A numerical method for a system of linear Boltzmann equations: Application to dose computation in radiotherapy

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We study the transport of relativistic particles interacting with a fixed medium. This problem emerges in radiotherapy. This type of cancer treatement consists in irradiating the patient with energetic particles (photons, electrons, postrons). Those particles are assumed to interact only with atoms in the medium and not with each others, their transport can be modeled by a system of linear Boltzmann equations ([1])

$$\begin{aligned}
\text{for} \quad b = 1, ..., N, \quad \Omega.\nabla_x \psi_b(x, \epsilon, \Omega) &= \rho(x) \left[\sum_{a=1}^{N} G_{a\to b}(\psi_a)(x, \epsilon, \Omega) - P_b(\psi_b)(x, \epsilon, \Omega) \right], \\
G_{a\to b}(\psi_a)(x, \epsilon, \Omega) &= \int_{S^2} \int_{\epsilon}^{\epsilon_{max}} \tilde{\sigma}_{a\to b}(\epsilon', \epsilon, \Omega'.\Omega) \psi_a(x, \epsilon', \Omega') d\Omega' d\epsilon', \\
P_a(\psi_a)(x, \epsilon, \Omega) &= \sigma_{T,a}(\epsilon) \psi_a(x, \epsilon, \Omega),
\end{aligned}$$

where ψ_b is the fluence of particles of type b. It depends on energy variable $\epsilon \in \mathbb{R}^+$, direction of flight $\Omega \in S^2$ and position $x \in \mathbb{R}^3$.
In order to reduce computational costs, we use the method of moments. This consists in studying the moments $\bar{\psi} = \int_{S^2} \bar{m}(\Omega) \psi d\Omega$, where $\bar{m}(\Omega)$ is a vector of polynomials of Ω, instead of studying the fluences. Moments depends on less variables than fluences, which makes their computation less costly. Moments equations take the form

$$\text{for} \quad b = 1, ..., N, \quad \nabla_x.F(\bar{\psi}_b)(x, \epsilon) = \rho(x) \left[\int_{\epsilon}^{\epsilon_{max}} \sum_{a=1}^{N} \sigma_{a\to b}(\epsilon', \epsilon) \bar{\psi}_a(x, \epsilon') d\epsilon' - \sigma_{T,b}(x, \epsilon) \bar{\psi}_b(x, \epsilon) \right],$$

where the fluxes $F(\bar{\psi})$ are computed using an entropy minimization principle ([2, 3]).
We propose a numerical approach to solve this system, based on relaxation methods for hyperbolic PDE ([4, 5]). It consists of an implicit scheme (in particular it is not constrained by a CFL condition), adapted to moments equations (because it preserves realizable states) and which satisfies conservation of mass, momentum and energy at the discrete level.

Références

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