Justification of macroscopic traffic flow model by specified homogenization of microscopic models

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The goal of this talk is to present and to justify Hamilton-Jacobi formulation for macroscopic traffic flow model. The idea is to show how it is possible to deduce macroscopic models of traffic flow from microscopic ones. The main advantage of microscopic models (in which we describe the dynamics of each vehicle in an individual way) is that one can easily distinguish each vehicle and then associate different attributes (like maximal velocity, maximal acceleration...) to each vehicle. It is also possible to describe microscopic phenomena like red lights, slowdown or change of the maximal velocity. The main drawback is for numerical simulations where we have to treat a large number of data, which can be very expensive for example if we want to simulate the traffic at the scale of a town.
On the contrary, macroscopic models consist in describing the collective behaviour of the vehicles for example by giving an evolution law on the density of vehicles. The oldest macroscopic model is the LWR model (Lighthill, Whitham [3], Richards [4]), which dates back to 1955 and is inspired by the laws of fluid dynamics. More recently, some macroscopic models propose to describe the flow of vehicles in terms of the averaged spacing between the vehicles. The main advantage of these macroscopic models is that it is possible to make numerical simulations on large portion of road. On the other side, it is more complicated to describe microscopic phenomena or attributes.
Generally speaking, microscopic models are considered more justifiable because the behaviour of every vehicle can be described with high precision and it is immediately clear which kind of interactions are considered. On the contrary, macroscopic models are based on assumptions that are hardly correct or at least verifiable. As a consequence, it is often desirable establishing a connection between microscopic and macroscopic models so to justify and validate the latter on the basis of the verifiable modelling assumptions of the former.
The goal of this talk is to show how to pass from microscopic models to macroscopic ones. As we will explain, this problem can be seen as an homogenization result on a non-local Hamilton-Jacobi equation. More precisely, at the microscopic scale, we will consider a first order model of the type follow the leader , i.e., the velocity of a vehicle depends only on the distance with the one in front of it and we will consider a local perturbation located at the origin which make slow down the vehicles. At the macroscopic scale, we attend to recover an Hamilton-Jacobi equation on the right and on the left of the origin and a condition of junction at the origin (as studied in the work of Imbert and Monneau [2]). This junction condition allows us to see the influence of the microscopic perturbation at the macroscopic scale. We will also consider the case of a simple junction, i.e., one road that separates in several roads.
This is joint works with W. Salazar and M. Zaydan

Références

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