In this talk, I will present some results concerning the study of large deviations for velocity jump processes from a PDE point of view. The Chapman-Kolmogorov equation of the process being a kinetic equation, I will show how to perform an Evans-Souganidis/Freidlin type of approach directly at the kinetic level. The talk will also underline the differences between the results we obtain, and the classical results obtained for the macroscopic limit of the process (the heat equation).