An all-regime Lagrange-Projection scheme for the gas dynamics equations

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We are interested in the simulation of subsonic compressible flows in a specific regime where the Mach number is small. More precisely, we consider the dimensionless system of gas dynamics

$$\begin{cases} \partial_t \rho + \nabla.\left(\rho \mathbf{u}\right) = 0, \\ \partial_t(\rho \mathbf{u}) + \nabla\left(\rho \mathbf{u} \otimes \mathbf{u}\right) + \frac{1}{M^2} \nabla p = 0, \\ \partial_t(\rho E) + \nabla.\left((\rho E + p)\mathbf{u}\right) = 0, \\ E = e + \frac{M^2 |\mathbf{u}|^2}{2} \end{cases} \tag{1}$$

where ρ, $\mathbf{u}$, E and M denote the density, the velocity, the total energy of the fluid and the Mach number ($M \ll 1$ at low Mach number). The pressure law $p = p(\rho, e)$ is assumed to be a given function of the density ρ and the internal energy e. To compute good numerical approximations of low Mach number flows, classic numerical schemes require a small mesh size $h = \mathcal{O}(M)$ and small time step $\Delta t = \mathcal{O}(Mh)$. Using such a fine discretization costs a lot of CPU time, especially in a multi-dimensional setting.

In this work, we propose an all-regime Lagrange-Projection scheme for the gas dynamics equations that is able to compute accurate approximations with a mesh size h and a time step Δt independent of the mach number M. We use a Lagrange-Projection algorithm to decouple the terms responsible for the acoustic waves and the material waves. This splitting of operator gives a natural mixed implicit-explicit strategy [1] to obtain a CFL stability condition driven by the (slow) material waves and not by the (fast) acoustic waves, which is especially important in the low Mach limit. Besides, an approximation based on a relaxation strategy provides a simple mean to circumvent the nonlinearities involved with the equation of state of the fluid. Then, a diffusion reduction technique inspired by the analysis of [2] is used to obtain the *all regime* property.

The good properties of the scheme include the all-regime property and a discrete entropy inequality in both classic and asymptotic regime. Numerical evidences show the robustness and good behavior of the scheme in both classic and asymptotic regime with a coarse mesh size and time step (see for instance Figure 1). A study of the benefit of the all-regime property in terms of CPU time is also given.

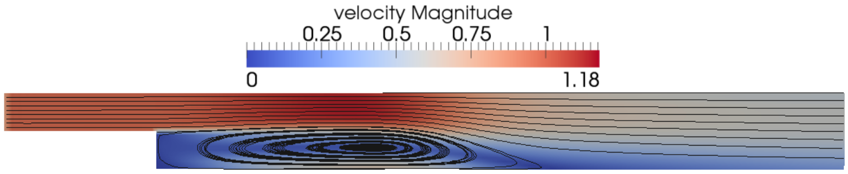

Figure 1: Backward facing step test case : profile at time t=50s of the velocity magnitude and stream lines for the all-regime scheme on a 220*20 cells Cartesian Mesh.

Références

[1] C. Chalons, M. Girardin and S. Kokh, *Large time step and Asymptotic Preserving numerical schemes for the gas dynamics equations with source terms*, SIAM J. Sci. Comput., 35(6) pp. A2874A2902, 2013.

[2] S. Dellacherie, *Analysis of Godunov type schemes applied to the compressible Euler system at low Mach number*, J. Comput. Phys., 229(4) pp. 978-1016, 2012.

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