High order stabilized finite element method for MHD and Reduced-MHD plasma modelling

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The understanding of Magnetohydrodynamic (MHD) instabilities is quite essential for the optimization of magnetically confined plasma. For example, the ITER (International Thermonuclear Experimental Reactor) scenario is expected to generate oscillations in the plasma core, modes around the outward limit of the plasma confinement zone, or local reconfiguration of the magnetic field topology. Numerical simulations play an important role in the investigation of the non-linear behavior of these instabilities and the interpretation of experimental observations. The study of plasma instabilities requires the use of high order methods which can be difficult in the context of finite volume (FV). Hence, the use of finite element methods seems to be reasonable for numerical precision, adaptability and flexibility to complex geometries. Moreover, High order smooth (C1) finite element can be combined with the potential formulation to achieve the solenoidal condition of the magnetic field. As MHD instabilities are usually dominated by convection, numerical schemes must take into account the effects of unresolved scales in order to insure stability of the numerical approach. In this context, stabilized finite element method (FEM) can provide a useful framework for the numerical approximation.

Galerkin finite element gives rise to a centered approximation of differential operators. This is suitable for elliptic like operators but can lead to nonphysical behaviors when flows are dominated by the effect of hyperbolic operators (convection). The variational multi-scale (VMS) formulation provides attractive guidelines for the development of stabilized schemes that take into account the hyperbolic nature of the considered systems. In this frame of work, stabilization is achieved by an additional contribution to the weak formulation which mimics the effects of the unresolved scales over the resolved ones. The critical point of this strategy is the design of a scaling matrix used to adjust the numerical dissipation such as to preserve the order of accuracy of the Galerkin method. This is very good for smooth solutions however it might generate spurious Gibbs oscillations associated to spectral truncation in the wave-number space. Hence, discontinuity capturing is also often used to enforce the total variation stability where the solution develops sharp gradients.

We will present applications to MHD and Reduced-MHD in tokamak (toroidal) geometry using the non-linear finite element code JOREK where the VMS stabilization will be discussed in terms of the Taylor-Galerkin formulation.

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