Some non-standard applications of the Raviart–Thomas–Nédélec element

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We consider here non-standard applications of the mixed finite element originally introduced in two space dimensions by Raviart and Thomas [5], and later extended to three space dimensions by Nédélec [6].

The first application is in the context of Hybrid High-Order (HHO) discretisations of the Brinkman equation. Specifically, we propose a novel discretisation of the Darcy term based on a Raviart–Thomas–Nédélec reconstruction of the velocity which warrants a robust behaviour in both the Darcy- and Stokes-dominated regimes, and also extends to the singular limit case corresponding to the Darcy problem. An important novelty in the analysis is that the local regime is identified by a dimensionless number which can be interpreted as a friction coefficient. An optimal error estimate in h^{k+1} (with h denoting, as usual, the meshsize) is proved.

The second application is the development of a stable gradient reconstruction in the context of HHO methods for linear and nonlinear scalar diffusion problems. HHO discretisations of such problems typically involve two types of contributions: standard Galerkin consistency terms involving a local gradient reconstruction, and stabilisation terms; see, e.g., [4, 2]. We show here that stabilisation can be incorporated into the gradient reconstruction by projecting suitable local residuals onto a Raviart–Thomas–Nédélec subspace. As a result, for both linear and nonlinear problems, a stable and convergent HHO discretisation can be obtained from the weak formulation of the continuous problem by simply replacing the continuous gradient operator with the corresponding discrete counterpart. This new construction also allows us to bridge HHO and other polytopal methods to the recently developed Gradient Discretisation framework.

Références

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