On the diffusion rate of Godunov type schemes

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Dissipation rates of Godunov upwind type schemes are often studied either through Fourier analysis or through the so-called modified equation technique. However, these techniques are limited to uniform cartesian meshes. In this work, we introduce a new methodology that allows to derive explicit lower bounds for the dissipation rate of the semi-discrete Godunov type schemes applied to non uniform or to triangular meshes. We illustrate our purpose by applying the theory to the one-dimensional transport equation discretized by the upwind scheme on non-uniform meshes and to the two-dimensional wave equation discretized by the Godunov scheme on triangular meshes. On rectangular meshes, we also study a variant of the Godunov scheme which is better adapted to the low-Mach regime. Several steps are used to obtain these results. First, the discrete solutions of the schemes are decomposed as sums of two elements: the first in the kernel of the discrete transport and wave operators and the other in the orthogonal of these kernels. Then it is verified whether these orthogonal subspaces are stable by the discrete operators. Finally, discrete Poincar inequalities in the orthogonal subspaces are used to obtain the dissipation lower bounds. The best provable lower bounds are in addition shown to be solutions of an eigenvalue problem. Numerical results illustrate the theory.

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