Geometrical Modelling of the Fibre Organization in the Human Left Ventricle

Ayman Mourad[1,2], Luc Biard[2], Denis Caillerie[1], Pierre-Simon Jouk[3,4], Annie Raoult[2], Nicolas Szafran[2], and Yves Usson[3]

[1] Laboratoire Sols, Solides, Structures, BP 53, 38041 Grenoble Cedex 9, France
[2] Laboratoire de Modélisation et Calcul/IMAG, BP 53, 38041 Grenoble Cedex 9, France
[3] Laboratoire TIMC, Domaine de de la Merci, 38076 La Tronche Cedex, France
[4] Unité fonctionnelle Biologie du développement et Génétique clinique, CHU, BP 217, 38043 Grenoble Cedex 9, France

Abstract. The aim of the present study is to check, by means of elementary mathematical tools, a conjecture according to which myocardial fibres are geodesic curves running on some surfaces. This conjecture was first stated and experimentally checked by Streeter (1979) for the equatorial part of the left ventricle free wall. Quantitative polarized light microscopy provides measurements on fibre orientation that could lead to evidence that the conjecture remains true for the whole of the left ventricle. Study of the right ventricle is under progress.

1 Introduction

It is commonly believed [3] that the myocardium design and structure allow maximal mechanical efficiency in the systole and diastole processes. The long-term purpose of our multi-disciplinary approach is to try and propose a model for the mechanical behaviour of the myocardium. In the long run, performance of the complete electro-mechanical system could be analyzed. For related works, see [1], [7]. It is well known that usual mechanical models for skeletal muscles are of no help for the myocardium. Obviously, the myocardium is not, as ordinary muscles, linked at both ends to a bone. Fibre micro-structure and fibre geometrical organization are quite different as well. We believe that the specific fibre organization in the myocardium should be taken into account in a complete model. Numerous anatomical studies, see *e.g.* [8], [9], have been devoted along the years to a description of the fibres arrangement and we refer to [5] for an extensive bibliography. What will be sufficient to recall here is that the dissection or peeling techniques are not precise enough, since apparently preferred fibre directions can be inferred by the experimental process. Data have been improved by means of several techniques in microscopy, such as photonic or electronic microscopy. In the present work, we use the data provided by the polarized light microscopy devices developed by some of the authors [4], [5].

More precisely, we intend to check the geometrical description proposed by Streeter [9] who introduced a topological representation of the left ventricle as a “nested set of toroidal bodies of revolution” on which myocardial fibres run as geodesics. This information would increase our understanding of the biomechanical properties and propagation of electrical stimuli in the heart.

The experimental technique, which is valid for both ventricles and for the septum, measures for a set of points located on several myocardial sections two angles from which the fibre orientation can be deduced. In other terms, the output of the experimental work is a discrete three-dimensional vector field. Assuming that the left ventricle has a structure of revolution, we use the invariance of Clairaut’s constant along geodesics as a first hint that the conjecture might be true.

Work under progress is devoted to a similar description of the right ventricle. Note however that the simple structure of revolution of the left ventricle is no longer true for the right ventricle.

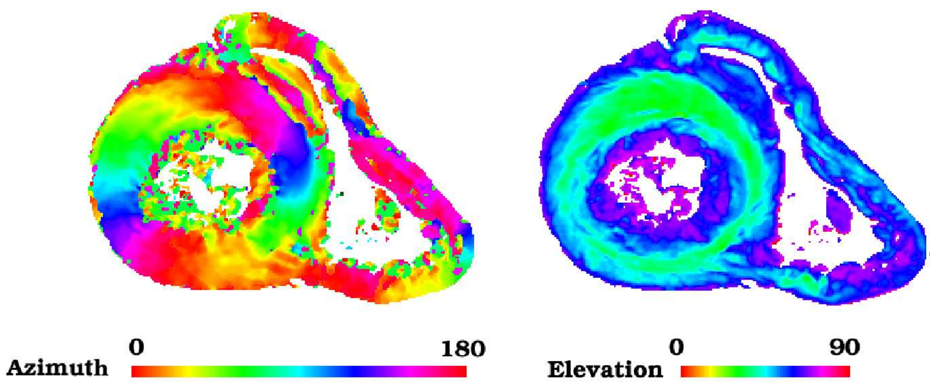

Fig. 1. Maps of the azimuth and elevation angles obtained by means of polarized light microscopy in a coronal section.

