Derivation and properties of a depth-averaged Euler system.

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The Saint-Venant system [2], often referred to as the non-linear Shallow Water equations, models the dynamics of a shallow, rotating layer of a homogeneous incompressible fluid and is typically used to describe vertically averaged flows in two or three dimensional domains in terms of horizontal velocity and depth variations. This set of equations is particularly well-suited for the study and numerical simulation of a large class of geophysical phenomena such as rivers, coastal domains, oceans or even run-off or avalanches depending on relevant source terms [1].
In one space dimension, the axis x corresponding e.g. to the favored direction of a river, the Saint-Venant system reads

$$\begin{aligned}
&\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}(H\overline{u}) = 0,\\
&\frac{\partial (H\overline{u})}{\partial t} + \frac{\partial}{\partial x}\left(H\overline{u}^2 + \frac{g}{2}H^2\right) = -gH\frac{\partial z_b}{\partial x},
\end{aligned}$$

where $H = \eta - z_b$ is the water depth, η the free surface height, z_b the bottom topography, g the gravity acceleration and $\overline{u}$, the mean fluid velocity averaged along the vertical direction
The derivation of the Saint-Venant system from the Navier-Stokes equations uses the hydrostatic assumption that consists in neglecting the vertical acceleration of the fluid. This assumption is valid for a large class of geophysical flows but is restrictive in various situations where the dispersive effects (like wave propagation) cannot be neglected.
During this presentation we present an extension of the Saint-Venant system including the non-hydrostatic/dispersive terms. The derivation process and the main properties of the proposed model are given. Especially we compare the model to the so-called Green-Naghdi model and confront them to analytical solutions. A numerical scheme for the resolution of the non-hydrostatic model and based on a kinetic finite volume solver is also presented.
Analysis results for this non-hydrostatic model will be given in the poster of Dena Kazerani.

Références

[1] F. Bouchut *Nonlinear stability of finite volume methods for hyperbolic conservation laws and well-balanced schemes for sources.* Birkhäuser, 2004.

[2] A.-J.-C. Barré de Saint-Venant Théorie du mouvement non permanent des eaux avec applications aux crues des rivières et à l'introduction des marées dans leur lit. *C. R. Acad. Sci. Paris*, 73:147–154, 1871.

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