Augmented Lagrangian Method for Optimal Partial Transport Problem

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The theory of optimal transport [7] deals with the problem to find the optimal way to move materials from a given source to a desired target in such a way to minimize a certain work. The problem was first proposed by Monge in 1781 and then Kantorovich made fundamental contributions to the problem in the 1940s. Since the late 80s, this subject has been investigated under various point of views with many applications in economics, fluid mechanics, PDE, optimization, geometry and other areas.
The standard optimal transport requires that the total mass of the source must be equal to the total mass of the target (balance condition of mass) and that all materials of the source must be transported. The balance condition of mass is excluded in optimal partial transport, which is a natural generalization of optimal transport problem, consisting in transporting effectively a prescribed amount of mass from the source to the target. More precisely, the optimal partial transport problem aims to study the case where only a part of the commodity (resp. consumer demand) of prescribed total mass $\mathbf{m}$, needs to be transported (resp. fulfilled). This generalization problem arose new variables and was first studied theoritically in [3] (see also [5]). However, numerical methods for the optimal partial transport are underdeveloped. In this work, we show how to reformulate unknown quantities (variables) of the optimal partial transport into an infinite dimensional minimization problem of the form:

$$\min_{v \in V} \mathcal{F}(u) + \mathcal{G}(\Lambda u),$$

where $\mathcal{F}, \mathcal{G}$ are l.s.c., convex functionals and $\Lambda \in \mathcal{L}(V, Z)$ is a linear operator between two Banach spaces. Thanks to peculiar properties of $\mathcal{F}$ and $\mathcal{G}$ in our situation, an augmented Lagrangian method is applied effectively. Numerical results validate the approach.

Références

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