Fast Variational Data Assimilation for Parametrized Partial Differential Equations: Weak Formulations, Error Estimates, and Greedy Methods

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We present a Parametrized-Background Data-Weak formulation of the variational data assimilation (state estimation) problem for partial differential equations. The main contributions are a constrained optimization/weak framework informed by the notion of experimentally observable spaces; *a priori* and *a posteriori* error estimates for the field and associated linear-functional outputs; stability-informed choice of observation functionals and related sensor locations; weak greedy construction of prior (background) spaces associated with an underlying and potentially high–dimensional parametric manifold; and finally, output prediction from the optimality saddle in $\mathcal{O}(M^3)$ operations, where M is the number of experimental observations. We present results for several examples: (synthetic) model problems to confirm the numerical properties suggested by theory; a (synthetic) Boussinesq unsteady natural convection problem to illustrate extension to time-dependent nonlinear equations; and finally, a (physical) raised-box acoustic resonator problem to demonstrate incorporation of real data. Work in collaboration with Yvon Maday, James Penn, Tommaso Taddei, and Masa Yano.

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