Finite element methods for imposing jump boundary conditions with a fictitious domain approach.

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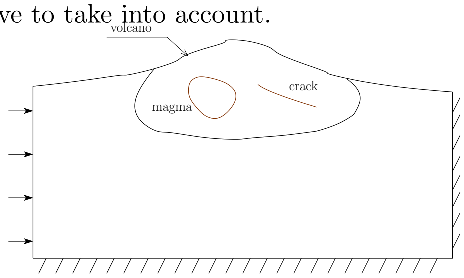
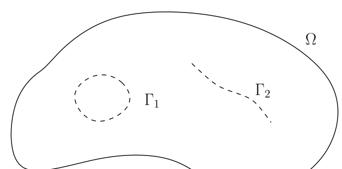
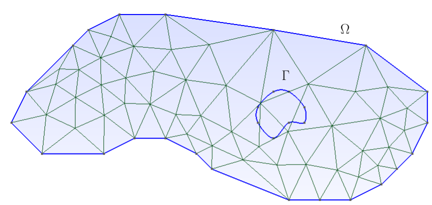
The aim of this work is to carry out a generic method for imposing jump conditions across interfaces (cracks for instance) whose the geometry would likely evolve through the time or more generally during iterations. The robustness with respect to this geometry is an important criterion, since the underlying purpose lies in solving inverse problems for which the geometry and the position of the crack is the unknown. That is why we consider a *fictitious domain* method, with which the mesh of the computational domain is independent of the crack.
In order to illustrate this approach, we consider a linearized elasticity problem governing the displacement inside a volcano submitted to internal cracks. The presence of these cracks induces jump conditions that we have to take into account.





Denoting by $\mathbf{u}$ the displacement inside the computational domain, and by $\sigma_L(\mathbf{u}) = \mu_L\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right) + \lambda_L(\mathrm{div}\ \mathbf{u})\mathrm{I}_{\mathbb{R}^d}$ the Lamé stress tensor (with $d = 2$ or 3), the linear elasticity problem we consider is the following

$$\left\{\begin{array}{rl} -\mathrm{div}\ \sigma_L(\mathbf{u}) = f & \text{in } \Omega, \\ \mathbf{u}_{|\partial\Omega} = g & \text{on } \partial\Omega, \\ [\mathbf{u}] = D & \text{on } \Gamma, \\ [\sigma_L(\mathbf{u})n] = 0 & \text{on } \Gamma. \end{array}\right.$$



The quantities f, g, D are given data, and $[\cdot]$ denotes the jump of a vector field across the crack Γ. Of course, mixed boundary conditions can be imposed on $\partial\Omega$ instead of the Dirichlet condition we consider here.
Several strategies will be presented, underlined by convergence curves and consideration of different types of geometries. They are based on non-conforming meshes, that is to say meshes which do not match to the geometry of the crack.

Références

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[2] Y. Gong, B. Li, Z. Li, *Immersed-interface finite-element methods for elliptic interface problems with nonhomogeneous jump conditions*, SIAM J. Numer. Anal., no. 46 (2008), pp. 472–495.

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