Boundary elements for contact problems: Stabilization, hp-methods, dynamics

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This talk considers recent work on boundary elements for the approximation of frictional contact problems. It focuses on stabilized hp-adaptive methods [1], as well as on first results for dynamic contact [2].
Boundary elements formulate the Signorini problem, possibly with friction, as a variational inequality for the Dirichlet-to-Neumann operator on the boundary. The resulting problem is then discretized and solved in a saddle point formulation.
As a basic problem of high-order polynomial ansatz functions, the discretized systems degenerate as the polynomial degree tends to infinity. We show that a basic idea by Barbosa and Hughes for the stabilization of finite element methods can be extended to boundary elements and obtain a priori and a posteriori error estimates for Tresca friction. Numerical experiments in $2d$ confirm the theoretical results and apply the approach to contact problems with Coulomb friction.
We then present first results with low-order ansatz functions for a dynamic contact problem for the wave equation. A priori estimates are obtained for Galerkin approximations in the case of a flat contact area, where the existence of solutions to the continuous problem is known. Numerical experiments in $3d$ demonstrate the performance of the proposed method. They indicate the stability and convergence beyond flat geometries. Future work will focus on space-time stabilization and adaptively generated meshes [3, 4].

Références

[1] Lothar Banz, Heiko Gimperlein, Abderrahman Issaoui and Ernst P. Stephan, *Stabilized mixed hp-BEM for frictional contact problems in linear elasticity*, Numerische Mathematik 135 (2017), 217 - 263.

[2] Heiko Gimperlein, Fabian Meyer, Ceyhun Ozdemir and Ernst P. Stephan, *Time domain boundary elements for dynamic contact problems*, Computer Methods in Applied Mechanics and Engineering 333 (2018), 147 - 175.

[3] Heiko Gimperlein, Fabian Meyer, Ceyhun Ozdemir, David Stark and Ernst P. Stephan, *Boundary elements with mesh refinements for the wave equation*, Numerische Mathematik (2018), 45 pages, to appear.

[4] Heiko Gimperlein, Ceyhun Ozdemir, David Stark and Ernst P. Stephan *A residual a posteriori estimate for the time domain boundary element method*, preprint (2018).

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