2 Data

The data are obtained on fetal hearts. We refer to [4] and [5] for a complete description of the protocol. Let us just recall that the ventricles are embedded in a transparent resin in which they can be clearly seen after polymerization. Sections that can be transversal, sagittal or coronal can then be cut. A section thickness is $500\mu m$, and, because of the thickness of the saw, adjacent sections are separated by a $250\mu m$ gap. The measurement technique relies on the birefringence properties of myocardial cells: in short, the velocity of the light is slower when travelling along the long axis than along the short axis of the fibre. Results are given pointwise: each section is dicretized in $130\mu m \times 130\mu m$ squares, and a mean angular information is collected for each of these elementary squares. For each voxel, the acquisition and representation processes result in two angles: the angle of elevation γ_{ele} which is the angle between the fibre and the plane of the section, and the azimuth angle γ_{azi} which is the angle between the projection of the fibre on the section plane and a fixed direction in this plane, namely the east-west axis. A local knowledge at point (x, y, z) of these two angles which should range from 0 to π for γ_{azi} and $-\pi/2$ to $\pi/2$ for γ_{ele} determines completely the direction $\boldsymbol{\tau}$ of the fibre

$$\boldsymbol{\tau} = \left| \begin{array}{l} \tau_x = \cos\gamma_{ele}\cos\gamma_{azi} \\ \tau_y = \cos\gamma_{ele}\sin\gamma_{azi} \\ \tau_z = \sin\gamma_{ele} \end{array} \right.$$

A drawback of the experimental device in its present state is that it provides the same value for γ_{ele} and $-\gamma_{ele}$. As a consequence and because of the averaging technique, values closed to $0°$ cannot be reached. Moreover, angles between $75°$ and $90°$ cannot be resolved. The accuracy of the method has been checked on myocardial samples which fibres are parallel: the resolution of the measurement method for both angles is $1°$.

From a mathematical point of view, we are given a discrete sample of a distributed vector field.

3 Geodesics

According to Streeter [9], in the equatorial part of the left ventricle free wall, fibres are organized into surfaces on which they run as geodesics. In order to check and extend this conjecture to the whole left ventricle, let us first recall some elementary properties of geodesics.

Definition: A regular curve of a given surface of $\mathbf{R^3}$ is a geodesic if and only if its principal normal at any point is normal to the surface.

This definition has well known consequences in terms of lengths [2].

Proposition: Any geodesic locally minimizes the arc length between two points. Conversely, when a regular curve is the shortest path between any two of its points, it is a geodesic.

Note that, in general, an arbitrary geodesic does not minimize globally the arc length between two of its points. For instance, an helix on a vertical cylinder links points on a same vertical line that can be more economically connected through a vertical straight line. Can we expect to find geodesics on any surface? Two answers are well known. First, local existence of a geodesic starting from a given point and tangent to a given vector belonging to the tangent plane at this point is an easy consequence of the fact that a geodesic on a parametrized surface is given by a system of two ordinary differential equations. Second, a global existence result is available in the case of closed surfaces: there exists a minimum geodesic joining two given points of a closed surface (particular case of the Hopf-Rinow theorem, [6]).

In the present study, we assume that the left ventricle has a structure of revolution. Namely, we see the outer surface as generated by rotating a meridian curve, described by Streeter as crescent-shaped, around a vertical axis, see Figure 2. The distance between this generating curve and the axis of revolution is close to 0, but not 0, at the apex which allows fibres to invaginate. Internal layers, if they exist, are also seen as rotation invariant. Computer Aided Design (CAD) approximate models using B-spline smooth surfaces have been constructed. They are out of the scope of the present paper and are used in a parallel work as test models for fibre reconstruction algorithms. This simplifying assumption allows us to use a specific property of geodesics of surfaces of revolution [2].

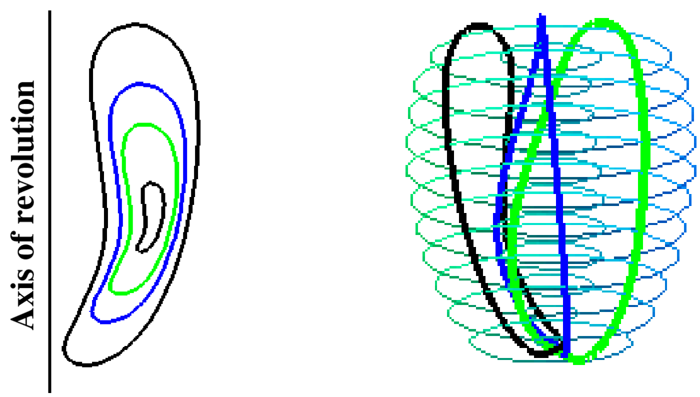

Fig. 2. An ideal setting. Left: nested meridian curves. Right: periodic geodesics deduced one from the other by rotations.

