Communication dans le cadre du mini-symposium “Ecoulements gravitaires”

Finite volume method for the shallow water system over the sphere on arbitrary grid

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The evolution of atmospheric or oceanic flows around the Earth can be modeled by the shallow water system over the sphere

$$\begin{cases} \partial_t h + \mathrm{div}_\xi(hW) = 0, \\ \partial_t W + W \cdot \nabla_\xi W + |W|^2 \frac{\xi}{R^2} + g\nabla_\xi(h+B) = -2\left(\frac{\xi}{R} \cdot \vec{\Omega}\right)\frac{\xi}{R} \wedge W, \end{cases} \tag{1}$$

where ξ is the position lying on the sphere of radius R, $|\xi| = R$, $h(t,\xi)$ is the width of the fluid layer, $W(t,\xi)$ is the velocity tangent to the sphere, $W(t,\xi)\cdot\xi = 0$, $B(\xi)$ is the topography, and $\vec{\Omega}$ is the angular rotation vector.

The classical plane shallow water system has been very well studied concerning its mathematical properties. Its numerical resolution via well-balanced finite volume methods is also now well understood. In comparison to the classical plane shallow water system, (1) presents some special features at the theoretical level and at the numerical level. At the theoretical level several difficulties come from the fact that the constants are not solutions to the system, contrarily to the plane case. In particular, the most simple discontinuous solutions available are piecewise smooth, but not piecewise constant. Indeed, the vector nature of $W(t,\xi)$ that needs to be tangent to the sphere at the point ξ makes it nonsense to consider constant data.

At the numerical level, additionally to the lack of notion of piecewise constant data, the integration of the equation over a control volume induces new geometric terms that are consequences of the curvature. The derivation of finite volume methods in this context is not straightforward. Several authors [2, 1, 3] have proposed methods that are attached to special grids and/or particular coordonate systems. Such approaches suffer from either singularities (poles), or matching grids that imply complexity, eventually loss of accuracy or stability, loss in cpu.

In this work, we propose a finite volume method that works for arbitrary grids, and that does not use any coordonate system. It is built via an interface solver between two cells, that resolves at the same time the physical terms (plane terms and source terms due to topography and Coriolis force) and the geometric source terms. It is well-balanced, positive, entropy satisfying. It is extended easily to second-order accuracy via slope reconstruction.

This work is partially financed by tha ANR project SVEMO.

Références

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