A numerical method for a depth averaged Euler system in two dimensions

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We propose a numerical method for a two dimensional dispersive shallow water system with topography (see [3] for the derivation of the model). This model is a depth averaged Euler incompressible system with free surface and takes into account a non-hydrostatic pressure which implies to solve an elliptic equation of type Sturm-Liouville. The contribution of the dispersive terms leads to having a difficult problem which requires to develop new numerical methods. A first approach in one dimension, based on an operator splitting procedure in the spirit of the prediction-correction method initially introduced by Chorin-Temam (see [5]), has been presented in [1] and [2] where stability properties together with the ability to treat dry/wet interfaces have been established. The prediction part leads to solving a shallow water system for which we use finite volume methods (see [4]), while the correction part leads to solving a mixed problem in velocity and pressure. Then, from the variational formulation of the mixed problem proposed in [2], the idea is to apply a finite element method with compatible spaces to the two dimensional problem on unstructured grids. In addition, to deal with the significant computational cost, an iterative method is used with appropriate boundary conditions. Taking the velocity/pressure in the spaces $\mathbb{P}_1$-iso-$\mathbb{P}_2/\mathbb{P}_1$, comparisons with analytical solutions and classical test cases are performed to evaluate the efficiency of our method. This analysis is done in collaboration with M.-O. Bristeau, E. Godlewski and J. Sainte-Marie.

Références

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