Almost Exact Simulation of Feller Processes using their Resolvent Kernels

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Feller processes [1] are involved in various applications such as geophysics, brain imaging, etc... While many numerical methods exist to simulate Feller processes with regular coefficients, it is a challenge to simulate them when singularities occur in the coefficients. A very relevant example is the case of piecewise continuous coefficients. This situation is encountered in geophysics when one wants to simulate the transport of pollutants in highly heterogeneous media [2, 4].
In this presentation, we provide a solution to the problem presented above. To be precise, we introduce an algorithm for simulating Feller processes which uses their resolvent kernel. This algorithm lets us deal with various type of irregular coefficients. Afterwards, we use it to provide an almost exact simulation scheme of a Feller process with piecewise constant coefficients. In effect, a recent method for the computation of resolvent kernels for singular coefficients which lets one obtain a closed form for piecewise constant coefficients was recently provided in [3].
At this point, one could consider this algorithm narrowly since the example we provided has a resolvent kernel with an analytical expression. This is without taking into account that the algorithm can be used with a truncated power series or a numerical approximation of the resolvent kernel. We discuss this point deeply after presenting the algorithm and the example.

Références

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