On the gap between RIP-properties and sparse recovery conditions

Guillaume LECUÉ, CNRS - ENSAE

Sjoerd DIRKSEN, RWTH Aachen University

Shahar MENDELSON, Technion and ANU

Holger RAUHUT, RWTH Aachen University

Mots-clés : Compressed Sensing, Random matrices, empirical processus

We prove that iid random vectors that satisfy a rather weak moment assumption can be used as measurement vectors in Compressed Sensing, and the number of measurements required for exact reconstruction is the same as the best possible estimate exhibited by a random Gaussian matrix. We then show that this moment condition is necessary, up to a log log factor.
In addition, we explore the noisy setup and consider the problem of recovering sparse vectors from underdetermined linear measurements via ℓ_p-constrained basis pursuit. Previous analyses of this problem based on generalized restricted isometry properties have suggested that two phenomena occur if $p \neq 2$. First, one may need substantially more than $s\log(en/s)$ measurements (optimal for $p = 2$) for uniform recovery of all s-sparse vectors. Second, the matrix that achieves recovery with the optimal number of measurements may not be Gaussian (as for $p = 2$). We present a new, direct analysis which shows that in fact neither of these phenomena occur. Via a suitable version of the null space property we show that a standard Gaussian matrix provides ℓ_q/ℓ_1-recovery guarantees for ℓ_p-constrained basis pursuit in the optimal measurement regime. Our result extends to several heavier-tailed measurement matrices. As an application, we show that one can obtain a consistent reconstruction from uniform scalar quantized measurements in the optimal measurement regime.
This talk is based on the two publications [1][2].

Références

[1] Lecué Guillaume and Shahar Mendelson, *Sparse recovery under weak moment property*, To appear in *Journal of the European mathematical society*, 2015.

[2] Sjoerd Dirksen, Guillaume Lecué and Holger Rauhut, *On the gap between RIP-properties and sparse recovery conditions*, To appear in *IEEE Transactions on Information Theory*, 2015.

Guillaume LECUÉ, CNRS - ENSAE - CREST 3 avenue Pierre Larousse 92240 Malakof. France
guillaume.lecue@ensae.fr
Sjoerd DIRKSEN, Lehrstuhl C für Mathematik (Analysis), RWTH Aachen University, Pontdriesch 10, 52062 Aachen, Germany
dirksen@mathc.rwth-aachen.de
Shahar MENDELSON, Technion - Israel Institute of Technology Technion City, Haifa 32000
shahar@techunix.technion.ac.il
Holger RAUHUT, Lehrstuhl C für Mathematik (Analysis), RWTH Aachen University, Pontdriesch 10, 52062 Aachen, Germany
rauhut@mathc.rwth-aachen.de