A 2D-kinetic scheme on staggered grids for the Euler system

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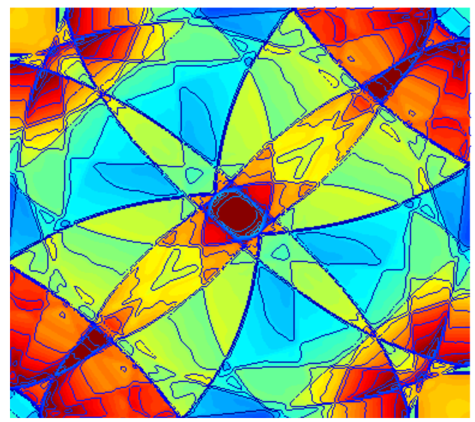
Motivated by the numerical simulation of multi-fluid models, F. Berthelin, T. Goudon, S. Minjeaud have introduced in [1, 2] a family of schemes for 1D barotropic Euler equations

$$\partial_t \begin{pmatrix} \rho \\ \rho u \end{pmatrix} + \partial_x \begin{pmatrix} \rho u \\ \rho u^2 + p(\rho) \end{pmatrix} = 0,$$

that works on staggered grids, with a kinetic flavor in the definition of the fluxes. The densities $\rho_{j+\frac{1}{2}}$ are evaluated at the centers $x_{j+\frac{1}{2}}$ of the cells and the velocities V_j at the edges x_j. The mass flux $\mathcal{F}_j$ is defined, by using some kinetic setting and the upwinding principle, as an explicit function of the variables $\rho_{j-\frac{1}{2}}$, $\rho_{j+\frac{1}{2}}$ and V_j. The momentum flux $\mathcal{G}_{j+\frac{1}{2}}$ is written by applying the upwinding principle, based on the sign of the mass fluxes $\mathcal{F}_j$ and $\mathcal{F}_{j+1}$, to the velocity field and centering the pressure. Our work is twofold: on one hand we upgrade this 1^{st} order accurate method to 2^{sd} order, and on the other hand we extend this scheme to the 2D framework on a Cartesian mesh.

We adapt the usual MUSCL procedure to the staggered grids and choose to work with the physical variables instead of the conservative ones. Concerning the discretisation of the mass flux, instead of $\rho_{j\pm\frac{1}{2}}$, we use a MUSCL reconstruction $\rho_j^{\pm}$ of the density at the interfaces x_j to define the upgraded mass flux $\mathcal{F}_j^{ML}$. To define the upgraded momentum flux $\mathcal{G}_{j+\frac{1}{2}}^{ML}$ we combine the flux $\mathcal{F}_j^{ML}$ and $\mathcal{F}_{j+1}^{ML}$ with a MUSCL reconstructed velocity $V_{j+\frac{1}{2}}^{\pm}$ at the interfaces $x_{j+\frac{1}{2}}$. The scheme we obtain -by replacing $\mathcal{F}$ and $\mathcal{G}$ by their upgraded MUSCL versions $\mathcal{F}^{ML}$ and $\mathcal{G}^{ML}$- is formally 2^{nd} order accurate.



Concerning the 2D structured framework, the densities $\rho_{i+\frac{1}{2},j+\frac{1}{2}}$ are evaluated at the centers of the cells while, for the velocity, we store only the normal components $U_{i,j+\frac{1}{2}}$ and $V_{i+\frac{1}{2},j}$ at the edges. We extend the 1D definition of the mass and momentum fluxes to this 2D framework.

Finally, using the Shallow Water model, we confront our results to falling water columns simulations inspired by N. Aguillon [3], see figure above.

Références

[1] Florent Berthelin, Thierry Goudon, Sebastian Minjeaud., *Kinetic schemes on staggered grids for barotropic Euler models: entropy-stability analysis*, hal-00858252v2, 2014.

[2] Florent Berthelin, Thierry Goudon, Sebastian Minjeaud., *Consistency analysis of a 1D Finite Volume scheme for barotropic Euler models*, hal-00961615, 2014.

[3] Nina Aguillon. , *Problemes d´ interfaces et couplages singuliers dans les systemes hyperboliques : analyse et analyse numérique*, NNT : 2014PA112248, 2014.

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