An edge-based scheme on polyhedral meshes for vector advection-reaction equations

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In this work [1], we present a low-order edge-based scheme on polyhedral meshes for the vector advection-reaction problem. This model problem is encountered in various situations, such as the representation of the Lie derivative in $\mathbb{R}^3$, the modelling of magnetic fields in moving plasmas or also as one of the building blocks of the Oseen problem. Our scheme is based on the use of a low order edge-based reconstruction map (see [2] and [3]), extending the notion of edge finite elements on polyhedral meshes. The well-posedness of our scheme is analyzed first under the classical hypothesis of Friedrichs systems that requires a strictly positive lower bound on the lowest eigenvalue of some tensor depending on the model parameters. We also prove stability when this quantity is null or slightly negative if the mesh size is small enough. Quasi-optimal a priori error estimates are established for solutions in $W^{1,p}$ with $p \in \left(\frac{3}{2}, 2\right]$ and numerical results are presented on three-dimensional polyhedral meshes.

Références

[1] Cantin, P. & Ern, A., *An edge-based scheme on polyhedral meshes for vector advection-reaction equations*, ESAIM: M2AN, 51. 2017.

[2] Gerristma, M., *An introduction to a compatible spectral discretization method*, Mechanics of Advanced Materials and Structures, 19, 2012.

[3] Codecasa, L., Specogna R. & Trevisan F. , *A new set of basis functions for the discrete geometric approach*, J. Comput. Phys. 223, 2010.

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