3D direct and inverse solver of eddy current testing.

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We consider the inverse problem of estimating the shape profile of an unknown deposit on the exterior of stream generator (SG) tubes from a set of eddy current impedance measurements due to coils located in the interior of the tubes. We shall address the problem in a 3D setting to treat the case where the deposits are located in the vicinity of the support plates. Numerical validating experiments on synthetic deposits with different shapes will be presented.

The ECT is based on the analysis and processing of impedance signal $\mathbf{Z}(\Omega_d)$ measured during a scan procedure of SG tube. Numerically, the impedance measured for the coil k in the electromagnetic field induced by the coil l is computed as follows: (see [2] for more details):

$$\begin{aligned} \Delta\mathbf{Z}_{kl} =& \frac{1}{|\mathbf{J}|}\left(\frac{\mu_0-\mu_d}{i\omega\mu_d\mu_0}\int_{\Omega_d}\left(\mathbf{curl}\mathbf{E}_k\,.\,\mathbf{curl}\mathbf{E}_l^0\right)\delta v\right. \\ & \left. +(\sigma_0-\sigma_d)\int_{\Omega_d}\mathbf{E}_k\,.\,\mathbf{E}_l^0\,\delta v\right). \end{aligned} \tag{1}$$

Where $\mathbf{E}=i\omega\mathbf{A}+\nabla\mathbf{V}_c$ stands for the electric field solution of the time Harmonic Eddy-current equation (see for instance [1]), with $\mathbf{A}$ and $\nabla\mathbf{V}_c$ are the magnetic vector potential and the electric potential respectively. In industrial applications one uses different combinations of $\Delta\mathbf{Z}_{kl}$ for a given frequency $\omega = 100kHz$. We shall use and compare different combinations of the impedance: $\mathbf{Z}_{FA}=\Delta\mathbf{Z}_{11}+\Delta\mathbf{Z}_{21}$ and $\mathbf{Z}_{F3}=\Delta\mathbf{Z}_{11}-\Delta\mathbf{Z}_{22}$. The inverse problem aims at minimizing the misfit cost function $\mathcal{J}(\Omega_d)=\int_{z_{\min}}^{z_{\max}}|\mathbf{Z}(\Omega_d;\zeta)-\mathbf{Z}_{mes}(\zeta)|^2d\zeta$, where $\mathbf{Z}$ is either $\mathbf{Z}_{FA}$ or $\mathbf{Z}_{F3}$ and Ω_d denotes the deposit domain. We shall present an inversion algorithm based on steepest gradient descent. Prior to this we shall rigorously define and characterize the shape gradient $\mathcal{J}'(\Omega_d)$. Using the adjoint technique this derivative is then expressed as $\mathcal{J}'(\Omega_d)(\boldsymbol{\theta})=-\frac{\omega}{I^2}\int_{\Gamma_0}(\boldsymbol{\nu}^t\boldsymbol{\theta})gds$ where the computation of the the function g involves the solution of the direct and the adjoint problem. In the shape gradient formulae, θ represents the transformation field and $\boldsymbol{\nu}$ stands for the outward normal. The solutions of the adjoint problem is expressed with P and W as the magnetic vector potential and the scalar electric potential respectively. The function g may have the form $g=g_{11}+g_{21}$ in the absolute mode or $g=g_{11}-g_{22}$ in the differential mode, with

$$\begin{aligned} g_{kl}=\int_{z_{\min}}^{z_{\max}} & \Re\left(\left(\overline{\mathbf{Z}(\Omega_D;\zeta)-\mathbf{Z}_{mes}(\zeta)}\right)\left\{\left[\tfrac{1}{\mu}\right]_{\pm}(\boldsymbol{\nu}\cdot\mathbf{curl}A_k)(\boldsymbol{\nu}\cdot\overline{P_l}-\boldsymbol{\nu}\cdot\mathbf{curl}\mathbf{A}_l^0)\right.\right. \\ & -[\mu]_{\pm}\left(\boldsymbol{\nu}\times(\tfrac{1}{\mu}\mathbf{curl}\mathbf{A}_k\times\boldsymbol{\nu})\right)\cdot\left(\boldsymbol{\nu}\times(\tfrac{1}{\mu}\mathbf{curl}\overline{P_l}\times\boldsymbol{\nu})-\boldsymbol{\nu}\times(\tfrac{1}{\mu^0}\mathbf{curl}\mathbf{A}_l^0\times\boldsymbol{\nu})\right) \\ & \left.\left.+\tfrac{1}{i\omega}[\sigma]_{\pm}(i\omega\mathbf{A}_{k\tau}+\nabla_\tau\mathbf{V}_k)\cdot(\overline{i\omega P_{l\tau}+\nabla_\tau W_l}+i\omega\mathbf{A}_{l\tau}^0+\nabla_\tau\mathbf{V}_l^0)\right\}\right)d\zeta. \end{aligned}$$

We shall present and compare two inversion strategies : the first one is based on a parametrized regularization of the shape and the second one is based on a regularized descent direction.

Références

[1] Ana Alonso Rodriguez and Alberto Valli. *Eddy current approximation of Maxwell equations*, volume 4 of MS&A. Modeling, Simulation and Applications. Springer-Verlag Italia, Milan, 2010. Theory, algorithms and applications.

[2] B. A Auld, J. C. Moulder. *Review of Advance in Quantitative Eddy Current Nondestructive Evaluation.* Journal of Nondestructive Evaluation, Vol. 18, No.1 1999.

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