Simulations of control for Navier-Stokes equations

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In many areas of science, we aim to drive a system (biological, chemical, physical, ...) to a desired state or behavior. The ability to do this in a manner is called controllability. In this communication, we address the problem of controlling the wake of an incompressible fluid past an obstacle (in 2D), by blowing–suction localised on the boundary of the obstacle.
More precisely, we consider the incompressible Navier-Stokes equation on $\Omega \subset \mathbb{R}^2$, an open bounded set, with control localised on a part of the boundary Γ, to stabilize the fluid on an unstable stationnary state.

The velocity z and the pressure q of a (newtonian) incompressible viscous fluid inside the domain Ω satisfy

$$\begin{cases} \dot{z} + (z \cdot \nabla)z - \nu \Delta z + \nabla q = 0, \\ \mathrm{Div}(z) = 0, \end{cases}$$

together with suitable initial and boundary conditions.

We consider the following configuration, from the CARPE Project[1].

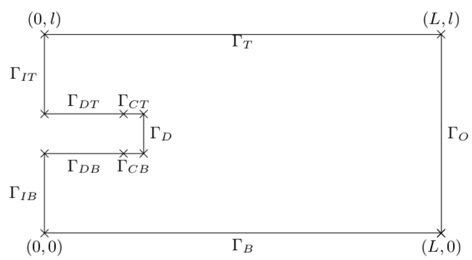

Figure 1: The domain Ω.

The boundary conditions are the following. On the inlet : $z_1 = g$ and $z_2 = 0$ on $\Gamma_{IT} \cup \Gamma_{IB}$, with a suitable profile g, choosen to be an approximation of the Blasius boundary layer profile. On the outlet: $\sigma(z,q) \cdot n = 0$ on $\Gamma_T \cup \Gamma_O \cup \Gamma_B$, where σ is the stress tensor. We complete boundary conditions by the Dirichlet one. In the uncontrolled case, this will only be homogeneous Dirichlet boundary condition. In the controlled case, we will apply a control on Γ_{CT} and Γ_{CB}, that is non-homogeneous Dirichlet condition.

In this work, we will briefly present the theoretical result of Raymond [1], which says that our problem is well-posed and gives a way to compute the (optimal) control (which will be of feedback form) and then focus on the discretization of the problem.

Références

[1] Raymond, JP, *Feedback boundary stabilization of the two dimensional navier-stokes equations*, (2006), SIAM Journal on Control and Optimization, 45(3), 790828.

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[1] The CARPE Project