Principal angles between subspaces and reduced order modelling accuracy in scalable robust optimization

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The performance of a system designed for given functioning conditions often seriously degrades when these conditions are modified. Typical situations of interest involve a few (typically one or two) parameters describing the functioning of the system. Multi-point optimization is widely used to address robustness issues in engineering and we showed [1, 2] how to use such formulation to address the robustness issue when a few functioning parameters are involved. We discussed optimal sampling of the functioning parameters intervals and quantitative confidence levels on the quality of our search direction. This naturally brought into light the question of the sampling size which needs to be monitored to maintain the calculation complexity low. The aim was to avoid the worst-case theoretical limitation indicating that the sampling size should be larger by one the size of the control space [1, 3, 4]. We showed that this is only necessary if all the associated gradients (i.e. evaluated at the sampling points) are linearly independent which is never the case in optimizations involving a state equation. Hence, we showed that large dimensional parametric optimization problems can be treated with very small sampling of the functioning parameters range with marginal losses on the gradient informations.

However, one often uses approximation in the definition of the gradients. And this is the starting point of this paper. One would like to analyze the impact of these using reduced order models on the search space defined by the gradients of the functional at the different sampling points of the functioning parameters. The mathematical tool we use for this analysis is the principal angles between subspaces [5]. We will discuss practical ways for their efficient evaluation [6, 7, 8]. Reduced order modelling also includes situations where lower accuracy and modelling complexity are accounted for in the linearization step than for the direct simulation chain used for the definition of the functional. This is often the situation where an approximate simulation chain is considered for linearization, droping or approximating some of the ingredients in the direct chain.

The approach appears to be an interesting tool for both aleatoric and epistemic uncertainties quantification in the presence of models of increasing complexity. We are also especially interested by the choices which bring into light efficient parallelism. The application of these concepts is illustrated in the design of the shape of an aircraft robust over a range of transverse winds.

Références

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