Unconditionally stable operator splitting method for a multiscale application in ophthalmology

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We are developing a Mathematical Virtual Simulator (MVS)[1] that combines multiscale mathematical modeling, advanced numerical techniques and efficient computational tools motivated by biomedical applications in the context of ophthalmology. The design of MVS is constituted by a three-dimensional poro-elastic system, representing the biomechanics and hemodynamics of the ocular posterior tissues, and a lumped-parameter circuit that describes the blood circulation in the retina and the blood flow in the central retinal vessels. Driven by the physics of the problem, which requires a blood systemic view, we do not impose artificial boundary condition for the hemodynamics; rather, we couple the description of the flow by means of a system of partial differential equations with a system of Ordinary Differential Equations (ODEs).

We have elaborated a novel splitting approach to numerically solve the multiscale problem in an efficient, accurate and affordable manner, which involves the coupling between Darcy equations and ODE systems. The algorithm is inspired by the work of Carichino et al.[2] and Glowinski[3] and it is based on a semi-discretization in time based on operator splitting in order to guarantee the physical energy balance at the discrete level. As a result, unconditional stability with respect to the time step choice is ensured by the implicit treatment of interface conditions within the Darcy sub-steps, whereas the coupling between the 3d and 0d sub-steps is enforced via appropriate initial conditions for each sub-step. Notably, unconditional stability is attained without the need of sub-iterating between Darcy and ODE sub-steps. In our case, the Darcy problem is solved using HDG (Hybridizable Discontinuous Galerkin) method to achieve high accuracy on fluxes, which have a fundamental role from a biological viewpoint.

In conclusion, we have formulated and implemented a numerical algorithm combining HDG discretization with an unconditionally stable operator splitting method for multiscale problems in order to simulate the physiological behavior of blood flow in the ocular posterior segment. More generally, the algorithm might be utilized for other problems arising in engineering and life sciences, where a robust and efficient coupling between a 3d poro-elastic model and a 0d fluid flow circuit described by a system of ODEs is required.

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Références

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