A fully justified proof of parameter identifiability in an ODE system

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Mots-clés : identifiability, parameter, HIV

First, we recall some usual definitions of identifiability (see [5]) and motivate them. Our work does not rely on the domain (biology here).
Then we take as an example the most classical model in HIV for which we define $T(t), U(t), V(t)$ the amount of uninfected lymphocytes, of infected lymphocytes and of virus (respectively) depending on time t. Using the parameter vector $\theta = (\lambda, \rho, \beta, \delta, N, c)$, one may state the ODE system (see [7] for instance, but this system appeared since 1995 : [1], [6]):

$$\begin{array}{ll} T'(t) = & \lambda - \rho T - \beta\, T\, V \\ U'(t) = & +\beta\, T\, V - \delta\, U \\ V'(t) = & N\delta U - c\, V \\ (IC) & T(0) = T_0, U(0) = U_0, V(0) = V_0. \end{array} \tag{1}$$

We discuss why this system is not yet a model in the sense of automatic.
Thanks to François Ollivier [2], we complete the proof of [7] that not all the parameters are (globally) identifiable. We use basic theorems so as to get a (very) reduced system of equations. We prove identifiability in a non-trivial model. In the case of local identifiability, one could use symbolic computation of [4] and even its associated software in MAPLE or DAISY (cf. [3]) in REDUCE.

Then we state a general theorem that provides a new method to prove identifiability (SGI and SLI) of a model's parameter.

Références

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