Clairaut's constant: For any point on a curve on a surface of revolution, let r be the distance from this point to the axis of revolution and let θ be the angle between the tangent to this curve and the parallel of the surface at this point. If the curve is a geodesic, then the quantity $r \cos \theta$ remains invariant along the curve. It is called the Clairaut constant.

Note that the above property is not sufficient for a curve to be a geodesic. Along all parallels, *e.g.*, the quantity $r \cos \theta$ remains constant since it is equal to r, but only some of the parallels are geodesics.

4 Experimental validation

As seems to be true from anatomical observations, assume that the left ventricle fibres are organized in nested toroidal layers with the same axis of revolution and that they are periodic geodesics of these layers, *i.e.*, the fibres follow closed curves with continuous tangents. Furthermore, assume that the fibres on a given layer can be deduced one from the other by rotations around the axis of revolution. Figure 2 describes an ideal setting where the geodesics have been computed on the abstract CAD model (the algorithm numerically solves the system of ordinary differential equations characterizing a geodesic). Then the Clairaut

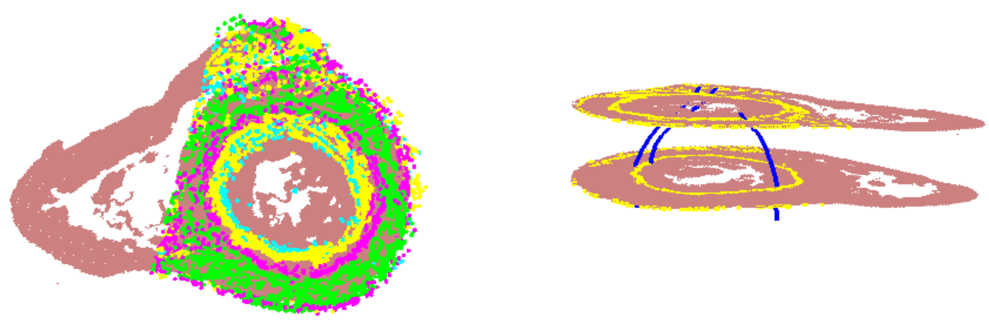

Fig. 3. Left: isovalues of $r\cos\theta$ on a given coronal section. Right: Fibre trajectories crossing traces of a given C on several coronal sections.

constant is the same for all fibres in the layer, but may vary from one layer to another. As a consequence, if the conjecture is true, then the isovalues of $r\cos\theta$, where θ denotes here the angle between the vector $\boldsymbol{\tau}$ at point (x, y, z) and the parallel at this same point, will be nested toroidal surfaces. The traces of these surfaces on each coronal section (a section which is orthogonal to the left ventricle axis) should be concentric circles. This comes from the fact that the fibres are supposed to be organized in layers and globally rotationally invariant on each layer. Moreover, a same constant value of $r\cos\theta$ should appear on two separate concentric circles translating the fact that the fibre invaginates and, then, makes at least one complete turn before closing back. As for the trace of each surface of isovalues on a meridian plane, it should consist of two symmetrical closed crescent-shaped curves.

With this in mind, we have computed $r\cos\theta$ for our experimental data. In order to use what is actually provided to us, namely the vector field $\boldsymbol{\tau}$, we have used the obvious identity

$$r\cos\theta = |-y\tau_x + x\tau_y|.$$

We have traced the lines of isovalues on several coronal section planes. We readily observe that on each section they are, as anticipated, concentric circles, and that a same constant value of $r\cos\theta$ appears on two separate concentric circles. In the same time, we have developed an algorithm which follows the fibres from section to section in the whole of the myocardium. By means of an interpolation procedure, the algorithm first transforms the given discrete data into a distributed vector field. Then, for a given h, the iterative procedure constructs a point from the previous one by moving with length h along its vector direction. In the left ventricle, the obtained trajectories cross the circles that correspond to a same Clairaut constant. In parallel, we have traced several isovalues of $r\cos\theta$ on two longitudinal section planes, one orthogonal and the other parallel to the interventricular septum (namely, transversal and sagittal sections), from the apex to the base. The result shows, as expected, a nested set of meridian curves which is similar to the abstract model described in Figure 2.

