Space-time domain decomposition for porous media flow and transport

Caroline JAPHET, Université Paris 13 & INRIA

Thi-Thao-Phuong HOANG, INRIA Rocquencourt

Michel KERN, INRIA Rocquencourt

Jean E. ROBERTS, INRIA Rocquencourt

Flow and transport problems in porous media are well-known for their high computational cost. In the simulation of an underground nuclear waste disposal site, one has to work with extremely different length and time scales, and highly variable coefficients while satisfying strict accuracy requirements. One strategy for tackling these difficulties is to apply a non-overlapping domain decomposition method which allows local adaptation in both space and time and makes possible the use of parallel algorithms. Two approaches, one using a time-dependent Steklov-Poincaré operator, another using the optimized Schwarz waveform relaxation have been proposed in [3] for solving a time-dependent diffusion problem in a mixed formulation.

In this work we extend these two methods for solving a time-dependent advection-diffusion equation in a mixed formulation (see [2]). Both methods are iterative methods that compute in the subdomains over the whole time interval, exchanging space-time boundary data. The first method uses a time-dependent Steklov-Poincaré operator and a generalized Neumann-Neumann preconditioner with weight matrices to handle the heterogeneities. In this approach the classical transmission conditions are enforced on the interfaces between subdomains. The second method uses the optimized Schwarz waveform relaxation (OSWR) with Robin transmission conditions. The OSWR algorithm uses more general (Robin or Ventcell) transmission operators in which coefficients can be optimized to improve convergence rates. For both approaches, an interface problem is derived on the space-time interface. Thus, different time steps can be used in different subdomains adapted to their physical properties. The time projections between subdomains are done by an optimal projection algorithm without any additional grid (see [1]). We use operator splitting to treat differently the advection and the diffusion. We show numerical experiments for various test cases, both academic and more realistic prototypes for nuclear waste disposal simulation, to investigate and compare the behavior of the two methods.

Références

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Caroline JAPHET, Université Paris 13, LAGA, UMR 7539, F-93430, Villetaneuse, France. INRIA Paris-Rocquencourt, BP 105, 78153 Le Chesnay, France.
japhet@math.univ-paris13.fr
Thi-Thao-Phuong HOANG, INRIA Paris-Rocquencourt, BP 105, 78153 Le Chesnay, France
phuong.hoang_thi_thao@inria.fr
Michel KERN, INRIA Paris-Rocquencourt, BP 105, 78153 Le Chesnay, France
Michel.Kern@inria.fr
Jean E. ROBERTS, INRIA Paris-Rocquencourt, BP 105, 78153 Le Chesnay, France
Jean.Roberts@inria.fr