomposition are possible:

$$\overline{(\rho E + p)\underline{u}} = (\overline{\rho E} + \breve{p})\underline{\widetilde{u}} - \underline{Q_H} \qquad (21)$$

with

Compressible LES Formulations (16/19)

- Density-weighted variables (cont'd): closure of total enthalpy flux $\overline{(\rho E + p)\underline{u}}$ (cont'd)

$$\underline{\mathcal{Q}_H} = \left[-\overline{(\rho E + p)\underline{u}} + (\overline{\rho E} + \check{p})\underline{\widetilde{u}} \right] \quad (22)$$

$$= \underbrace{\left[-\overline{(\rho e + p)\underline{u}} + (\overline{\rho e} + \check{p})\underline{\widetilde{u}} \right]}_{\underline{\mathcal{Q}_h}} + \underbrace{\left[-\tfrac{1}{2}\overline{\rho(\underline{u}.\underline{u})\underline{u}} + \tfrac{1}{2}(\overline{\rho \underline{u}.\underline{u}})\underline{\widetilde{u}} \right]}_{\underline{\mathcal{W}}} \quad (23)$$

Compressible LES Formulations (17/19)

- Density-weighted variables (cont'd): closure of total enthalpy flux $\overline{(\rho E + p)\underline{u}}$ (cont'd)

$$\underline{\mathcal{Q}_h} = \underbrace{\left[-\overline{(\rho e)\underline{u}} + (\overline{\rho e})\underline{\widetilde{u}} \right]}_{\underline{\mathcal{Q}_e}} + \left[-\overline{p\underline{u}} + \check{p}\underline{\widetilde{u}} \right] \quad (24)$$

$\rho e = \rho C_v T = p/(\gamma - 1)$: internal energy.

- $\underline{\mathcal{Q}_h}$ and $\underline{\mathcal{Q}_e} \propto$ grad $\widetilde{T}$ (in Erlebacher *et al.*, 1992, and Moin *et al.*, 1991, resp.)
- $\underline{\mathcal{Q}_H} \simeq \overline{\rho} C_p(\nu_t/Pr_t)$grad $\check{T}$ with $\check{p} = \check{p}$ yields Normand and Lesieur (1992):

Compressible LES Formulations (18/19)

- Normand and Lesieur (1992) heuristic form

$$\frac{\partial \overline{\rho}}{\partial t} + \text{div } (\overline{\rho}\underline{\widetilde{u}}) = 0$$

$$\frac{\partial \overline{\rho}\underline{\widetilde{u}}}{\partial t} + \underline{\text{div}} \left(\overline{\rho}\underline{\widetilde{u}} \otimes \underline{\widetilde{u}} + \check{p}\underline{\underline{I}} - 2\left[\mu(\check{T}) + \overline{\rho}\nu_t(\underline{\widetilde{u}})\right] \underline{\underline{S_0}}(\underline{\widetilde{u}}) \right) = 0$$

$$\frac{\partial}{\partial t}\left(\frac{\check{p}}{\gamma - 1} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}} \right) + \text{div} \left[\left(\tfrac{\gamma}{\gamma-1}\check{p} + \tfrac{1}{2}\overline{\rho}\underline{\widetilde{u}}.\underline{\widetilde{u}} \right)\underline{\widetilde{u}} - C_p\left(\frac{\mu(\check{T})}{Pr} + \frac{\overline{\rho}\nu_t(\underline{\widetilde{u}})}{Pr_t} \right) \underline{\text{grad}}\ \check{T} - 2\mu(\check{T})\underline{\underline{S_0}}(\underline{\widetilde{u}}).\underline{\widetilde{u}} \right] = 0$$

