Evaluation of singular integrals arising from boundary element method in electromagnetism

Soumaya OUESLATI, University of Cergy-Pontoise

Imen BALLOUMI, University of Cergy-Pontoise

Christian DAVEAU, University of Cergy-Pontoise

Abdessatar KHELIFI, University of Carthage

Abstract

Our main interest is to develop a new method to evaluate singular integrals derived from boundary integral method for the resolution of boundary problems. We consider as a model the scattering problem with Leontovich impedance boundary conditions. Indeed, an integral formulation of Maxwell equation [3], gives rise to singular integrals in the form of singular and double layer operators. The evaluation of these integrals has been the subject of a large number of works and publications. In our numerical code we deal with three different methods. The first one is a semi analytical method based on a change of variable [1]. The second one is purely numerical method in which we apply the Duffy transformation [4]. We are particularly interested in the method introduced by Lenoir and Salles in [2]. We are inspired by this work to calculate the double layer potential appearing in the variational formulation.

$$\iint_{\Gamma} [\mathbf{f}_i(x) \times \mathbf{f}_j(y)] \cdot \frac{x-y}{|x-y|^{1+\xi}}\ dx\ dy; \xi = 0 \text{ or } 2. \tag{1}$$

where f_i and f_j are basis functions. The method developed in this work allows us to evaluate analytically and with great precision, this singular integral. It order to apply a boundary element method, it was necessary to discretize the edge of the domain using plan polygons. Here we deal with plan triangles and Rao-Wilton-Glisson basis functions. To better understand this singularity problem, it should be noted that the domain of integration for these integrals, is the product of two elements of the mesh, in 3-D, the product of two triangles S and T. When the two triangles are distant, it is possible to evaluate the integral with standard numerical integration methods. But the problematic cases occurs when the intersection of the two triangles is not empty(adjacent triangles and triangles with a common vertex). These two cases require separate treatment. The presented method makes it possible to reduce a 4-D integral to a linear combination of mono-dimensional integrals whose integrand is regular. It is then possible to numerically evaluate these 1-D integrals, but also explicitly. We developed a library in Fortran 90 to implement the three methods. The exact solution is approximated with Mie series. Then, we compare numerical results with differents sphere-mesh.

Références

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Soumaya OUESLATI, Department of Mathematics, CNRS (UMR 8088), University of Cergy-Pontoise, 2 avenue Adolphe Chauvin, 95302 Cergy-Pontoise
`soumaya.oueslati1@u-cergy.fr`
Imen BALLOUMI, Department of Mathematics, CNRS (UMR 8088), University of Cergy-Pontoise, 2 avenue Adolphe Chauvin, 95302 Cergy-Pontoise
`imen.balloumi@u-cergy.fr`
Christian DAVEAU, Department of Mathematics, CNRS (UMR 8088), University of Cergy-Pontoise,2 avenue Adolphe Chauvin, 95302 Cergy-Pontoise
`christian.daveau@u-cergy.fr`
Abdessatar KHELIFI, Department of Mathematics, University of Carthage, Bizerta, Tunisia
`abdessatar.khelifi@fsb.rnu.tn`