Spectral method for hypocoercive operators

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We consider the numerical approximation of the solution of a Poisson problem:

$$-\mathcal{L}\Phi = \psi, \tag{1}$$

for a hypocoercive operator $\mathcal{L}$. The well-posedness of the Poisson equation can be proven using dedicated techniques under certain structural assumptions on the symmetric and anti-symmetric parts of $\mathcal{L}$ [3].
We prove the consistency of Galerkin method for such problems by proving the exponential decay of the semigroup associated with the Galerkin discretization of the operator $\mathcal{L}$. Given a discretization basis, some work is indeed needed to prove that the rigidity matrix is invertible and to provide an equivalent of Céa's lemma. We also show uniform bounds on the inverse of the rigidity matrix and deduce error estimates. We show in particular that the consistency error decays faster than any polynomial for smooth data ψ. It can be convenient to use a non-conformal basis, in which case a saddle-point formulation is used. Our results are extended to this setting.
In a second part we use Monte Carlo methods to compute the expectation of the observable Φ. We show how an approximation of the solution Φ of (1) is used to reduce the variance of the estimator and thus fastens the computation.

Références

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