Stokes and Navier-Stokes equations with Navier boundary condition

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We focus on the study of the incompressible fluid in a bounded domain in $\mathbb{R}^3$. We consider the stationary Stokes equation

$$-\Delta \boldsymbol{u} + \nabla \pi = \boldsymbol{f}, \quad \text{div } \boldsymbol{u} = 0 \ \text{ in } \Omega \tag{1}$$

and the stationary Navier-Stokes equation

$$-\Delta \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla \pi = \boldsymbol{f}, \quad \text{div } \boldsymbol{u} = 0 \ \text{ in } \Omega \tag{2}$$

where Ω be a bounded domain in $\mathbb{R}^3$ with boundary Γ, possibly not connected, of class $\mathcal{C}^{1,1}$ and $\boldsymbol{u}$ and π are the velocity field and the pressure of the fluid respectively, $\boldsymbol{f}$ is the external force acting on the fluid. A boundary condition was suggested by Navier (in 1823) which states that on one hand, the normal component of the fluid velocity is zero at the boundary (impermeability condition) and on the other hand, the amount of slip in the tangential part of the velocity, rather than being zero, is proportional to the tangential part of the normal stress exerted by the fluid on the boundary i.e.

$$\boldsymbol{u} \cdot \boldsymbol{n} = 0, \quad 2\left[(\mathbb{D}\boldsymbol{u})\boldsymbol{n}\right]_{\boldsymbol{\tau}} + \alpha\, \boldsymbol{u}_{\boldsymbol{\tau}} = \boldsymbol{0} \ \text{ on } \Gamma \tag{3}$$

where $\boldsymbol{n}$ and $\boldsymbol{\tau}$ are the unit outward normal and tangent vectors on Γ respectively and $\mathbb{D}\boldsymbol{u} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$ is the rate of strain tensor. Here, α is the coefficient which measures the tendency of the fluid to slip on the boundary, called *friction coefficient*. We are interested to discuss the well-posedness of the problem (1) and (2) along with (3), in particular existence, uniqueness of weak and strong solutions in $\boldsymbol{W}^{1,p}(\Omega)$ and $\boldsymbol{W}^{2,p}(\Omega)$ for all $1 < p < \infty$ considering minimal regularity on the friction coefficient α. Moreover, we deduce estimates to analyze the behavior of the solution with respect to α which indicates in some sense, an inverse of the derivation of the Navier boundary conditions from no-slip boundary condition for rough boundaries.

Références

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