Nonlinearity and interactions in bosonic systems

Jean-Claude GARREAU, Laboratoire PhLAM, Université Lille 1

Many interesting physical systems, e.g. condensed matter systems, are hard to study. From the experimental point of view, in such systems most parameters are "frozen" for a given sample. From the theoretical point of view, they often cannot be simply modelled from first principles. In contrast, laser-matter interactions are very well known theoretically, and a very high degree of control of laser beams is available for the experimentalist. In the last years, one learned to "engineer" quantum hamiltonians using ultracold atoms interacting with electromagnetic fields. For example, laser beams may form an interference pattern that is "seen" by the atoms as a periodic potential, thus creating an analog of a crystal. However, the analog has great advantages over the original system, as one has a larger control of its properties: One can vary the parameters of the potential by changing the properties of the laser beams, one can make a "mesoscopic" quantum system by using a Bose-Einstein condensate, one can change it to a degenerate Fermi gas by using a fermionic isotope, one can control atom-atom interactions by using the so-called *Feshbach resonances*. Building on seminal ideas of Feynman, the notion of "quantum simulator" – a "simpler" system able to mimic the physics of a complicate one – recently became a reality.

In a variety of situations of experimental interest, weakly interacting degenerate boson gases (Bose-Einstein condensates) are very well described by the Nonlinear Schödinger Equation with a cubic nonlinearity (called, in the present context, the *Gross-Pitaevskii equation*), which is much simpler than the underlying many-body problem. This opened a new field for both theoretical and experimental studies: The *nonlinear* quantum mechanics, where the familiar techniques like the spectral analysis in terms of eigenvalues and eigenvectors, which derive from the linearity of the usual Schödinger equation, cannot be used. The mathematical framework of this new quantum mechanics is still largely to be invented. Meanwhile, one can resort to numerical simulations – and to experiments.

In this talk, I will discuss two simple yet important examples illustrating these ideas. First, I will show how one can "quantum simulate" with ultracold atoms the celebrated Anderson model [1, 2, 3], which describes quantum disordered systems. One can even enrich this system by adding particle-particle interactions in a controlled way. Second, I will show a simple model displaying "quasiclassical" chaos, that is, chaotic behavior related to sensitivity to the initial conditions, which cannot exist in *linear* quantum mechanics. I will show that the observed behavior, although intrinsically quantum, can be described by the KAM theorem [4].

Références

[1] P. W. Anderson, *Absence of Diffusion in Certain Random Lattices*, Phys. Rev. **109**, 1492 (1958).

[2] G. Casati *et al.*, *Anderson Transition in a One-Dimensional System with Three Incommensurate Frequencies*, Phys. Rev. Lett. **62**, 345 (1989).

[3] J. Chabé *et al.*, *Experimental Observation of the Anderson Metal-Insulator Transition with Atomic Matter Waves*, Phys. Rev. Lett. **101**, 255702 (2008).

[4] Q. Thommen *et al.*, *Classical Chaos with Bose-Einstein Condensates in Tilted Optical Lattices*, Phys. Rev. Lett. **91**, 210405 (2003).

Jean-Claude GARREAU, Bat. P5 - UFR de Physique, F-59655 Villeneuve d'Ascq cedex
`jean-claude.garreau@univ-lille1.frl`