Reaction-diffusion models, constraints and control

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Reaction-diffusion equations are ubiquitous in a variety of fields including combustion and population dynamics.

There is an extensive mathematical literature addressing the analysis of steady state solutions, traveling waves, and their stability, among other properties.

Control problems arise in many applications involving naturally these models. Often times, control and/or state constraints emerge as intrinsic requirements of the processes under consideration.

There is also a broad literature on the control of those systems, addressing, in particular, issues such as the possibility of driving the system to a given final configuration in finite time. But, the necessity of preserving the natural constraints of the process are rarely taken into account.

In this lecture we shall present the recent work of our team on the Fisher-KPP and Allen-Canh or bistable model, showing results of two different types. First, the fact that constrained controllability for large enough time can be achieved. And, second, as a consequence, in particular, of the classical comparison principle for parabolic problems, negative results showing the existence of threshold effects so that some targets can never be achieved, or the existence of a waiting time for them to be reached. This is in sharp contrast with the unconstrained case where parabolic systems can be controlled in an arbitrarily small time.

This presentation is based on joint work with Jérôme Lohéac (CNRS-Nancy), Camille Pouchol and Emmanuel Trélat (LJLL-Sorbonne Univ.), Dario Pighin (UAM-Madrid) and Jiamin Zhu (Toulouse).

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