

# An $\mathcal{H}$ -matrix based direct solver for Boundary Element Method in 3D elastodynamics

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The Boundary Element Method (BEM) is well suited to treat seismic wave propagation problems in semi-infinite regions. Although the resulting matrices are fully-populated, it is possible to find evaluation procedures that are fast and avoid their explicit storage, in order to compress the information and give a data-sparse representation. In the present work, we propose a fast method to accelerate the direct solution of the BEM based analysis of 3D frequency-domain elastodynamic problems, using the  $\mathcal{H}$ -matrix arithmetic and low-rank approximations (performed with Adaptive Cross Approximation, ACA). We assess the numerical efficiency and accuracy on the basis of numerical results obtained for problems having known solutions. In particular, we study the efficiency of the low-rank approximations when the frequency is increased. We show that the existing complexity analysis of the  $\mathcal{H}$ -matrix approximation, which is based on a constant rank, is not proper for analyzing elastodynamic problems. The efficiency of the method is also illustrated to study seismic wave propagation in 3D-domains..

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