Adaptive Collocation Schemes for Conservation Laws

Erwan DERIAZ, IJL - CNRS - Univérsité de Lorraine

Mots-clés : Adaptive Mesh Refinement, Collocation method, Interpolets, Multigrid, Wavelets

When applying adaptive collocation schemes [1] to conservative laws, we face the following problem: how to conserve the circulating quantity? Collocation schemes, as for instance Finite Differences, compute the solution to a partial differential equation through point values. Hence, contrarily to the Finite Volumes schemes, they do not make fluxes appear naturally. And generally they do not ensure the mass conservation.

However, Collocation schemes have the advantage to easily open the way to high order schemes with an adaptive mesh refinement [2, 3], while increasing the order of Finite Volumes inside an adaptive mesh refinement is a very difficult challenge.

If we consider a set of points (x_i) associated to volumes (l_i) containing a quantity (u_i) circulating according to the conservation law

$$\partial_t u + \partial_x (a(x)u) = 0 \tag{1}$$

and discretized in space as

$$\forall i, \qquad \partial_t u_i + \frac{1}{l_i} \sum_j \alpha_{ij} a(x_j) u_j = 0, \tag{2}$$

where (α_{ij}) corresponds to p-th order finite differences –for instance, assuming $l_i = h$ for all i, and second order finite differences $\alpha_{i\,i+1} = 1/2$, $\alpha_{i\,i-1} = -1/2$, this is written:

$$\frac{1}{l_i} \sum_j \alpha_{ij} a(x_j) u_j = \frac{a_{i+1}u_{i+1} - a_{i-1}u_{i-1}}{2h},$$

then we are able to derive conditions on (x_i, l_i) which garanty both the mass conservation and the order of the scheme.

The interpolation schemes used in an adaptive mesh refinement enforce the values of volumes (l_i). And a second result shows that, given a q-th order interpolation scheme, it is possible to provide a $(q-1)$-th order conservative collocation scheme.

Applications to Plasma Physics and Astrophysics simulations illustrate this approach.

Références

[1] A. Baeza, A. Martnez-Gavara, P. Mulet, *Applied Numerical Mathematics* **62** 278–296 (2012).

[2] S. Bertoluzza, *Wavelet Analysis and Its Applications* **6** 109–135 (1997).

[3] E. Deriaz et V.Perrier, *SIAM Multiscale Modeling and Simulation*, **7**(3) 1101–1129 (2008).

Erwan DERIAZ, Institut Jean Lamour, Matériaux-Métallurgie-Nanosciences-Plasmas-Surfaces UMR 7198 - CNRS - Université de Lorraine, Faculté des Sciences et Technologies, Campus Victor Grignard - BP 70239, 54506 VANDOEUVRE-LES-NANCY CEDEX
erwan.deriaz@univ-lorraine.fr