Clearly, an arbitrary vector field $\boldsymbol{\tau}$ would not satisfay the above properties.

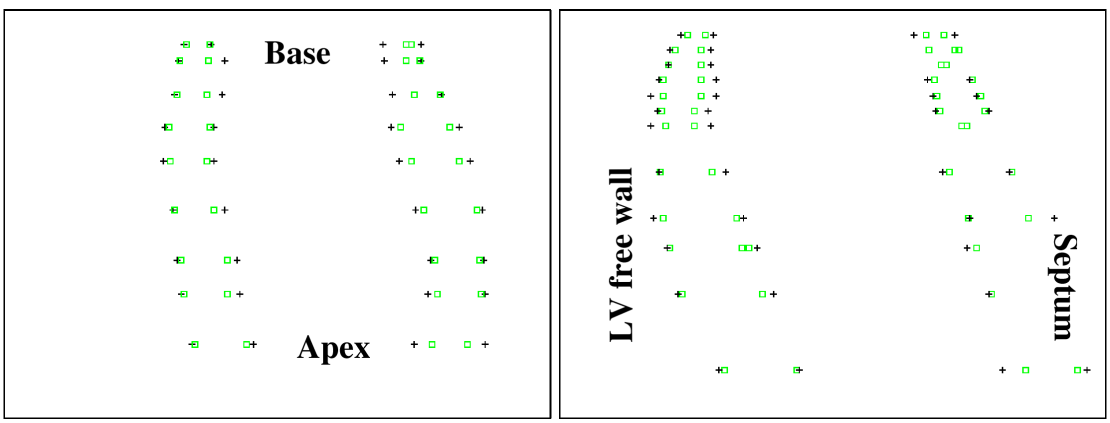

Fig. 4. Traces of some values of C on longitudinal section planes parallel (left) and orthogonal (right) to the interventricular septum.

The conclusion of this study is that checking the invariance of the Clairaut number against experimental values backs up the geodesic conjecture, at least for the left ventricle. In a future work, we intend to study the conjecture for the right ventricle. We will use the fibres tracking algorithm that has already been developed and used for the left ventricle. Simultaneously, an explanation of the geodesic structure in terms of mechanical efficiency will be investigated.

Acknowledgements This work was supported by the Fonds National de la Science (ACI Modélisation mathématique, mécanique et numérique du myocarde) and by the Région Rhône-Alpes (Projet Mathématiques pour ADéMo).

References

[1] Cai H. (1998) - *Loi de comportement en grandes déformations du muscle à fibres actives. Application à la mécanique du cœur humain et à sa croissance*, Thèse de l'Université de Savoie.

[2] Do Carmo M.P. (1976) - *Differential geometry of curves and surfaces*, Prentice-Hall.

[3] Hutchins G.M. et al (1978) - Shape of the human cardiac ventricles, *Am. J. Cardiol.*, Vol. 41, pp. 646-654.

[4] Jouk P.S., Usson Y., Michalowicz G., and Parazza F. (1995) - Mapping of the orientation of myocardial cells by means of polarized light and confocal scanning laser microscopy, *Microsc. Res. Tech.*, Vol. 30, pp. 480-490.

[5] Jouk P.S., Usson Y., Michalowicz G., and Grossi L. (2000) - Three-dimensional cartography of the pattern of the myofibres in the second trimester fetal human heart, *Anat. Embryol.*, Vol. 202, pp. 103-118.

[6] Hopf H., Rinow W. (1931) - Über den Begriff der vollständigen differentialgeometrischen Flächen, *Comm. Math. Helv.*, Vol. 3, pp. 209-225.

[7] Ohayon J., Usson Y., Jouk P.S., and Cai H. (1998) - Fibre orientation in human fetal heart and ventricular mechanics : A small perturbation analysis, *Comput. Methods Biomechanics Biomedic. Eng.*, Vol. 2, pp. 83-105.

[8] Sanchez-Quintana D., Garcia-Martinez V., Climent V., and Hurle J.M. (1995) - Morphological changes in the normal pattern of ventricular myoarchitecture in the developing human heart, *Anat. Rec.*, Vol. 243, pp. 483-495.

[9] Streeter D.D. (1979) - Gross morphology and fiber geometry of the heart, in *Handbook of Physiology. The cardiovascular system*, Berne R.M., Sperelakis N., Geiger S.R. eds, Am. Phys. Soc., Williams & Wilkins, Baltimore.