A first order mixed formulation for the numerical controllability of waves.

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In this communication we consider an optimization approach for the numerical approximation of boundary controls for the wave equation. It is well known that the minimal L^2-norm boundary control supported on $\mathcal{J} \subseteq \partial\Omega \times (0,T)$ which drives a certain initial data $(u_0, u_1) \in L^2(\Omega) \times H^{-1}(\Omega)$ —initial position and velocity distributions— to the zero state under the evolution governed by the wave equation can be found as $v = \varphi|_{\mathcal{J}}$, where φ is the solution to the optimization problem

$$\min_{\varphi} J^\star(\varphi) = \frac{1}{2}\int_{\mathcal{J}} |\tfrac{\partial\varphi}{\partial\nu}|^2 \, d\sigma \, dt + (u_0, \varphi(0))_{L^2(\Omega)} - \langle u_1, \varphi_t(0)\rangle_{H^1, H^{-1}} \tag{1}$$

and satisfies

$$\partial_{tt}\varphi - \Delta\varphi = 0, \quad \varphi = 0 \text{ on } \partial\Omega \times (0,T), \quad (\varphi(0), \varphi_t(0)) \in H_0^1(\Omega) \times L^2(\Omega). \tag{2}$$

In order to handle the constrained optimization problem (1)-(2), a mixed formulation which introduces a Lagrange multiplier was proposed in [1]. In that work, the classical observability and direct inequalities were employed to prove the existence of minimal L^2-norm controls by means of the Ladyzhenskaya-Babuska-Brezzi Theorem. Moreover, the authors of [1] obtained numerical evidence showing that the mixed formulation they proposed is well suited for finite elements discretizations.
In this communication we want to explain a first order mixed formulation related to the method employed in [1]. Our approach is based on the following observation: if φ solves the wave equation $\varphi_{tt} - \Delta\varphi = 0$, then the new variables $v := \varphi_t$ and $p := \nabla\varphi$ solve the system

$$v_t - div\, p = 0, \quad p_t - \nabla v = 0. \tag{3}$$

This kind of first order formulation have been already considered in the finite element literature [2], and has the advantage of allowing to reduce the regularity of the finite elements spaces, which entails a simpler implementation of the discretization.
In this communication we will explain some of the results we have obtained regarding to the continuous problem and some numerical experiments which support this approach for the numerical approximation of controls.

Références

[1] N. Cîndea, A. Münch, *A mixed formulation for the direct approximation of the control of minimal L^2-norm for linear type wave eaquations*, *Calcolo* **52** (2015), no. 3, 245–288.

[2] E. Bécache, P. Joly, C. Tsogka, *An analysis of new mixed finite elements for the approximation of wave propagation problems*, *SIAM J. Numer. Anal.* **37** (2000), no. 4, 1053–1084.

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