AP schemes for kinetic models of mixtures with diffusion limit

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Kinetic and fluid equations are among the main tools for modelling complex phenomena occurring in most sciences. Their mathematical and numerical study is a major scientific challenge since they often contain small parameters. Asymptotic expansions on these parameters usually yields asymptotic equations that describe the averaged, effective, or macroscopic equations. These asymptotic regimes imply computational challenges since one needs to numerically solve these small scales which can be very expensive. If the asymptotic or macroscopic equations are uniformly valid in the entire domain of interest, it is much more efficient just to solve them. However, there are many problems where the macroscopic models break down in part of the domain, thus the microscopic models are needed, at least locally.
The asymptotic-preserving (AP) approach has been developed for a wide range of time-dependent kinetic and fluid (mainly hyperbolic) equations. The basic idea is to develop numerical methods that preserve the symptotic limits from the microscopic to the macroscopic models, in the discrete setting. AP schemes allow to solve one set of equations, the microscopic ones, and avoid to tackle the coupling of different kind of models. They provide robust macroscopic solvers automatically when, in the asymptotic regimes, the small scales are not numerically solved.
In particular here we will focus our study on a kinetic model of mixtures in a diffusive scaling of the form

$$\varepsilon \partial_t f_i^\varepsilon + v \cdot \nabla_x f_i^\varepsilon = \frac{1}{\varepsilon} Q_{ii}(f_i^\varepsilon, f_i^\varepsilon) + \frac{1}{\varepsilon} \sum_{j \neq i} Q_{ij}(f_i^\varepsilon, f_j^\varepsilon), \quad \text{on} \quad \mathbb{R}_+^* \times \Omega \times \mathbb{R}^3,$$

with classical mono-species and bi-species Boltzmann-like collisional operators, where the aim is to develop suitable AP schemes that are able to capture the correct hydrodynamical limit of Maxwell-Stefan type for these equations (see [1] and [2]).

Références

[1] L. Boudin, B. Grec, F. Salvarani, *The Maxwell-Stefan diffusion limit for a kinetic model of mixtures*, Acta Applicandae Mathematicae, 2014.

[2] F. Filbet, S. Jin, *A class of asymptotic-preserving schemes for kinetic equations and related problems with stiff sources*, Journal of Computational Physics, 2010.

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