Modeling and Calculating Hydrodynamic Solitary Waves Generated by a Piston Wave Maker in a Horizontal Channel

Abdelhamid LAOUAR,
Department of Mathematics, Faculty of Sciences,.Badji Mokhtar University, P.O. Box 12, 23000 Annaba, Algeria.
e-mail: laouar.abdelhamid@univ-annaba.org

Abstract

In this work, we investigate hydrodynamic solitary waves (solitons) and their characteristics (amplitude, velocity,...) generated by a piston wave maker placed at upstream of a horizontal channel. The mathematical model uses both the irrotational flow of incompressible no viscous fluid, the boundary conditions, the wave maker condition, and the Lagrangian variables. Introducing a double distortion and approximating at fourth order lead to KDV equation

$$\frac{\partial f}{\partial s}(r,s) - 6f\frac{\partial f}{\partial r}(r,s) + \frac{\partial^3 f}{\partial r^3}(r,s) = 0, \tag{1}$$

the indices s and r indicate differentiation with respect to space and time variables respectively.

It is known that the KdV equation admits as particular solution a solitary wave (see [7, 8]). Using the initial potential $f(r,0)$ and the inverse Scattering transformation approach bring us to the Sturm-Liouville spectral problem. The latter can be solved numerically by Runge-Kutta method. For illustration, we consider two types of wave maker movement which generate same waves. Finally, the numerical results are presented to support the theory.

Références

[1] M. J. Ablowitz and P. A. Clarkson, Solitons, nonlinear evolution equations and inverse scattering, Cambridge University Press, Cambridge, 1991.

[2] R. Courant and D. Hilbert, Methods of Mathematical Physics, Vol (I). New York, intersciences Pub, 1953.

[3] C. S. Gardner, J. M. Greene, M. D. Kruskal and R. M. Miura, Method for solving the Korteweg-de Vries equation, Phys. Rev. Lett. 19, 1095–1097 (1967).

[4] R. Grimshaw,Solitary Waves in Fluids. Advances in Fluid Mechanics , Vol 47, WIT Press, UK.(ed.) 2007.

[5] A. Laouar and A. Guerziz, Numerical Simulation of the Field Velocities and Local Disturbances of a Long Gravity Wave Passing above an Immersed Vertical Barrier, Differential Equations and Nonlinear Mechanics, Volume 2008, Article ID 135982, pages11, 2008

[6] X. Li, M. Wang, A sub-ODE method for finding exact solutions of a generalized KdV–mKdV equation with high-order nonlinear terms, Phys. Lett. A 361 (2007) 115, 2007.

[7] A. Miranville and R.Temam, Modélisation mathématique et mécanique des milieux continus, Springer

[8] A. Temperville, Contribution à la théorie des ondes de gravité en eau peu profonde, Thèse d'état de mathématiques, Université de Grenoble (1985).

[9] E. Tzirtzilakis, V. Marinakis, C. Apokis, T. Bountis, Soliton-like solutions of higher order wave equations of Korteweg–de Vries type, Journal of Mathematical Physics 43 (12) (2002) 6151–6161.

Allaoua GUERZIZ , Department of Physics, Faculty of Sciences.Badji Mokhtar University, P.O. Box 12, 23000 Annaba, Algeria.
allaoua.guerziz@univ-annaba.org
Aicha BOUSSAHA , Department of Mathematics, Faculty of Sciences.Badji Mokhtar University, P.O. Box 12, 23000 Annaba, Algeria.
boussahaaicha@yahoo.fr