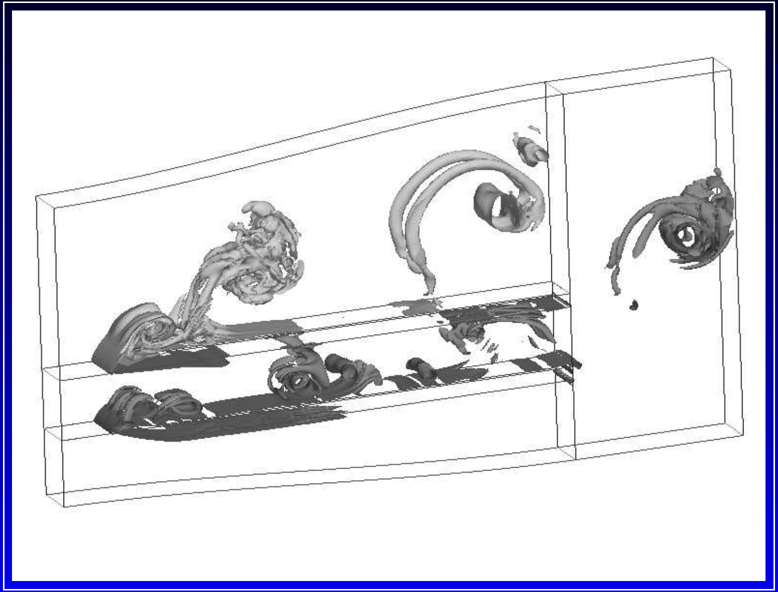
(25)

still with $\check{T} = \check{p}/(\overline{\rho}R)$.

Compressible LES Formulations (19/19)

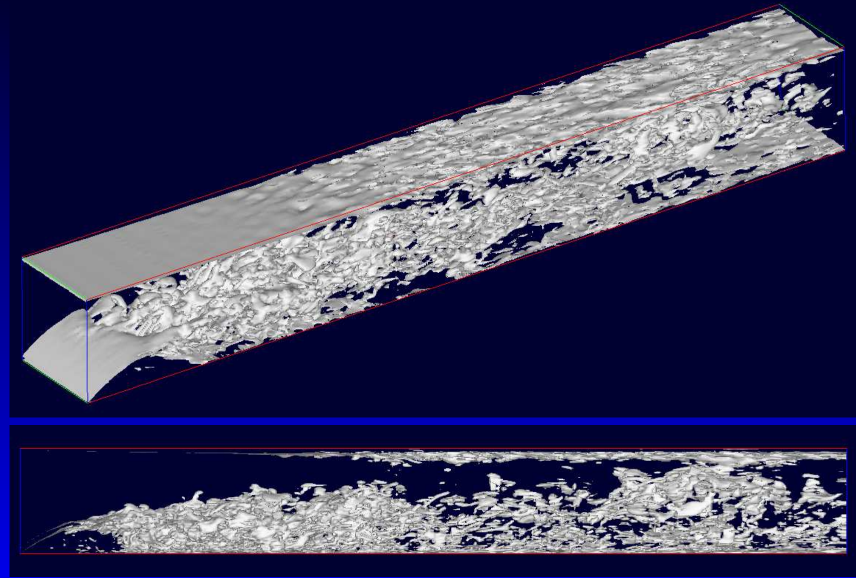
- Normand and Lesieur (1992) heuristic form (cont'd)
  - ▷ amounts to adding $\overline{\rho}\nu_t$ and $\overline{\rho}C_p(\nu_t/Pr_t)$ to their molecular counterpart in (25) except in the last term of the energy equation.
  - ▷ This exception disappears when option (62b) is taken, with $\underline{\mathcal{Q}_h} \simeq \overline{\rho}C_p\ (\nu_t/Pr_t)$grad $\check{T}$ and the RANS type model $\underline{\mathcal{W}} \simeq \underline{\tau}.\underline{\widetilde{u}}$.
  - ▷ used succesfully by Knight *et al.* (1998, see also Okong'o & Knight, 1998) on unstructured grids.
  - ▷ $|\underline{\mathcal{W}} - \underline{\tau}.\underline{\widetilde{u}}|$ small in constant-density RANS filtering.

Air intake, Arnaudon & Tsen, 1974 (1 of 5)



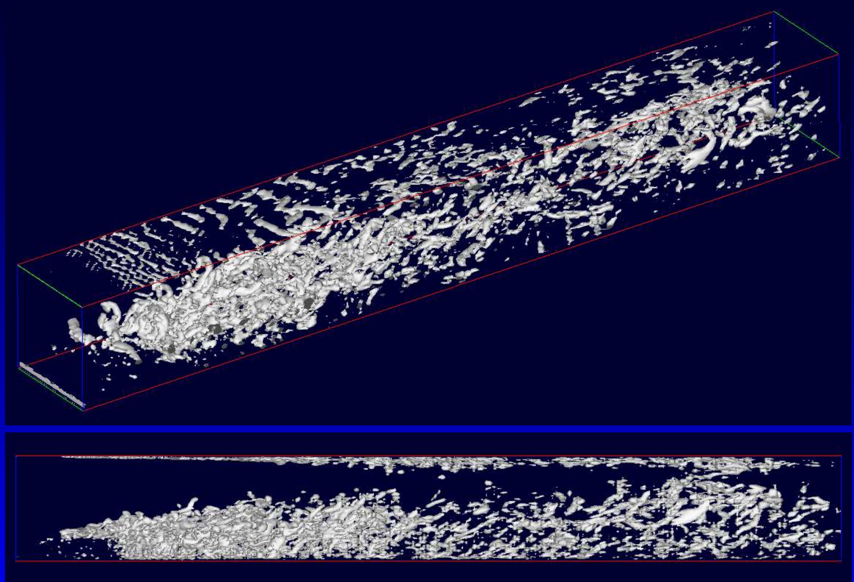
Low Re number. Isosurface of vorticity magnitude for $\theta = 30°$.

