Shape optimisation of an obstacle with respect to the linear wave-making resistance

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In this study, our goal is to minimise the wave-making resistance (and hence the amplitude of the wake) of an obstacle moving in a fluid with a constant velocity. This problem arises in various situations, both in theoretical physics and in engineering. The application in theoretical physics we are interested in is the experimental setup of the hydrodynamic analogue to the Hawking radiation phenomenon [1]. In the field of naval architecture, the reduction of the wave-making resistance by the optimisation of the hull's shape is an approach to the reduction of fuel consumption [2].

In this research, we simulate the solution of the free surface problem with a smooth body immersed in a semi-infinite domain with a uniform motion. The fluid is assumed to be inviscid, incompressible and irrotational. Furthermore, for the sake of simplicity, we use a linearised version of the steady free-surface Euler equations with a uniform motion known as the Kelvin-Neumann equations:

$$\begin{cases} \Delta\phi = 0\,, & \text{in } (\mathbb{R}\times\mathbb{R}^-)\setminus\overline{\Omega}\,, \\ \partial^2_{xx}\phi + (g/U^2)\partial_z\phi = 0\,, & \text{on } z=0\,, \\ \partial_{\vec{n}}\phi = -U\,n_x\,, & \text{on } \Gamma\,, \end{cases} \tag{1}$$

where ϕ is the velocity potential, U the velocity of the obstacle Ω (with respect to the fluid) whose border is denoted Γ and $\vec{n}$ is the outward unit normal vector.

As in [3] and [4], we will use a boundary element approach in order to solve numerically the above problem. This method deals with a Green function which is designed to satisfy the linearised free surface condition. Hence the resulting integral formulation only involves the boundary of the obstacle. Once we recover the velocity potential ϕ on the boundary, we can compute the wave-making resistance easily with an integral on Γ:

$$R_w(\Gamma) = \int_\Gamma |\nabla\phi - U\vec{e_x}|^2 n_x \,\mathrm{d}\Gamma$$

Our approach for the shape optimisation problem follows the method of boundary variations described in [5]. A gradient of the wave-making resistance R_w with respect to the shape Γ is computed. This allows us to obtain a descent direction, which in our case provides a deformation field for the obstacle.

Références

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