Air intake (2 of 5)



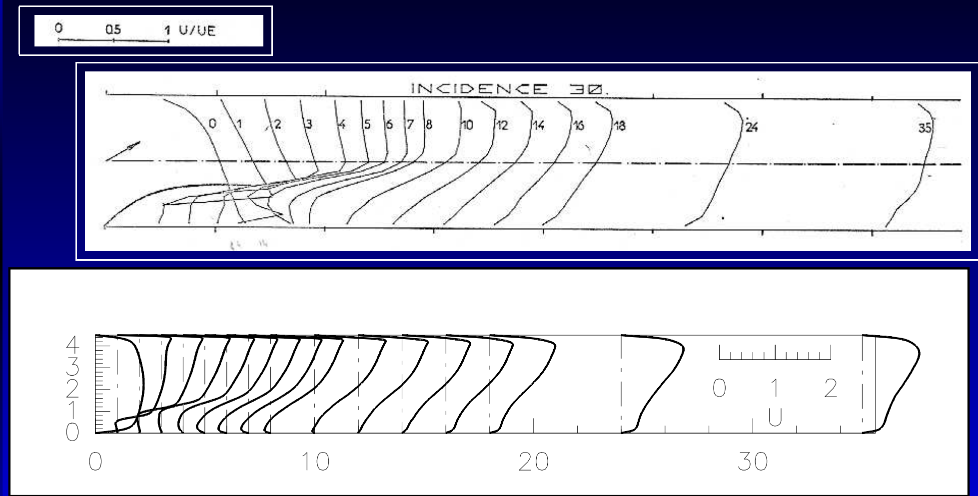
$Re = 50000$. Isosurface of vorticity magnitude: $\|\vec{\omega}\| = 1.1\ U_\infty/H$

Air intake (3 of 5)



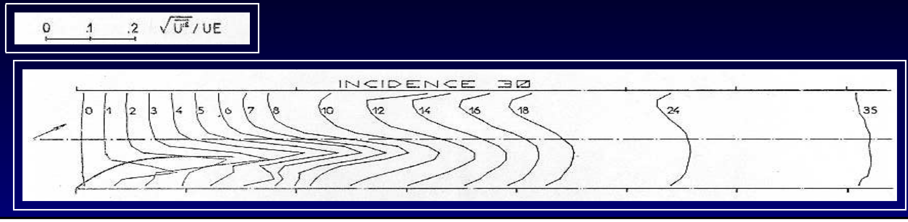
$Re = 50000$. Iso-surface $Q = \frac{1}{2}(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij}) = 0.4\ (U_\infty/H)^2$

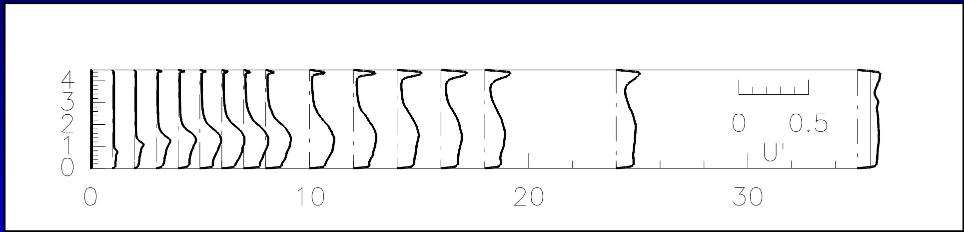
Air intake (4 of 5)



$\theta = 30°$: $\overline{u}$ Arnaudon & Tsen (1974) top

Air intake (5 of 5)





$\theta = 30°$: u' Arnaudon & Tsen (1974) top